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**Intuitionistic Fuzzy Multiset (IFMS) In Binomial Distributions**

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**Abstract:** We proposed the application of IFMS in binomial distributions. This is possible as we assumed theoretically that the probability of the membership degrees is constant for each of the trials and that the intuitionistic fuzzy multiset index is negligible i.e. \( \pi = 0 \). We made use of the Bernoulli trial formula to find the binomial probability of intuitionistic fuzzy multisets.

**Keywords:** binomial probability, fuzzy sets, intuitionistic fuzzy sets, fuzzy multisets, intuitionistic Fuzzy multisets.

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1. **Introduction**

Modern Set theory formulated (or invented) by a German mathematician George Cantor (1845-1918) is fundamental and indispensable for the whole of Mathematics. In fact, Set Theory (ST) is the language of Science, Mathematics, Logic and Philosophy. However, one cardinal issue associated with the notion of a set is the concept of vagueness. Obviously, every one believes that, in accordance to the tenet of Mathematics, all mathematical notions including set must be exact. This vagueness or the representation of imperfect knowledge has been a challenge for a long time for philosophers, logicians, scientists and mathematicians. Notwithstanding, of recent it became a crucial issue for computer scientists, particularly in the area of artificial intelligence. To handle situation like this, many tools were suggested viz: fuzzy sets, multisets, rough sets, soft sets, fuzzy multisets and intuitionistic fuzzy sets [8]. Considering the unpredictable factors in decision-making, Zadeh [1] introduced the idea of fuzzy set which has a membership function that assigns to each element of the universe of discourse, a number from the unit interval [0,1] to indicate the degree of belongingness to the set under consideration. Atanassov [2] subsequently proposed the concept of intuitionistic fuzzy set (IFS) by bringing a non-membership function together with the membership function of the fuzzy set introduced earlier by Zadeh [1]. Among the various notions of higher-order fuzzy sets, IFS proposed by Atanassov[4] provides a flexible framework to elaborate uncertainty and vagueness. This idea of IFS seems to be resourceful in modeling many real life situations like negotiation processes, psychological investigations, reasoning, medical diagnosis among others [8]. Nevertheless, it is worthy to note that many field of modern Mathematics have been emerged by violating a basic principle of a given theory only because useful structures could be defined this way. For instance, set which is a well-defined collection of distinct and definite objects, that is, the elements of a set are pair-wise different.

2. **Preliminaries**

**Definition 1:** Let \( X \) be a nonempty set. A fuzzy set \( A \) drawn from \( X \) is defined as \( A = \{(x, \mu_A(x)): x \in X\} \), where \( \mu_A(x) : X \rightarrow [0, 1] \) is the membership function of the fuzzy set \( A \).

**Definition 2:** Let \( X \) be a nonempty set. An intuitionistic fuzzy set \( A \) in \( X \) is an object having the form \( A = \{(x, \mu_A(x), \nu_A(x)): x \in X\} \), where the functions \( \mu_A(x), \nu_A(x) : X \rightarrow [0, 1] \) define respectively, the degree of membership and degree of non-membership of the element \( x \in X \) to the set \( A \), which is a subset of \( X \), and for every element \( x \in X, 0 \leq \mu_A(x) + \nu_A(x) \leq 1 \). Furthermore, \( \pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \) is called the intuitionistic fuzzy set index or hesitation margin of \( x \) in \( A \). \( \pi_A(x) \) is the degree of non-determinacy of \( x \in X \) to the IFS \( A \) and \( \pi_A(x) \in [0,1] \) i.e., \( \pi_A(x) : X \rightarrow [0,1] \) for every \( x \in X \). \( \pi_A(x) \), expresses the lack of knowledge of whether \( x \) belongs to IFS \( A \) or not.

