Some Convergence Theorems On Linear Models Generating A Pair Of Related Time Series

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Abstract: The main aim of this paper is to establish some convergence theorems on Linear Models Generating a Pair of Related Time Series of certain covariance type functions relating to the model specified. The estimates of residual are obtained on using the estimators defined under different placements of the roots ρ_1 and ρ_2 of P (z). This work is motivated by similar studies on linear stochastic difference equations for scalar time series. The pivotal lemmas concerned with the statements and proofs of some lemmas.

Keywords: Time series definition, Stationary process, non-stationary process, assumptions of stationary, stochastic models for time series, some of the pivotal lemmas.

Time series:

A time series is a sequence of numerical data points in successive order. In investing, a time series tracks the movement of the chosen data points, such as a security's price, over a specified period of time with data points recorded at regular intervals. There is no minimum or maximum amount of time that must be included, allowing the data to be gathered in a way that provides the information being sought by the investor or analyst examining the activity

Applications: The usage of time series models is twofold:

- Obtain an understanding of the underlying forces and structure that produced the observed data
- Fit a model and proceed to forecasting, monitoring or even feedback and feed forward control.

Time Series Analysis:

Time series analysis can be useful to see how a given asset, security or economic variable changes over time. It can also be used to examine how the changes associated with the chosen data point compare to shifts in other variables over the same time period. Time series data often arise when monitoring industrial processes or tracking corporate business metrics. The essential difference between modelling data via time series methods and using the process monitoring methods discussed earlier in this chapter is the following: "Time series analysis accounts for the fact that data points taken over time may have an internal structure (such as autocorrelation, trend or seasonal variation) that should be accounted for". This section will give a brief overview of some of the more widely used techniques in the rich and rapidly growing field of time series modelling and analysis. For example, suppose you wanted to analyze a time series of daily closing stock prices for a given stock over a period of one year. You would obtain a list of all the closing prices for the stock from each day for the past year and list them in chronological order. This would be a one-year daily closing price time series for the stock. Delving a bit deeper, you might be interested to know whether the stock's time series shows any seasonality to determine if it goes through peaks and valleys at regular times each year. Analysis in this area would require taking the observed prices and correlating them to a chosen season. This can include traditional calendar seasons, such as summer and winter, or retail seasons, such as holiday seasons. Alternatively, you can record a stock's share price changes as it relates to an economic variable, such as the unemployment rate. By correlating the data points with information relating to the selected economic variable, you can observe patterns in situations exhibiting dependency between the data points and the chosen variable. Time Series Analysis is used for many applications such as:

- Economic Forecasting
- Sales Forecasting
- Budgetary Analysis
- Stock Market Analysis
- Yield Projections
- Process and Quality Control
- Inventory Studies
- Workload Projections
- Utility Studies
- Census Analysis and many, many more...

Components of Time Series:

The factors that are responsible to bring about changes in a time series, also called the components of time series, are as follows:

1. Secular Trend (or General Trend)
2. Seasonal Movements
3. Cyclical Movements
4. Irregular Fluctuations

1. Secular Trend:

The secular trend is the main component of a time series which results from long term effect of socio-economic and political factors. This trend may show the growth or decline in a time series over a long period. This is the type of tendency which continues to persist for a very long period. Prices, export and imports data, for example, reflect obviously increasing tendencies over time.

2. Seasonal Trend:

These are short term movements occurring in a data due to seasonal factors. The short term is generally considered as a period in which changes occur in a time series with variations in weather or festivities. For example, it is commonly observed that the consumption of ice-cream during summer us generally high and hence sales of an ice-cream dealer would be higher in some months of the year while relatively lower during winter months. Employment, output, export etc. are subjected to change due to variation in weather. Similarly sales of garments, umbrella, greeting cards and fire-work are subjected to large variation during festivals like Valentine's Day, Eid, Christmas, New Year etc. These types of variation

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in a time series are isolated only when the series is provided biannually, quarterly or monthly.

3. Cyclic Movements:
These are long term oscillation occurring in a time series. These oscillations are mostly observed in economics data and the periods of such oscillations are generally extended from five to twelve years or more. These oscillations are associated to the well known business cycles. These cyclic movements can be studied provided a long series of measurements, free from irregular fluctuations is available.

4. Irregular Fluctuations:
These are sudden changes occurring in a time series which are unlikely to be repeated, it is that component of a time series which cannot be explained by trend, seasonal or cyclic movements .It is because of this fact these variations sometimes called residual or random component. These variations though accidental in nature, can cause a continual change in the trend, seasonal and cyclical oscillations during the forthcoming period. Floods, fires, earthquakes, revolutions, epidemics and strikes etc,. are the root cause of such irregularities.

Stationary Process:
In statistics, a stationary process is a stochastic process whose joint probability distribution does not change when shifted time. Consequently, parameters such mean and variance, if they are present, also does not change over time.

Definition:
Formally, let $\{\mathbf{X_t}\}$ be a stochastic process and let $F_X\{x_{t_1+\tau}, ----, x_{t_k+\tau}\}$ represent the cumulative distribution function of the joint distribution of $\{\mathbf{X_t}\}$ at times $t_1+\tau, ----, t_k+\tau$. Then, $\{\mathbf{X_t}\}$ is said to be strictly (or strongly) stationary if, for all k, for all τ , and for all $\mathbf{t_1}$, -----, $\mathbf{t_k}$, $F_X(x_{t_1+\tau}, ----, x_{t_k+\tau}) = F_X(x_{t_1}, ----, x_{t_k})$. Since τ does not affect $F_X(.)$, F_X is not a function of time.

Non-Stationary Process:
A non-stationary time series will have a time-varying mean or a time-varying variance or both.

A white noise process: $\in_t$ ~ N $(0,\boldsymbol{\sigma^2})$ such that it is independently and identically distributed (i.i.d) with $\text{cov}(\in_t, \in_{t-s}) = 0$.

Example:
A Random Walk Model (RWM) is a non-stationary process. There are two types:
(i) Without a drift and (ii) with a drift.

