Longitudinal And Lateral Dynamic System Modeling Of A Fixed-Wing UAV

Pann Nu Wai Lin, Nang Lao Kham, Hla Myo Tun

Abstract: In this paper presents work completed for flight characteristics, mathematical model of an aircraft are the focus. To construct the mathematical model, type of UAV and flying mode quality must be chosen firstly.Longitudinal command hold outputs and lateral outputs (slide slip velocity, yaw rate, heading angle, and roll angle) must be considered to control the desired flight conditions.

Index Terms: Longitudinal, Lateral, Transfer function, State Space System, Stability

1 INTRODUCTION

UMANNED Aerial Vehicle (UAV) has been done for research in almost Technological University around the world. An Unmanned Aerial Vehicle (UAV) is an autonomous flying vehicle. The UAV system is composed of several subsystems. During recent years with the development of new technologies like Global Positioning Systems (GPS) and smaller, faster processors there has been a huge increase in the development of Unmanned Aerial Vehicle. The advantages of unmanned vehicles include better maneuverability, decreased size and obviously the lack of risk to a pilot. Unmanned Aerial Vehicles are currently being produced by many different parties including governments and private companies. The sizes and purposes of these UAVs vary greatly.Unmanned Air vehicles would be less expensive to develop and manufacture than manned aircraft, and that UAVs will reduce the demand for the supporting facilities and manpower that modern aircraft require. As a result of technological advances in flight control, data and signal processing, off-board sensor, communications links, and integrated avionics, unmanned aerial vehicles are now a serious option. There are far more than technological advances that are accelerating the development of UAVs. UAVs are better suited for dull, dirty, or dangerous missions than manned aircraft. The Smart One C is used for surveillance and personal aerial mapping system. It could be controlled in three modes: manual, assisted control and automatic mode. Mathematical model is the most fundamental of an aircraft. Mathematical model is based on the laws of physics and can be derived from either Euler-Lagrange method or Newton approach. Mathematical model can be evaluated by both analytical and empirical method. But analytical method is more accurate than by using the computer software (i.e. DIGITAL DATCOM and Advance Aircraft Analysis). The aerodynamic coefficients and derivatives can be achieved from DATCOM software.

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2 IDENTIFICATION OF AIRCRAFT

The motion of aircraft is necessary to define a suitable coordinate system for formulations of the equation of motion. It has two axis systems: earth axis system and body axis system. Earth axis system means that it is fixed to the earth surface. Body axis system is attached to the aircraft body. Moreover, the stability axis system is used as a reference for aerodynamic moment and forces. Lift forces and drag forces are transformed to normal forces and axial force respectively. In dynamic modeling of aircraft, the forces acting on the aircraft are mainly:

- The weight located at the center of gravity
- The thrust of the propeller acting in the x-direction.
- The aerodynamic forces of each part of the airplane (mainly the wing).

The aircraft equations of motions in order to formulate that must be considered as the follows;

- There is the flat earth.
- There is non-rotation mass.
- The aircraft is rigid body.
- The aircraft is symmetric.
- There is a constant wind.
- There is no rotating earth.

The symmetric flight produces the zero bank angle and all moments are zero in steady straight flight conditions. In dynamic modeling of aircraft, the forces acting on the aircraft are mainly:

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$$\sum_{abs} F = ma$$

$$\sum_{abs} M = I\omega$$

The velocity vector is related with six parameters: forward velocity, sideway velocity, vertical velocity, roll rate, pitch rate and yaw rate. The velocity vector, **v** in the body frame is defined as, where the angular velocity is expressed with respect to the north-east-down frame:

Velocity vector, **v=**

$$\begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \text{forward velocity} \\ \text{sideway velocity} \\ \text{vertical velocity} \\ \text{roll rate} \\ \text{pitch rate} \\ \text{yaw rate} \end{bmatrix}$$

6 DOF rigid-body forces and moments in the body frame are defined as follows:

$$\begin{bmatrix} X \\ Y \\ Z \\ L \\ M \\ N \end{bmatrix} = \begin{bmatrix} \text{forward force} \\ \text{sideway force} \\ \text{vertical force} \\ \text{rollmoment} \\ \text{pitchmoment} \\ \text{yawmoment} \end{bmatrix}$$