**Definition 3:** Let \( X \) be a nonempty set. A fuzzy multiset (FMS) \( A \) drawn from \( X \) is characterized by a function, ‘count membership’ of \( A \) denoted by \( CM_d : X \rightarrow \mathbb{Q} \) where \( \mathbb{Q} \) is the set of all crisp multisets drawn from the unit interval \([0,1] \). Then for any \( x \in X \), the value \( CM_d(x) \) is a crisp multiset drawn from \([0,1] \). For each \( x \in X \), the membership degree is defined as the decreasingly ordered sequence of elements in \( CM_d(x) \). It is denoted by \((\mu_A^1(x), \mu_A^2(x), ..., \mu_A^n(x))\) where \( \mu_A^1(x) \geq \mu_A^2(x) \geq ... \geq \mu_A^n(x) \).
3. Concept of Intuitionistic Fuzzy Multisets (IFMSs)

Definition 4: Let X be a nonempty set. An IFMS A drawn from X is characterized by two functions: “count membership” of A denoted as CM(A) and “count non-membership” of A denoted as CN(A) given respectively by CM: X → Q and CN: X → Q where Q is the set of all crisp multisets drawn from the unit interval [0, 1] s.t. for each x ∈ X, the membership degree of element in CM(A) is defined as a decreasingly ordered sequence and it is denoted as (µ_A(x), ν_A(x), ..., µ^n_A(x)). Where µ_A(x) ≥ µ_2(x) ≥ ... ≥ µ^n_A(x) whereas the corresponding non-membership degrees of element in CN(A) is denoted by (ν_A(x), ν_2(x), ..., ν^n_A(x)) s.t. 0 ≤ ν_A(x) + ν_2(x) ≤ 1 for every x ∈ X and i = 1, 2, ..., n. This means, an IFMS A is defined as: A = {(x, CM_A(x), CN_A(x)) : x ∈ X} or A = {(x, µ_A(x), ν_A(x)) : x ∈ X}, for i = 1, 2, ..., n. For each IFMS A in X, π_A(x) = 1 - µ_A(x) is the intuitionistic fuzzy multisets index or hesitation margin of x in A. The hesitation margin π_A(x) for each i = 1, 2, ..., n is the degree of non-determinacy of x in X, to the set A and π_A(x) ∈ [0, 1]. The function π_A(x) expresses lack of knowledge whether x in X or not. In general, an IFMS A is given as A = {(x, µ^n_A(x), ν^n_A(x)), π^n_A(x)) : x ∈ X}.

Note: µ^n_A(x) + ν^n_A(x) + π^n_A(x) = 1.

4. Operations in Intuitionistic Fuzzy Multisets

For any two IFMSs A and B drawn from X, the following operations hold:

[Complement] A' = {(x, ν_A(x), µ_A(x)) : x ∈ X}
[Union] A ∪ B = {(x, max(µ_A(x), µ_B(x))), min(ν_A(x), ν_B(x)) : x ∈ X}
[Intersection] A ∩ B = {(x, min(µ_A(x), µ_B(x))), max(ν_A(x), ν_B(x)) : x ∈ X}
[Addition] A + B = {(x, µ_A(x) + µ_B(x)) - (ν_A(x) + ν_B(x)) : x ∈ X}
[Addition] A Λ B = {(x, µ_A(x) Λ µ_B(x)), (ν_A(x) + ν_B(x)) : x ∈ X}

5. Cardinality of Intuitionistic Fuzzy Multisets

Definition 5: The length of an element x in an IFMS A is defined as the cardinality of CM(A) or CN(A) for which 0 ≤ µ(x) ≤ 1 and it is denoted by L(x: A) i.e. the length of x in A for each x ∈ X. Then, L(x: A) = |CM(A)(x)|.

Definition 6: If A and B are IFMSs drawn from X, then

L(x: A, B) = max{L(x: A), L(x: B)} or L(x) = V[L(x: A), L(x: B)] where L(x) = L(x: A, B) and V denotes maximum.