(i) Without a drift:

$$Y_t = Y_{t-1} + \in_t$$

Where value Y at time t is equal to its previous value and random stock. Efficient market hypothesis states that stock prices in efficient markets follow a random walk process without a drift such that there is no scope for profitable speculation in the stock market, the change in the in the stock price from one period to the next essentially random and unpredictable. We can write: $Y_1 = Y_0 + \in_1, Y_2 = Y_1 + \in_2, Y_3 = Y_2 + \in_3$ and replacing $Y_3 = Y_0 + \in_1 + \in_2 + \in_3$ such that $Y_t = Y_0 + \sum \in_t$. Therefore, $E[Y_t] = Y_0$ since errors have zero expectation, i.e., $E[\in_t] = 0$. Similarly, $\text{var}(Y_t) = t\sigma^2$, i.e., it is dependent on time, not time invariant. Hence, RWM without drift is a non-stationary process. Although, it's mean is constant over time, its variance increase over time. In this model, shocks persist as the current value is equal to the initial value plus a series of random shocks over time. A random walk has a infinite memory.

(ii) With a drift:

$$Y_t = \alpha + Y_{t-1} + \in_t$$

We write $Y_1 = \alpha + Y_0 + \in_1, Y_2 = \alpha + Y_1 + \in_2, Y_3 = \alpha + Y_2 + \in_3$ and replacing $Y_3 = \alpha + \alpha + \alpha + Y_0 + \in_1 + \in_2 + \in_3$ such that $Y_t = \sum \alpha + Y_0 + \sum \in_t$. Therefore, $E[Y_t] = \alpha t + Y_0$ since errors have zero expectation, i.e., $E[\in_t] = 0$. Similarly, $\text{var}(Y_t) = t\sigma^2$. Hence, RWM with a drift is non-stationary process. Both its mean and its variance increase over time such that it is again a non-stationary process. A time series X={X (t); t ϵ I}, (I being the set of integers) is said to be strictly stationary if for any k ≥ 1, $\mathbf{t_1}$,-------, $\mathbf{t_k}$ distinct members of I and for any t∈I. [1]. A time series X={X (t); t ϵ I} is said to be stationary in the wide sense if [2], Where Q (.) Is an everywhere finite real valued function and Q (.) is also known as the auto covariance function associated with the wide sense time series X. Stationarity is statistically convenient assumption. Although many time series do not possess this property, the assumption of stationarity is a common practice in the statistical analysis of time series. If one were to classify a time series as non-stationary whenever it is not stationary (in either sense), a usual approach is to identify and isolate the non-stationary part, so that the residual behaves as a stationary time series. From this context, the assumption of stationarity helps indirectly to solve some problems relating to time series which is non-stationary in some sense. Further the fact that strict stationarity is preserved when taking in probability or in mean square of a sequence of strictly stationarity time series, the asymptotic theories can be developed without much difficulty. Towards covering same non-stationary time series which behave as stationary time series, when't' is sufficiently large, one can introduce the concept of asymptotic stationarity. A time series X={X (t); t ε I} is said to be asymptotically stationarity in the wide sense if [3], Where Q (.) is the auto covariance function of wide sense stationary time series.

Stationary Assumptions on Time Series

An immediate advantage of this assumption over wide sense stationary is that one can restrict I to be the set of positive integers, (on defining, if necessary, X (t) to be arbitrary constant whenever t≤ 0). Further the assumption of wide sense stationary provides a new dimension to time series analysis in terms of Spectral theories. The spectral representation theorem asserts the existence of a unique non-negative, non-decreasing and right continuous function F (λ) on [-π, π] such that [4], μ_F being the Borel Measure induced by F on the Borel class associated with R. F(λ) is called the spectral distribution function of the time series X, and, if F(λ) is such that [5], Then F (λ) is called the spectral density function of X and is given by [6], Whenever $\sum_{r\in I} Q(r)$ is absolutely convergent. The problems associated with the estimation of F (λ) have been widely discussed in the literature (Anderson (1971), Bartlett (1966), Fuller (1976), Hannan (1960)). One can define the asymptotic spectral distribution function and the asymptotic spectral density function for an asymptotically wide sense stationary time series, using Q (.) introduced in (6). Venkataramana (1972) has highlighted the estimation methods relating to such spectral density function.

Stochastic Models for Time Series

What distinguishes a time series from other discrete parameter stochastic processes is the prevalence of dependence between successive components of the time series, with the nature of dependence varying as one move along the time axis. Stochastic models are constructed with the primary objective to highlight mathematically and form a probabilistic angle, such a dependence that exists in a time series. The dependence among the components may be either self induced (endogenous) or due to extraneous forces (exogenous). A general stochastic model for exposing such dependence may include lagged variables to explain the endogenous dependence, and, regressors to explain the exogenous dependence, in addition to error terms to introduce the stochastic nature of the time series. Linear versions of such models have been widely discussed in the literature, a typical model having specification [7] Where (i)$\boldsymbol{f_i}$ (t) are known real valued functions of t satisfying certain regularity conditions and (ii) Z (t); t≥ $\mathbf{1}$ is an error process with a specified probability distribution, but for a finite number of constants occurring in the distribution. The dynamic stability of the time series that flows from [7] depends on the location of the roots of its characteristic polynomial [8]. with reference to the unit circle. If all the roots lie within the unit circle, the time series is said to be autoregressive in nature. Such time series, with regression component have been discussed in the literature, under varying assumptions on Z(t) [(Anderson (1971), Fuller (1976), Hannan and Nicholls (1972), Fuller, Hasza and Goebel (1981)] , and Venkataramana and Viswanathan (1981-(a))]. Fuller etal (1981), and Venkataramana and Viswanathan (1981)-(a), (b)) have also studied the explosive situations when some or all the roots of $\bar{\boldsymbol{P}}$(Z) = 0 lie outside the unit circle. Stochastic models for non-stationary time series are constructed so as to identify and isolate the non-stationary part of the time series, so that their elimination will result in a process which is stationary in some sense. Estimation techniques associated with models, have been developed, solely resting on such possibility. In many applications, X (t) will turn out to be vector valued, resulting in X being a vector (multiple) time series. Stochastic models for vector time series will then have to be a vector version of the models of the type [7]. In this context, simultaneous linear models, with as many equations as there are components in X (t), which include lagged variables, find their applications in quantitative econometric research. The simultaneous linear model, generating a pair $\boldsymbol{X_1}$={$\boldsymbol{X_1(t)}$, $\boldsymbol{t\epsilon I}$}, $\boldsymbol{X_2}$={$\boldsymbol{X_2(t)}$, $\boldsymbol{t\epsilon I}$} of related time series, considered in this , has the specification.[9], Where f (t) and g (t) are linear functions of known real valued functions of t, and{$\boldsymbol{\epsilon_1(t)}$, $\boldsymbol{t\epsilon I}$}, and {$\boldsymbol{\epsilon_2(t)}$, $\boldsymbol{t\epsilon I}$} are error processes governed by some probabilistic assumptions. This simple simultaneous linear model of first order in two variables, is considered in this only to expose the problems (and methods of solving them), encountered in the process of statistical estimation of the unknown parameters in [12].One can easily generalise the techniques and arguments put forth, without any loss of rigour in such generalisation. However the underlying algebra gets aggravated when either the number of variables is increased or the order of the lag is increased. The linear stochastic model generating a pair of related time series $X_1 = \{X_1(t); t = 1,2, \dots.\}$ And $X_2 = \{X_2(t); t = 1,2 \dots.\}$ discusses in this chapter has the following specifications [10] on being governed by the following basic and explanatory assumptions.