Dynamic pressure, $Q = \frac{1}{2}\rho V_T^2 = \frac{1}{2}\rho u_o^2$

On component form:

- $m(\dot{u}+qw-rv+g\sin\theta)=X$
- $m(\dot{v}+ur-pw+g\cos\theta\sin(\varphi))=Y$
- $m(\dot{w}+pv-qu+g\cos\theta\sin(\varphi))=Z$
- $I_x\dot{p} + I_{xz}(\dot{r} + pq) + (I_z - I_y)qr = L$
- $I_y\dot{q} + I_{xz}(p^2 - r^2) + (I_x - I_z)pr = M$
- $I_z\dot{r} - I_{xz}(\dot{p} + qr) + (I_y - I_x)pq = N$

Airflow angles: The two angles β slideslip angle and α angle of attack that are related to the flight direction deals with the air are expressed as follows:

Inverse $\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} v_T\cos\alpha\cos\beta \\ v_T\sin\beta \\ v_T\sin\alpha\cos\beta \end{bmatrix}$ relationship:

$$\begin{bmatrix} v_T \\ \beta \\ \alpha \end{bmatrix} = \begin{bmatrix} \sqrt{u^2 + v^2 + w^2} \\ \sin^{-1}(v/v_T) \\ \tan^{-1}(w/u) \end{bmatrix}$$

$$\alpha = \tan^{-1}(\frac{w}{u})$$

$$\beta = \tan^{-1}(\frac{v}{\sqrt{u^2 + w^2}})$$

3 Physical Measurement and Mass Properties

Smart Fly is the airframe with small-size flying wing UAV that is used for aerial mapping system. The airframe is assumed to have aerodynamically efficient as the conventional stabilizers areremoved in Flying wings UAVs. The major dimensions of Smart One are presented in Table 4.1. Aspect ratio is calculated by using $AR= b^2/S$ and it has the cruise speed with 11.5 m/s. The aircraft is 5m/s of climb rate with the pitch angle of 7 degrees (or) 0.122 radians. The dynamic pressure is also calculated by using $1/2\rho u^2$.

4 Stability and Control Derivatives

The formulae for coefficient of X-axis and Z-axis are as follows:

- $C_{x0} = -C_{D0}\cos\alpha+C_{L0}\sin\alpha=-0.0237$
- $C_{x\alpha}=- C_{D\alpha}\cos\alpha+C_{L\alpha}\sin\alpha=0.2972$
- $C_{x\delta e} = -C_{D\delta e}\cos\alpha+C_{L\delta e}\sin\alpha =-0.0143$
- $C_{xq} = - C_{Dq}\cos\alpha+C_{Lq}\sin\alpha =0$
- $C_{z0} = -C_{D0}\sin\alpha+C_{L0}\cos\alpha =0.0226$
- $C_{z\alpha}= -C_{D\alpha}\sin\alpha+C_{L\alpha}\cos\alpha =0.4956$
- $C_{z\delta e} =-C_{D\delta}e\sin\alpha+C_{L\delta e}\cos\alpha=0.573$
- $C_{zq} = - C_{Dq}\sin\alpha+C_{lq}\cos\alpha=0$

TABLE 1

UAV SPECIFICATIONS

Symbol	Values	Properties
b	1.99m	Wing span
S	0.076m²	Wing area
m	0.0973kg	Empty weight
u	9.67m/s	Forward velocity along the X-axis
w	1.16m/s	Vertical velocity along the Z-axis
ρ	1.225kg/m³	Air pressure density
α	0.119rads	Angle of attack
θ	0.119rads	Pitch angle
δ_e	0.0279rads	Elevator deflections angle
δ_T	0.57rads	Throttle angle
c	0.0381m	Chord
I_x	0.1124	Rolling moment of inertia
I_y	0.0709	Pitching moment of inertia
I_z	0.0562	Yawing moment of inertia
Q	296.45kg/m³	Dynamic pressure

Longitudinal stability derivatives formulae are presented as follow:

