Development Of Students’ Worksheet Based On APOS Theory Approach To Improve Student Achievement In Learning System Of Linear Equations

I Made Arnawa, Yerizon, Sri Nita, Roni Tri Putra

Abstract: This study aims to develop student worksheet based on APOS theory approach to improve student achievement in learning matrix and vector spaces, particularly in systems of linear equations (SLE). This research is a research & development by using Plomp model. This research was conducted at Andalas University. Instruments used in this study were guidelines of the interview, validation instrument, lecturers' and students' response questionnaire, and SLE test. The results showed that the students' worksheets designed already met the criteria valid, practical, and effective.

Keywords: Students' Worksheet, APOS Theory, System of Linear Equations, research & development, Plomp Model

1. Introduction

Linear algebra is one of main mathematics course at university level that is very important to learn, because it has a very wide application, particularly in modeling the problems in the fields of science, engineering, and even in economics ([1],[2]). Unfortunately, some researcher showed that most students have difficulties in learning linear algebra (see [3],[4]). Particularly in learning SLE, some misconceptions to be the cause of student have difficulties in learning SLE ([5]). In addition, the learning approach that does not fit cause the students failed in the transition from school mathematics to the university mathematics ([6]). One of the key factors in the success of students in understanding a mathematics concept is determined by learning material that lectures used, good learning materials are learning materials that can help students in the construction of own mathematical knowledge. Based on the result of students’ questionnaire in the preliminary research to students majoring in industrial engineering who were taking matrix and vector space course academic year 2017/2018 was obtained information that 50% of the 40 students stated that the text books that used in the course was difficult to understand, so that students are very dependent upon the explanation lecturer in understanding the content of the textbook. In addition, all students do not understand the background to the emergence of a theorem that is associated with properties in linear algebra.

According to the doctrine of constructivism, the theorem should be found by students through their learning experience either singly or in collaboration with other students, in the collaboration, students will be able to increase the social skills and communication skills ([7]). One way of helping students in constructing own mathematical knowledge is by providing worksheets that containing learning activities to be experienced so that the student can eventually find some concepts or theorems in mathematics ([8],[9]). The use of the worksheet in learning mathematics can improve learning outcomes ([10], [11]). There are several approaches that can be used in developing a worksheet (see [12],[8],[9]). APOS theory is an approach to learning that is specifically for the learning mathematics at the college level, which integrate the use of computers, cooperative learning, and pay attention to the mental construction in understanding a concept or theorem in mathematics. APOS theory basically embraced the doctrine of konstruktivism, particularly social constructivism. According to Dubinsky & Mcdonald in [13], students’ mathematical knowledge is a result of the construction and the students interaction with others in the understanding of mathematical ideas. Mathematical knowledge is constructed by the student through the stages of mental construction process, actions, objects, and schemas are abbreviated with APOS. Interactions between students and student interaction with lecturers meant to develop what is referred to as Vygosky by zone of proximal development ([14]). There are many models of research in research and development, e.g. 4-D model ([15]) the ADDIE model ([16]), and Plomp models ([17]). This research uses Plomp model, if compared to other development models, the Plomp model has several advantages, e.g. for assessment of the practicability of its products is done through three stages i.e. at the stage one-to-one evaluation, small group, and field test.

2. Method

This study is research and development using Plomp model which is aimed to develop valid, practice, and effective worksheet based on APOS theory approach to improve student achievement in learning SLE. The development process of this worksheet consists of three phases:
3. Result and Discussion

3.1 Result

3.1.1 Result in the preliminary research

Based on interviews conducted against the two lecturer who teach matrix and vector space was obtained information that the examples and exercises found in the text book have not been adequate both in the amount or quality, especially in the topic of SLE, so it is seen necessary to enrich the examples and exercises, especially to show that not all method can be used to determine the solution of SLE. Based on results of interviews against three students was obtained information that they do not know the background appears a theorem, so hope to have learning materials which present a theorem by first reviewing the background to the emergence of these theorems, for example by proposing the phenomena-phenomena or samples so they can be generalized into a theorem. In the syllabus of matrix and vector spaces course, particularly for the topic of SLE looks that still need to be added to some of the learning outcomes (LO), so the complete LO were: (1) students can explain the link between the method of elimination and substitution on linear equations with elementary row operations on the augmented matrix; (2) student can determine the solution of SLE that has a single solution and has many solutions; (3) students can determine the condition that SLE have a single solution, no solution, and has a lot of solutions; (4) student can modeling everyday life problem solving in the form of SLE; (5) the student can explain the link between the methods Gaussian, Gauss-Jordan Elimasi, and Cramer with the kinds of solutions of an SLE. Based on APOS Theory framework, students LO about SLE enlisted in Table 1.