ASSUMPTION 1: $\{U_1(t); t = 1,\ 2, \dots.\}, \{U_2(t);\ t = 1, 2, \dots\}$are linear processes generated by the two independent families of i.i.d random variables $\{\epsilon_1\ (t);\ t = 1, 2, \dots\}$ and $\{\epsilon_2\ (t)\ ;\ t = 1,\ 2, \dots\}$say, such that (i) E $(\epsilon_i\ (t))$; i = 1, 2 is zero and (ii) $0 < \sigma_i^{\ 2} = E\ (\epsilon_i^2\ (t)) < \infty\ ; i = 1,\ 2.$ To be specific, let for i= 1, 2

$$U_i(t) = \sum_{r=0}^{t-1} p_i(r)\varepsilon_{l,i}(t-r); t \geq 1$$

$$\varepsilon_{l,i}(t) = \sum_{r=0} b_i(r)\varepsilon_i(t-r); b_i(0) = 1$$

On setting $\epsilon_i\ (t) = 0$ for t ≤ 0, and under the stipulation that $b_i(r);\ r = 0,\ \dots,$ and $P_i(s);\ s \geq 0$ are real numbers such that

$$0 < \sum_{s=0}^{\infty} |pi(s)| < +\infty$$

That is, $\sum\ p_i\ (s)$ is absolutely summable.

Solving for $X_1(t)$ and $X_2\ (t)$ in terms of $U_1(t)$ and $U_2(t)$ from [10], one has [11].

Equations [2]-(a), (b) lead to the explicit solutions [12] on identifying [13].

And defining λ(r) =0 for r<0, λ (0) =1, and for r≥1, [14].

The dynamic stability or otherwise of the solution in [12] depend on the placements of the roots of the characteristic polynomial associated with the stochastic difference equations in [11].

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ASSUMPTION 2: The roots P_1 and P_2 of the polynomial
$P(z) = Z^2 - (\alpha_{11} + \alpha_{22})\ Z + (\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21})$ have either of the following placements.
(i) $|P_i| < 1\ ;\ i = 1,\ 2$
(Auto regressive)
(ii) $|P_1| > 1 > |P_2|$
(partially explosive)
(iii) $|P_1| > |P_2| > 1$
(purely explosive)

However, under partially explosive or purely explosive situations the following additional assumption is imposed.

ASSUMPTION 3: whenever the two infinite series $\sum \alpha_r$ and $\sum c_r$ are absolutely summable it follows the random variable $\sum_{r=1}^{\infty} a_r \varepsilon_1(r) + \sum_{r=1}^{\infty} c_r \varepsilon_2(r)$ is continuous at zero.

NOTE : All random variables represented by infinite series in this thesis are presumed to be the mean square limits of absolutely mean square convergent series.

The model specified in [10] has wide applications in Econometrics. Our basic assumptions imply that the identifiability conditions are necessarily satisfied so that estimation and testing problems associated with the model [10] can be carried out without much theoretical limitations. Estimation of the parameters in [10] under autoregressive placements on the roots ρ_1 and ρ_2 of P (z) have been extensively discussed in the literature (Hannan (1970), Anderson (1971), Fuller (1976)), viewing the model [10] as a first order vector process. A detailed investigation on the models of the type [10] is due to Venkataraman (1974) wherein he has studied, in depth, the asymptotic properties of the least squares estimators of the parameters under the both explosive placement of the roots ρ_1 and ρ_2 specified in assumption (ii). It is interesting to note that generally, the least squares estimator $(\hat{\alpha}_1, \hat{\alpha}_2) = (\hat{\alpha}_{11}, \hat{\alpha}_{12}, \hat{\alpha}_{13}, \hat{\alpha}_{21}, \hat{\alpha}_{22}, \hat{\alpha}_{23})$ of the parameter vector $(\alpha_1, \alpha_2) = (\alpha_{11}, \alpha_{12}, \alpha_{13}, \alpha_{21}, \alpha_{22}, \alpha_{23})$ occurring in [10] obtained by minimizing the sum of squares [15]. With respect to (α_1, α_2) is asymptotically well behaved under all placements of the roots ρ_1 and ρ_2 specified in assumption (ii). Further, under the purely explosive placements of the roots ρ_1 and ρ_2 in assumption (ii), i.e., when $|\rho_1| > |\rho_2| > 1$ the modified least squares estimator $(\tilde{\alpha}_1, \tilde{\alpha}_1) = (\tilde{\alpha}_{11}, \tilde{\alpha}_{12}, \tilde{\alpha}_{13}; \tilde{\alpha}_{21}, \tilde{\alpha}_{22}, \tilde{\alpha}_{23})$ of (α_1, α_2) defined below is also asymptotically well behaved.
(a) Let $(\tilde{\alpha}_{1i}, \tilde{\alpha}_{2i})$; i=1,2 be the least squares estimators of $(\alpha_{1i}, \alpha_{2i})$; i=1,2 respectively by minimising the sum of squares

$$\sum_{t=1}^{N-1} (X_1(t+1) - \alpha_{11} X_1(t) - \alpha_{12} X_2(t))^2 + \sum_{t=1}^{N-1} (X_2(t+1) - \alpha_{21} X_1(t) - \alpha_{22} X_2(t))^2$$
[16].