Note: 1. In an IFMS, |µ_i(x) | = |ν_i(x) | for each i = 1, 2, ..., n.
2. Whenever i = 1, an IFMS becomes IFS.

3. IFMS and FMS of the same length have equal cardinality.

For example, for the set X = {x} with A = ((0.3, 0.0, (0.4, 0.6)) and B = ((0.5, 0.4, 0.3, 0.2), (0.4, 0.6, 0.6, 0.7)). Then L(x: A) = 2, L(x: B) = 4 and L(x) = 4.

Definition 7 (Simple IFMS): Let X be nonempty. An IFMS A drawn from X is said to be simple if all its membership degrees are the same or have the same degree i.e. µ^n_A(x) = µ^n_B(x) = ... = µ^n_A(x). Since µ_A(x) and ν_A(x) are complementary as π_A(x) = 0, it then implies that ν_A(x) = ν^n_A(x) = ... = ν^n_A(x).

For example, A = ((0.4, 0.4, 0.4), (0.6, 0.6, 0.6)).

Definition 8 (Multiple of IFMS): Let X be nonempty. The multiplicity of an IFMS A is the number of time the membership degree (with the non-membership degree) appears. For example, A = ((0.3, 0.4, 0.4, 0.7, 0.6, 1.0)) has the multiplicity of 3 i.e. i = 1, 2, 3 where i is the step.

6. Application of Intuitionistic Fuzzy Multisets in Binomial Distribution

The binomial distribution arises from a repeated random experiment which has two possible outcomes:
1. that an event will occur i.e. degree of membership µ.
2. that an event will not occur i.e. degree of non-membership ν.

For example, in the experiment of tossing a fair coin repeatedly, there are two possible outcomes namely; a head or not a head (i.e. membership degree or non-membership degree). For each repetition of tossing of the fair coin is called a trial. The two possible outcomes of the random experiment are usually called success and failure. If the probability that a membership degree µ shows up is a success, the probability that a non-membership degree ν shows up is a failure, and conversely. Since membership degree and non-membership degree are complementary, it implies that µ + ν = 1 or ν = 1 - µ i.e. π = 0.

Note: 1. In the binomial distribution, the hesitation margin is negligible i.e. π = 0 as in normalization of IFS [9].
2. We assume theoretically that the probability of membership degree is the same for each of the trials i.e. simple IFMS.

The independent trials discussed above are often called Bernoulli trials. If we denote the probability that in n-Bernoulli trails, we have i successes and n - i failures by f(i), then f(i) is defined as thus: f(i) = C(n, i)µ^iν^(n-i), where C(n, i) = n! / (n-i)! and n is the number of trials.

Note: f(i) is called binomial probability (or frequency or density function) or binomial distribution [3].

Example: Given that A = ((0.3, 0.3, 0.3, 0.3), (0.7, 0.7, 0.7, 0.7)), the binomial probability of A in at least step 2 is thus A has the...
multiplicity of 4 i.e. $n = 4$ and 4 steps since $i = 1, 2, 3, 4$. In at least step 2 means in step 2, 3 and 4.

\[ f(i) = \binom{n}{i} \mu^i \nu^{n-i}, \mu = 0.3 \text{ and } \nu = 0.7. \]

\[
f(\text{at least in step 2}) = f(2) + f(3) + f(4).
\]

\[
f(2) = \binom{4}{2}0.3^20.7^2 = 0.2646
\]

\[
f(3) = \binom{4}{3}0.3^30.7^1 = 0.0756
\]

\[
f(4) = \binom{4}{4}0.3^40.7^0 = 0.0081
\]

Therefore, the required binomial probability of $A$ in at least step 2 is 0.3483.

7. Conclusion

The notion of intuitionistic fuzzy multiset is useful in binomial distributions whenever the intuitionistic fuzzy multiset index is negligible.

References


[7]. W.D. Blizzard, Dedekind multisets and function shell, Theoretical Computer science 110 (1993) 79-98.