- Stability Derivative, X_u = -6.68
- Angle of Attack Derivative, X_w = 4.1754
- Elevator Deflection, $X_{\delta e}$ = -0.649
- Thrust Deflection ,$X_{\delta T}$= 0
- Compressibility Effect Derivative ,
- M_u = -0.01376
- Dimensional Pitching Moment ,
- Derivative, M_w = 0.05852

- Pitching moment (Elevator Deflection) ,$M_{\delta e}$ = -1.1526
- Dimensionless Pitching Moment Derivative, M_q= -0.1179
- Pitching moment (Thrust Deflection) ,$M_{\delta T}$= 0
- Pitch Rate Derivative X_q = -1.16
- Stability Derivative, Z_u = -0.6276
- Angle of Attack Derivative, Z_w =-3.0503
- Elevator Deflection ,$Z_{\delta e}$ = 26.0063
- Thrust Deflection, $Z_{\delta T}$= 0
- Pitch Rate Derivative, Z_q= 9.67

Lateral stability derivatives formulae are presented as follows:

- Roll Rate ,Y_p = -0.05579
- Aileron Deflection Derivative ,$Y_{\delta a}$ = 0
- Yaw Rate Derivative, Y_r = 0
- Sideslip Derivative Y_β = -4.5129
- Rolling Moment , L_p = -0.3295
- Rolling Moment L_r ,= 0.0205
- Rolling Moment,$L_{\delta a}$ = 3.6299
- Roll Acceleration ,L_β = 3.7096
- Yawing Moment, $N_{\delta a}$ = 3.0316
- Yawing Moment , N_p = 0.02025
- Yawing Moment ,N_r = -0.10266
- Yaw Acceleration ,N_β = 0.79937

5 State Space System

Non –Linear equation of motion which had been derived based on Newton's second law in Chapter 3, are difficult to be used for control system design purpose. The linearized dynamic equations had been calculated by using small-disturbance theory in Chapter 3. Linear differential equation with constant coefficients, it is possible to write as a set of first-order differential equations in the form of a State – Space Model. The equation of motion or state equation of the linear time invariant multivariable system in the matrix form is written:

- $\dot{x}(t)=Ax(t)+Bu(t)$
- where;
- x(t) = the column vector of n state variables called the state vector

u(t) = the column vector of m input variables called the input vector
A = the (n/n) state matrix
B = the (n/m) input matrix

The corresponding output equation is written as follows.

- y(t) = Cx(t) +Du(t)
- where, y(t) = the column vector r of output variable called the output vector
- C = the (r/n) output matrix
- D = the (r/m) feed forward matrix

The matrices A, B, C and D have the constant elements for an LTI system

5.1 Longitudinal–Directional State Space System

Since four states variables u, w, q and θ in longitudinal motion of the aircraft, four transfer functions are required. Therefore, the longitudinal equation of motion can be written in the form of state space mode:

5.2 Lateral–Directional State Space System

There are four lateral directional motion of aircraft: side force, rolling moment, and yawing moment. The lateral motion can be written by using state space form in Equation

$$\begin{bmatrix}\Delta \dot{u}\\ \Delta \dot{w}\\ \Delta \dot{q}\\ \Delta \dot{\theta}\end{bmatrix}=\begin{bmatrix}X_u & Xw & Xq+w_0 & -g\cos\theta_0\\ Z_u & Zw & Zq+w_0 & -g\sin\theta_0\\ M_u & Mw & Mq & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}\Delta u\\ \Delta w\\ \Delta q\\ \Delta \theta\end{bmatrix}+\begin{bmatrix}X\delta e & X\delta T\\ Z\delta e & Z\delta T\\ M\delta e & M\delta T\\ 0 & 0\end{bmatrix}\begin{bmatrix}\Delta\delta_e\\ \Delta\delta T\end{bmatrix}$$