The limit theorems that are established in this chapter depend on these estimators and their limit distribution properties. To this end, some of the well known results on these estimators are recorded for easy reference.

PROPOSITION (I): Let (i) $|\rho_1| < 1$ (ii) $U_i(t) = \varepsilon_i(t)$ for i=1, 2. Then under basic assumptions the following statements holds.
$(N^{1/2}(\hat{\alpha}_{11}-\alpha_{11}), N^{1/2}(\hat{\alpha}_{12}-\alpha_{12}), N^{1/2}(\hat{\alpha}_{13}-\alpha_{13}), N^{1/2}(\hat{\alpha}_{21}-\alpha_{21}), N^{1/2}(\hat{\alpha}_{22}-\alpha_{22}), N^{1/2}(\hat{\alpha}_{23}-\alpha_{23}))$
Converges in law ($\xrightarrow{L}$) as $N \to \infty$, to a normal vector $\xi_1 = (\xi_1(11), \xi_1(12), \xi_1(13), \xi_1(21), \xi_1(22), \xi_1(23))$, say with mean zero and a well specified non-singular covariance matrix. Towards stating the asymptotic properties of the estimators under explosive situations it is necessary to introduce, for i=1, 2. [17].

PROPOSITION (II): Let (i) $|\rho_1| > 1 > |\rho_2|$, (ii) $U_i(t) = \varepsilon_i(t)$, (iii) P(G $_i$=0)=0 for i=1, 2. Then under basic assumptions, the following statements hold.
$(N^{1/2}(\hat{\alpha}_{11}-\alpha_{11}), N^{1/2}(\hat{\alpha}_{12}-\alpha_{12}), N^{1/2}(\hat{\alpha}_{13}-\alpha_{13}), N^{1/2}(\hat{\alpha}_{21}-\alpha_{21}), N^{1/2}(\hat{\alpha}_{22}-\alpha_{22}), N^{1/2}(\hat{\alpha}_{23}-\alpha_{23}))$
Converges in law ($\xrightarrow{L}$) as $N \to \infty$, to a normal vector $\xi_2 = (\xi_2(11), \xi_2(12), \xi_2(13), \xi_2(21), \xi_2(22), \xi_2(23))$ say, with mean zero, such that
(i) $M_0 \xi_2(11) + \xi_2(12) = 0$
(ii) $M_0 \xi_2(21) + \xi_2(22) = 0$

M_0 Being defined as $G_1 / G_2 = \alpha_{12} / (\rho_1 - \alpha_{11}) = (\rho_1 - \alpha_{22}) / \alpha_{21}$

PROPORSITION (III): Let (i) $|\rho_1| > |\rho_2| > 1$, (ii) $P(G_i = 0) = 0$ for i=1, 2, (iii) $P(H_i = 0) = 0$ for i=1, 2.

Then under the basic assumptions following statements hold.
(a) $(\rho_2^N(\hat{\alpha}_{11}-\alpha_{11}), \rho_2^N(\hat{\alpha}_{12}-\alpha_{12}), \rho_2^N(\hat{\alpha}_{21}-\alpha_{21}), \rho_2^N(\hat{\alpha}_{22}-\alpha_{22}))$
Converges in law ($\xrightarrow{L}$) as $N \to \infty$, to a normal vector $\xi_3 = (\xi_3(11), \xi_3(12), \xi_2(21), \xi_3(22))$; say such that
- $M_0 \xi_3(11) + \xi_3(12) = 0$
- $M_0 \xi_3(21) + \xi_3(22) = 0$

(b) $(N^{1/2}(\hat{\alpha}_{13}-\alpha_{13}), N^{1/2}(\hat{\alpha}_{23}-\alpha_{23}))$ Converges in law ($\xrightarrow{L}$) as $N \to \infty$, to a normal vector $\xi_4 = (\xi_4(13), \xi_4(23))$ say, with mean zero and a non-singular covariance matrix.
(c) $(\rho_2^N(\tilde{\alpha}_{11}-\alpha_{11}), \rho_2^N(\tilde{\alpha}_{12}-\alpha_{12}), \rho_2^N(\tilde{\alpha}_{21}-\alpha_{21}), \rho_2^N(\tilde{\alpha}_{22}-\alpha_{22}))$
Converges in law ($\xrightarrow{L}$) as $N \to \infty$,
$\xi_5 = (\xi_5(11), \xi_5(12), \xi_5(21), \xi_5(22))$ Say, such that
(i) $M_0 \xi_5(11) + \xi_5(12) = 0$
(ii) $M_0 \xi_5(21) + \xi_5(22) = 0$
(d) $(N^{1/2}(\tilde{\alpha}_{13}-\alpha_{13}), N^{1/2}(\tilde{\alpha}_{23}-\alpha_{23}))$ Converges in law ($\xrightarrow{L}$) as $N \to \infty$, to a normal vector $\xi_6 = (\xi_6(13), \xi_6(23))$ Say, with mean zero and a non-singular covariance matrix.

The main aim of this chapter is to establish some pivotal lemmas certain covariance type functions relating to the model [10]. The estimates of residual are obtained on using the estimators defined in [15], and [16] under different placements of the roots ρ_1 and ρ_2 of P (z). This work is motivated by similar studies on linear stochastic difference equations for scalar time series. The preliminary ground work that is necessary to establish the basic results of this chapter is presented in the next section. Section, pivotal lemmas concerned with the statements and proofs of two pivotal lemmas.

SOME OF THE PIVOTAL LEMMAS

The following lemma, which is elementary and classical, providing the essential ingredient for the validity of propositions (i),(ii) and (iii) finds repeated reference throughout this chapter.

LEMMA (1): Under the basic assumption on $\varepsilon_i(t)$; i=1, 2.

$$\begin{pmatrix} N^{1/2}(\sigma_i\sigma_j)^{-1}\sum_{t=1}^{N}\in_i(t)\in_j(t+r), r=1,2,---; i=1,2 \\ N^{1/2}(\sigma_i)^{-1}\sum_{t=1}^{N}\in_i(t)\in_j(t+k), i=1,2 \end{pmatrix}$$

Converges in law,as $N\to\infty$, to a normal vector $\left(\xi_0(i,j,r), r=1,...,T, i=j=1,2; \xi_0(i), i=1,2\right)$ with mean zero and unit covariance matrix.

NOTE: The validity of the statement is a direct consequence of the standard result due to Diananda (1953) as applied to the vector process $\left(\xi_i(t)\xi_j(t+r), r=1,...,T, \xi_i(t+k), i=j=1,2\right)$.

The following lemma which is referred to in this, time and again, plays the role of the necessary probabilistic tool, towards establishing the limit theorems related to time series under study.

LEMMA (2): Let $(U_{iN}),(V_{iN}(n))$ and $(W_{iN}(n))$; i=1, -----, T, n=1, 2, ------- be a sequences of random variables such that

(a) For N≥N_0 (n) and n≥n_0 (say)

$U_{iN} = V_iN(n) + w_{iN}(n)$ For i =1, 2, ---, T.

(b) For fixed n(≥n_0)

$\lim_{N\to\infty} p(V_{iN}(n)\le x_i; i=1,...,T) = p.d.fF_{n,T}(x_1...x_T)$ And

$\lim_{n\to\infty} p.d.fF_{n,T}(x1...xT) = F_T(x_1...x_T)$ for all

(X_1... x_T) lying in a set $D_{(T)}$ (say) dense in the T-dimensional Euclidean space R_T (say); and

(c) $\lim_{n\to\infty}\lim_{N\to\infty} \sup p\left(|w_{iN}(n)|\ge\partial\right)=0$ For any $\partial>0$ and for i =1, ---, T.

Then it follows that

$\lim_{N\to\infty} p(V_{iN}(n)\le x_i; i=1,...,T) = F_T(x_1...x_T)$ at every continuity point $F_{.T}$

NOTE: The proof of this useful convergence is found in Venkataraman (1968).

LEMMA (3): Under the basic assumption on $\varepsilon_i(t)$ for i =1, 2 the following statement holds:

$$(N^{-1/2}\sum_{t=1}^{N-k}Ui(t+k); i=1,2)$$

Converges in law, as $N\to\infty$, to (θ_1,θ_2), say, where

$\theta_i = (\sum_{r=0}^{\infty}p_i(r))(\sum_{s=0}^{\ell}b_i(s))Z_i$ for i =1, 2.

And Z_1 and Z_2 are independent normal variables with means zero and variance equal to ${\sigma_1}^2$ and ${\sigma_2}^2$ respectively.

PROOF: For $n\ge1$ and N>n+k+2 it is possible to represent that [18], Where $w_{iN}(n)$; i =1, 2 satisfy the requirement (c) in Lemma (2). The validity of the statement in the lemma follows from [18] on invoking Lemmas (1) and (2). Let [19].

LEMMA (4): Let $|P_i| < 1$; $i =$ 1, 2 and $U_i(t)=\varepsilon_i(t)$; i=1, 2. Then under the basic assumptions, the following statements holds for i=1, 2, as $N\to\infty$.

(a) For fixed integrals p, q and $h(\le N)$

(i) $N^{-1}\sum_{t=1}^{N-h}X_i(t+q)\xrightarrow{p}\mu_i$

(ii) $N^{-1}\sum_{t=1}^{N-h}X_1(t+q)X_1(t+q)\xrightarrow{p}{\mu_1}^2+{\sigma_1}^2[Q(q-p)-\alpha_{22}(Q(q-p-1)+Q(q-p+1))+\alpha_{22}^2Q(q-p)]+{\sigma_2}^2(\alpha_{12}^2Q(q-p))$

(iii) $N^{-1}\sum_{t=1}^{N-h}X_1(t+q)X_2(t+q)\xrightarrow{p}\mu_1\mu_2+{\sigma_1}^2[\alpha_{12}Q(q-p+1)-\alpha_{21}\alpha_{22}(Q(q-p)]+{\sigma_2}^2(\alpha_{12}Q(q-p-1))-\alpha_{11}\alpha_{12}Q(q-p))$

(iv) $N^{-1}\sum_{t=1}^{N-h}X_2(t+q)X_2(t+q)\xrightarrow{p}{\mu_2}^2+{\sigma_1}^2[\alpha_{21}^2Q(q-p)]+{\sigma_2}^2[Q(q-p))-\alpha_{11}(Q(q-p-1)+Q(q-p+1))+\alpha_{11}^2Q(q-p)]$

(b)

(i) For any p≥0 and for any o<h≤N

$N^{1/2}E\left|\sum_{t=1}^{N-h}X_i(t+p)\right|\le A_0$

(ii) For any p, q $\ge$ 0 and for any o≤h<N

$N^{-1}E\left|\sum_{t=1}^{N-h}X_i(t-p)X_j(t+h-q)\right|\le A_0$

(iii) For any fixed q≥0, for any p≥0 and o≤h<N

$N^{1/2}E\left|\sum_{t=1}^{N-h}U_i(t+p)X_j(t+h-q)\right|\le A_0$

(iv) For any integer p and q and o≤h(<N)

$N^{1/2}E\left|\sum_{t=1}^{N-h}U_i(t+p)X_j(t+q)\right|\le A_0$ With p>q

$Q(\mu)$ being defined in [10].

NOTE: In this A_0 is being used as a generic notation for any positive memorising constant.

PROOF: The validity of the statements in the lemma follows directly on a substitution evaluation based on [12] and assumption (I), on remembering that $|\rho_i|$<1.
It follows from assumption (II) and (V) that

An alternative representation for $X_i(t)$; i =1, 2 can now be deduced from [12] and [20] under the case: $|\rho_1|>1>|\rho_2|$ as

[21].

Further if ρ_* is a real root of P (Z) such that $|\rho_*|$>1, and if [22],

It is known that, on suitable rearrangement of terms [23].

Next, the discussions in the sequel are based on the properties of the function [24]

Thus settled, the following is an important result for the case: $|\rho_1|>1>|\rho_2|$.