$$\begin{bmatrix}\Delta \dot{u}\\ \Delta \dot{w}\\ \Delta \dot{q}\\ \Delta \dot{\theta}\end{bmatrix}=\begin{bmatrix}-6.68 & 4.1754 & 0 & -32.174\\ -0.6276 & -3.0503 & 19.34 & 0\\ -0.01376 & 0.05852 & -0.1179 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}\Delta u\\ \Delta w\\ \Delta q\\ \Delta \theta\end{bmatrix}+\begin{bmatrix}-0.649\\ 26.0063\\ -1.1526\\ 0\end{bmatrix}\begin{bmatrix}\Delta\delta_e\\ \Delta\delta T\end{bmatrix}$$

$$\begin{bmatrix}\Delta\beta\\ \Delta p\\ \Delta\phi\\ \Delta r\end{bmatrix}=\begin{bmatrix}Y\beta & Yp & -(u0-Yr) & g\cos\theta_0\\ L\beta & Lp & Lr & 0\\ N\beta & 1 & Nr & 0\\ 0 & Np & \tan\theta_0 & 0\end{bmatrix}\begin{bmatrix}\Delta\beta\\ \Delta p\\ \Delta\phi\\ \Delta r\end{bmatrix}+\begin{bmatrix}0\\ L\delta a\\ N\delta a\\ 0\end{bmatrix}[\Delta\delta a]$$

$$\begin{bmatrix}\Delta\beta\\ \Delta p\\ \Delta\phi\\ \Delta r\end{bmatrix}=\begin{bmatrix}-0.2051 & -0.05579 & -21.9543 & 32.174\\ -0.1686 & -0.3295 & 0.0205 & 0\\ 0.03633 & 0.02025 & -0.10266 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}\Delta\beta\\ \Delta p\\ \Delta\phi\\ \Delta r\end{bmatrix}+\begin{bmatrix}0\\ 3.6299\\ 3.0316\\ 0\end{bmatrix}[\Delta\delta a]$$

6 Transfer Function for Longitudinal and Lateral Dynamic System

Transfer functions are more convenient than state space system because its denominator gives the system pole locations in order to decide the system either stable or unstable condition. Pole location in s-plane is significant in control system because it can generate the corresponding response of a plant with step input. The response indicates the system is either dynamically stable or unstable curves. Therefore, the state space system can be transformed into four transfer functions

6.1 Longitudinal Transfer Function

$$G_p(s) = \frac{-1.153\, s2\ -9.684s\ -17.78}{s4+9.848\, s3+23.01\, s2-4.181\, s-2.532}$$

The pole locations of longitudinal motions are:-5.6842, -4.292, 0.3930,-0.2641

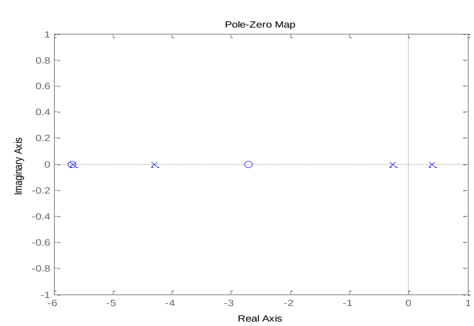

Fig. 1.*Pole-Zero Map for longitudinal transfer function*

6.2 Lateral Transfer Function

$$G_p(s) = \frac{3.63\,s^2 + 1.179s + 14.21}{s^4 + 0.6373\,s^3 + 0.9102\,s^2 + 5.618s + 0.5329}$$

The pole locations of lateral motions are:-0.6244 + 1.6441i, 0.6244 - 1.6441i,-1.7897,-0.0963.

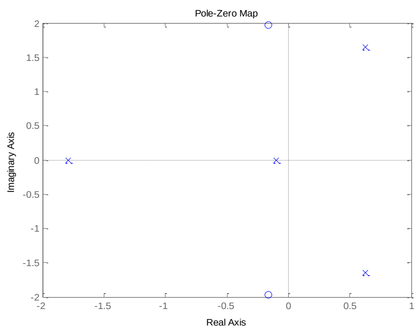

Fig. 2 *Pole-Zero Map for lateral transfer function*

7 Conclusion

The system is approached to unstable condition the fact that the right hand pole in the s-plane gives the dynamically unstable with sinusoidal oscillations in the exponentially increasing components. The system must be required controller improvement.

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