LEMMA (5): Let (i) $|\rho_1|>|\rho_2|>1$,(ii) $U_i(t)=\varepsilon_i(t)$ (iii) $F(G_i=0)=0$, i=1, 2.

Then under the following statements hold for i=1, 2, as $N\to\infty$.

(a) (i) For fixed integers p, q and $0\le h(<N)$

$$\rho_1^{-2N}\sum_{t=1}^{N-h}X_i(t+p)X_j(t+q)\xrightarrow{p}(\rho_1-\rho_2)^{-2}G_iG_j\rho_1^{p+q-2h+4}(\rho_1^2-1)^{-1}$$

For i, j=1, 2.

(ii)For fixed integers p and 0≤ h (<N)

$\rho_1^{-N}\sum_{t=1}^{N-h}X_i(t+p)$ is bounded in probability.

(iii) For fixed non-negative integers p and 0≤ h (<N)

$$N^{-1}\sum_{t=1}^{N-h}\phi(t+p)\xrightarrow{p}(G_2\mu_1-G_1\mu_2)$$

(iv) For fixed non-negative integers p, q and 0≤ h (<N)

$$N^{-1}\sum_{t=1}^{N-h}\phi(t+p)\phi(t+q)\xrightarrow{p}(G_2\mu_1-G_1\mu_2)^2+(G_2^2\sigma_1^2+G_1^2\sigma_2^2)\rho_2^{(p-q)}(1-\rho_2^2)^{-1}$$

(b) For any integer p and 0≤ h (<N), each of the following terms has absolute expectation bounded by A_0

(i) $\rho_1^{-N}\sum_{t=1}^{N-h}X_i(t+p)$

(ii) $N^{-1/2}\sum_{t=1}^{N-h}\phi(t+p)$

(c) For any integral p, q≥0 and 0≤ h (<N) the following terms has absolute expectation bounded by A_0

(i) $N^{-1}\sum_{t=1}^{N-h}\phi(t+p)\phi(t+q)$

(ii) $\rho_1^{-N}\sum_{t=1}^{N-h}\phi(t+p)X_i(t+h-p)$

(iii) $\rho_1^{-N}\sum_{t=1}^{N-h}\phi(t+h-p)X_i(t-p)$

(iv) $\rho_1^{-2N}\sum_{t=1}^{N-h}X_i(t-p)X_i(t+h-q)$

(v) $\rho_1^{-N}\sum_{t=1}^{N-h}U_i(t+p)X_j(t+h-p)$

(vi) $N^{-1/2}\sum_{t=1}^{N-h}U_i(t+p)\phi(t+h-q)$

PROOF: It follows from [23] that when $|\rho_1|>1$ [25]

This information together with [21] implies that [26].

The validity of the statements (a),(b) and (c) of the lemma follows directly on a substitution evaluation of the relevant terms based on [12] and [17], together with an apple to lemma(1) on remembering the basic assumptions.

A parallel result for the case: $|\rho_1|>|\rho_2|>1$ is given below;

LEMMA (6): Let (i) $|\rho_1|>|\rho_2|>1$ (ii) $p(G_i=0)=0$ (iii) $p(H_i=0)=0$, i=1, 2. Then under basic assumptions the following statements hold, as $N\to\infty$.

(a) (i) For fixed integers p, q and 0≤ h (<N)

$$\rho_1^{-2N}\sum_{t=1}^{N-h}X_i(t+p)X_j(t+q)\xrightarrow{p}(\rho_1-\rho_2)-2G_iG_j\rho_1^{p+q-2h+4}(\rho_1^2-1)^{-1};i=j=1,2.$$

(ii)For fixed integers p and 0≤ h (<N)

$$\rho_1^{-N}\sum_{t=1}^{N-h}X_i(t+p)\xrightarrow{p}(\rho_1-\rho_2)^{-1}G_i\rho_1^{p-h+2}(\rho_1-1)^{-1};i=1,2.$$

(iii) For fixed non-negative integers p and 0< h (<N)

$$\rho_2^{-N}\sum_{t=1}^{N-h}\phi(t+p)\xrightarrow{p}(\rho_1-\rho_2)^{-1}(G_2H_1-G_1H_2)\rho_2^{p-h+2}(\rho_2-1)^{-1}$$

(iv) For fixed non-negative integers p, q and 0≤ h (<N)

$$\rho_2^{-2N}\sum_{t=1}^{N-h}\phi(t+p)\phi(t+q)\xrightarrow{p}(\rho_1-\rho_2)^{-2}(G_2H_1-G_1H_2)^2\rho_2^{p+q-2h+4}(\rho_2^2-1)^{-1}$$

(v) For fixed integers p and q ≥ 0 and 0≤ h (<N)

$$\rho_1^{-N}\rho_2^{-N}\sum_{t=1}^{N-h}X_i(t+p)\phi(t+q)\xrightarrow{p}(\rho_1-\rho_2)^{-1}(\rho_2-\rho_1)^{-1}G_i(G_1H_2-G_2H_1)\rho_1^{p-h+2}\rho_2^{p-h+2}(\rho_1\rho_2-1)^{-1}$$

(b) For any integer p and 0≤ h (<N), each of the following terms has absolute expectation bounded by A_0

(i) $\rho_1^{-N}\sum_{t=1}^{N-h}X_i(t+p)$

(ii) $\rho_2^{-N}\sum_{t=1}^{N-h}\phi(t+p)$

(c) For any integral p, q>0 and 0≤ h (<N) the following terms have absolute expectations by A_0

(i) $\rho_1^{-2N}\sum_{t=1}^{N-h}X_i(t-p)X_j(t+h-q);i=1,2.$

(ii) $\rho_1^{-N}\rho_2^{-N}\sum_{t=1}^{N-h}X_i(t-p)\phi(t+h-q);i=1,2.$

(iii) $\rho_2^{-2N}\sum_{t=1}^{N-h}\phi(t-p)\phi(t+h-q)$

(iv) $\rho_1^{-N}\sum_{t=1}^{N-h}U_i(t+p)X_j(t+h-q)$

(v) $\rho_2^{-N}\sum_{t=1}^{N-h}U_i(t+p)\phi(t+h-q)$

PROOF: It follows from [12] and [20] that for $t\geq 1$[27],

Further from [17], [24], [25] and [27], it can be checked on routine manipulations that [28], and the basic assumptions that the expectation within the square brackets on the R.H.S. of [19] has absolute expectation which is uniformly expressions based on [18] and [19] together with an appeal to lemma(2) and to standard probabilistic results established the validity of the statements (a), (b) and (c) of the lemma. From [6] it can be seen that [29],

Where

(i) $\hat{\Delta}_{10}=\begin{vmatrix} <X_1^2(t)> & <X_1(t)X_2(t)> & <X_1(t)> \\ <X_1(t)X_2(t)> & <X_2^2(t)> & <X_2(t)> \\ <X_1(t)> & <X_2(t)> & N-1 \end{vmatrix}$

(ii) $<f(t)>=\sum_{t=1}^{N-h}f(t).$

(iii) $\hat{\Delta}_{1i};i=1,2,3$ is obtained from $\hat{\Delta}_{10}$ on replacing its 'i[th]' column by the column vector $\left(<U_1(t+1)X_1(t)>,<U_1(t+1)X_2(t)>,<U_1(t+1)>\right)$

(iv) $\hat{\Delta}_{2i};i=1,2,3$ is obtained from $\hat{\Delta}_{1i};i=1,2,3$ respectively,

On replacing $U_1(t+1)$ by $U_2(t+1)$

NOTE: let $R_{ij}(f)$ denote the operation on a determinant of adding f times j[th] row to its i[th] row and $C_{ij}(f)$, a similar operation on columns. Successive operations of $R_{ij}(.)$ and $C_{ij}(.)$ are read from right to left to indicate order of application.

Equations:

(ii) $\boldsymbol{t_1},\_\_\_\_\_\_\_,\boldsymbol{t_k}$ are distinct members of I, and

(iii) $\boldsymbol{X(t_1)},------,\boldsymbol{X(t_k)}$ are real numbers.

$(\boldsymbol{X(t_1)},------,\boldsymbol{X(t_k)})$ And

$(\boldsymbol{X(t_1+t)},------,\boldsymbol{X(t_k+t)})$ --------- [1]

are identically distributed.

$$\left.\begin{array}{l} \mathrm{E}\,(\mathrm{X(t)})=\mu \\ \mathrm{E(X(t)-}\mu)\,(\mathrm{X(s)-}\mu)=\mathrm{Q(|\,t-s\,|)} \end{array}\right\} \text{ ----------------- [2]}$$

$$\left.\begin{array}{ll} \mathrm{E(X))}\rightarrow\mu & \text{as t}\rightarrow\infty \\ \mathrm{Cov\,(X\,(t),X(\,t+s))}\rightarrow\mathrm{Q(s)} & \text{as t}\rightarrow\infty \end{array}\right\} \text{ -------- [3]}$$

$$\mathrm{Q(s)} = \int_{-\pi}^{\pi}\exp(\lambda)d\mu_F \quad \text{--------------------------------- [4]}$$

$$\mathrm{F}\,(\boldsymbol{\lambda}) = \int_{-\pi}^{\lambda} f(\mu)d_\mu \quad \text{------------------------------------ [5]}$$

$$\mathrm{F}\,(\boldsymbol{\lambda}) = (\boldsymbol{2\pi})^{-1}\sum_{r=-\infty}^{\infty}Q(r)\ \boldsymbol{cos\,\lambda_r}\ ;\ -\boldsymbol{\pi}\leq\boldsymbol{\lambda}\leq\boldsymbol{\pi} \text{--------- [6]}$$

X (t + k) - $\boldsymbol{\beta_1\,X(t+k-1)}-\ \dots\dots\dots.-\boldsymbol{\beta_k X(t)}-$

$\sum_{i=0}^{p}\alpha_i f_i(t+k)=z(t+k)$; t≥ $\mathbf{1}$ ---- [7]

$\bar{\boldsymbol{P}}$(z) = $\boldsymbol{Z^k}-\boldsymbol{\beta_1}\,\boldsymbol{Z^{k-1}}-\ \dots\dots\dots-\boldsymbol{\beta_k}$ ------------------ [8]

$\boldsymbol{X_1(t+1)-\alpha_{11}X_1(t)-\alpha_{12}X_2(t)-f(t+1)=\ \epsilon_1(t+1)}$

$\mathbf{X_2(t+1)-\alpha_{21}X_1(t)-\alpha_{22}X_2(t)-f(t+1)=\ \epsilon_2(t+1)}$ ------------ [9]

$$\left.\begin{array}{l} X_1(t+1)-\alpha_{11}X_1(t)-\alpha_{12}X_2(t)-\alpha_{13}=U_1(t+1) \\ X_2(t+1)-\alpha_{21}X_1(t)-\alpha_{22}X_2(t)-\alpha_{23}=U_1(t+1) \end{array}\right\}----[10]$$

(a) $\mathrm{X_1(t+2)\ -\ (\alpha_{11}+\alpha_{22})\ X_1\,(t+1)\ +\ (\alpha_{11}\alpha_{22}\ -\ \alpha_{12}\alpha_{21})\ X_1\,(t)}\ -$
$[(1-\alpha_{22})\ \alpha_{13}\ +\ \alpha_{12}\ \alpha_{23}]$
$=\mathrm{U_1(t+2)\ -\alpha_{22}\ U_1(t+1)+\alpha_{12}U_2(t+1)}$
$=\mathrm{L_1\ \ (t+2)}$ (say)

(b) $\mathrm{X_2(t+2)-(\alpha_{11}+\alpha_{22})\ X_2\ (t+1)\ +\ (\alpha_{11}\alpha_{22}-\alpha_{12}\alpha_{21})\ X_2\ (t)}\ -$
$[(1-\alpha_{11})\ \alpha_{23}\ +\ \alpha_{21}\ \alpha_{13}]$
$=\mathrm{U_2(t+2)\ -\ \alpha_{11}U_2(t+1)+\alpha_{21}U_1(t+1)}$
$=\mathrm{L_2\ (t+2)}$ (say) ----- [11]

$$\left.\begin{array}{l} X_1(t)=\mu_1+\sum_{r=0}^{t-1}\lambda(r)L_1(t-r) \\ X_2(t)=\mu_2+\sum_{r=0}^{t-1}\lambda(r)L_2(t-r) \end{array}\right\}----[12]$$

$$\left.\begin{array}{l} \mu_1=(1-\alpha_{11}-\alpha_{22}+\alpha_{11}\alpha_{22}-\alpha_{12}\alpha_{21})^{-1}[(1-\alpha_{22})\alpha_{13}+\alpha_{12}\alpha_{23}] \\ \mu_2=(1-\alpha_{11}-\alpha_{22}+\alpha_{11}\alpha_{22}-\alpha_{12}\alpha_{21})^{-1}[(1-\alpha_{11})\alpha_{23}+\alpha_{21}\alpha_{13}] \end{array}\right\}---[13]$$

$$\lambda(r)-(\alpha_{11}+\alpha_{22})\lambda(r-1)+$$
$$(\alpha_{11}\alpha_{11}-\alpha_{12}\alpha_{21})\lambda(r-2)=0------[14]$$

$$\sum_{t=1}^{N-1}[X_1(t+1)-\alpha_{11}X_1(t)-\alpha_{12}X_2(t)-\alpha_{13}]^2+\sum_{t=1}^{N-1}(X_2(t+1)-\alpha_{21}X_1(t)-\alpha_{22}X_2(t)-\alpha_{23})^2---[15]$$

$$\left.\begin{array}{l} \tilde{\alpha}_{13}=(N-1)^{-1}\sum_{t=1}^{N-1}(\mathrm{X_1(t+1)}-\tilde{\alpha}_{11}X_1(t)-\tilde{\alpha}_{12}X_2(t)) \\ \tilde{\alpha}_{23}=(N-1)^{-1}\sum_{t=1}^{N-1}(X_2(t+1)-\tilde{\alpha}_{21}X_1(t)-\tilde{\alpha}_{22}X_2(t)) \end{array}\right\}---[16]$$

$$\left.\begin{array}{l} G_i = \sum_{r=1}^{\infty} \rho_1^{-r} L_i(r), when |\rho_1| > 1 \\ H_i = \sum_{r=1}^{\infty} \rho_2^{-r} L_i(r), when |\rho_2| > 1 \end{array}\right\} ---[17]$$

$$N^{-1/2} \sum_{t=1}^{N-k} U_i(t+k) = N^{-1/2} \sum_{r=0}^{n} p_i(r) \sum_{s=0}^{\ell} b_i(s) \sum_{t=1}^{N} \varepsilon_i(t) + w_{iN}(n) \quad -- - [18]$$

$$Q(\mu) = \sum_{r=0}^{\infty} \lambda(r) \lambda(r + |\mu|) ----[19]$$

$$\lambda(r) = (\rho_1 - \rho_2)^{-1} \rho_1^{r+1} + (\rho_2 - \rho_1)^{-1} \rho_2^{r+1}; \rho_1 \neq \rho_2 -----[20]$$

$$X_i(t) = (\rho_1 - \rho_2)^{-1} \rho_1^{r+1} G_i + (\rho_2 - \rho_1)^{-1} \rho_2^{r+1} L_i(t-r) - (\rho_1 - \rho_2)^{-1} \sum_{r=1}^{\infty} \rho_1^{-r+1} L_i(t+r) + \mu_i$$

; i=1, 2; t≥1 ----- [21]

$$J_i = \sum_{r=1}^{\infty} \rho_*^{-r} L_i(r); i = 1,2 ------[22]$$

$$\left.\begin{array}{l} (\rho_* - \alpha_{11}) J_1 - \alpha_{12} J_2 = 0 \\ (\rho_* - \alpha_{22}) J_2 - \alpha_{21} J_1 = 0 \end{array}\right\} ----[23]$$

$$\left.\begin{array}{l} \phi(t) = G2X1(t) - G1X2(t); t \geq 1 \\ \phi(t) = 0; t \leq 0 \end{array}\right\} ---[24]$$

$$\left.\begin{array}{l} (\rho_1 - \alpha_{11}) G_1 - \alpha_{12} G_2 = 0 \\ (\rho_1 - \alpha_{22}) G_2 - \alpha_{21} G_1 = 0 \end{array}\right\} ----[25]$$

$$\phi(t) = (G_2 \mu_1 - G_1 \mu_2) + G_2 \sum_{r=0}^{t-1} \rho_2^r U_i(t-r) - G_1 \sum_{r=0}^{t-1} \rho_2^r U_2(t-r) ----[26]$$

$$X_i(t) = (\rho_1 - \rho_2)^{-1} \rho_1^{t+1} G_i + (\rho_2 - \rho_1)^{-1} \rho_2^{t+1} H_i - (\rho_1 - \rho_2)^{-1} \sum_{r=1}^{\infty} \rho_1^{-r+1} L_i(t+r) - (\rho_2 - \rho_1)^{-1} \sum_{r=1}^{\infty} \rho_2^{-r+1} L_i(t+r) + \mu_i; i = 1,2. ---[27]$$

$$\phi(t) = (\rho_2 - \rho_1)^{-1} \rho_2^{t+1} (G_2 H_1 - G_1 H_2) + (G_2 \mu_1 - G_1 \mu_2) + [G_2 \sum_{r=1}^{\infty} \rho_2^{-r} \varepsilon_2(t+r) - G_1 \sum_{r=1}^{\infty} \rho_2^{-r} \varepsilon_1(t+r)] ----[28]$$

$$\left.\begin{array}{l} (\hat{\alpha}_{11} - \alpha_{11}) = \hat{\Delta}_{11} / \hat{\Delta}_{10}; \hat{\alpha}_{21} - \alpha_{21} = \hat{\Delta}_{21} / \hat{\Delta}_{10} \\ (\hat{\alpha}_{12} - \alpha_{12}) = \hat{\Delta}_{12} / \hat{\Delta}_{10}; \hat{\alpha}_{22} - \alpha_{22} = \hat{\Delta}_{22} / \hat{\Delta}_{10} \\ (\hat{\alpha}_{13} - \alpha_{13}) = \hat{\Delta}_{13} / \hat{\Delta}_{10}; \hat{\alpha}_{23} - \alpha_{23} = \hat{\Delta}_{23} / \hat{\Delta}_{10} \end{array}\right\} ---[29]$$

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