<table>
<thead>
<tr>
<th>APOS Theory Framework</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Action</td>
<td>Student can declare SLE in the form of matrices and augmented matrix</td>
</tr>
<tr>
<td></td>
<td>student can determine the solution of SLE that has a single solution or has many solutions</td>
</tr>
<tr>
<td></td>
<td>students can determine the condition that SLE have one solution, no solution, and has a lot of solutions</td>
</tr>
<tr>
<td></td>
<td>Student can check if SLE has no solution, one solution, or many solution and can solve if a SLE have a solution</td>
</tr>
<tr>
<td>Process</td>
<td>Student can solve for a, b, c in a SLE sack that the SLE: (1) has no solution., (2) has one solution, or (3) have many solution</td>
</tr>
<tr>
<td>Object</td>
<td>student can modeling everyday life problem solving in the form of SLE and can solve if a SLE have a solution</td>
</tr>
<tr>
<td>Schemas</td>
<td>Student can describe in the form of concept maps link between SLE, the kinds of solutions, and the appropriate method to solve the SLE</td>
</tr>
</tbody>
</table>

An analysis of the concepts in the topic of SLE is obtained that the concept of elementary row operations, determinant, echelon matrix, and the reduced row-echelon matrix was the essential concepts in SLE, so need to get the emphasis in learning process of SLE. Many studies show that lack of understanding against the essesial concepts in SLE impact to low achievement in SLE ([18]).

3.2 Result in the In prototyping phase

a. Prototype 1

an initial draft of student worksheet in the early stages are continuously evaluated by the researcher during the process of design is called a prototype 1. The following was outlined the process and results of the product. Teaching and learning SLE based on APOS Theory begins with asking the students pay attention to the problems in everyday life are associated with SLE was contained within the worksheet, the student intended to find out the advantage of learning SLE and then was expected more motivated in learning. According to Middleton & Spanias in [19], indicate the usefulness of a topic is one of many strategies for motivating students in mathematics. Students must understand that the mathematics instruction they receive is useful, both in immediate terms and in preparing them to learn more in the fields of mathematics and in areas in which mathematics can be applied (e.g., physics, business, etc.). This strategie providing opportunities for students to develop intrinsic motivation in mathematics. Introduces the concept of SLE begins with asking the students pay attention to the definition 1 about Three forms of SLE, i.e. form 1, 2, and 3 as follows.

Definition 1

Let \( \mathbb{R} \) be the set of real number, the arrangement of linear equations in the form

\[
\begin{align*}
& a_1 x_1 + a_2 x_2 + \ldots + a_n x_n = b_1 \\
& a_1 x_1 + a_2 x_2 + \ldots + a_n x_n = b_2 \\
& \vdots \\
& a_1 x_1 + a_2 x_2 + \ldots + a_n x_n = b_m
\end{align*}
\]

with \( b_k, a_i \in \mathbb{R}, \quad k = 1, 2, \ldots, m \) and \( j = 1, 2, \ldots, n \) is called system of linear equation (SLE) that consist of \( m \) equations and \( n \) variables.
In the form of matrix (form 2) and in the form of augmented matrix (form 3), SLE in definition 1, each is expressed as follow.

\[
\begin{pmatrix}
  a_{11} & a_{12} & \cdots & a_{1n} \\
  a_{21} & a_{22} & \cdots & a_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  a_{m1} & a_{m2} & \cdots & a_{mn}
\end{pmatrix}
\begin{pmatrix}
  x_1 \\
  x_2 \\
  \vdots \\
  x_n
\end{pmatrix}
= 
\begin{pmatrix}
  b_1 \\
  b_2 \\
  \vdots \\
  b_m
\end{pmatrix}
\]

(2)

and

\[
\begin{pmatrix}
  a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\
  a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\
  \vdots & \vdots & \ddots & \vdots & \vdots \\
  a_{m1} & a_{m2} & \cdots & a_{mn} & b_m
\end{pmatrix}
\]

(3)

Furthermore, it is presented examples of SLE in forms 1, 2, and 3, and then the student is assigned to change an SLE from form 1 to form 2, form 2 to form 3 and the converse from form 3 to form 2, and from form 2 to form 1, then student conclude that form 3 is the most efficient form of SLE. Introduces the concept of elementary row operations (ERO) begins by presenting an SLE, then students are assigned to: (1) add an equation with multiple other equations; (2) multiply the equation with a real number that is not zero; (3) interchanging two equations. Next, students were assigned to state the results of (1), (2) and (3) in the form of a augmented matrix and made a conclusion that the ERO on augmented matrix equivalent to the elimination and substitution on the SLE in the form 1. Solution of an SLE is introduced by assigning students to pay attention to the definition 2, example 1, and example 2.

Definition 2
Let \( \mathbb{R} \) be the set of real number, \( x_1 = s_1 \), \( x_2 = s_2, \ldots, x_n = s_n \) with \( s_1, s_2, \ldots, s_n \in \mathbb{R} \), that meets the SLE
\[
a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n = b_1 \\
a_{21} x_1 + a_{22} x_2 + \cdots + a_{2n} x_n = b_2 \\
\vdots \\
a_{m1} x_1 + a_{m2} x_2 + \cdots + a_{mn} x_n = b_m
\]
is called the solution of the SLE.

Next is presented an example solution of some SLE as follows.

Example 1
Note that \( x = 1 \) and \( y = 2 \) is solution of the SLE
a. (i): \( x + y = 3 \)  
   b. (ii): \( x + 2y = 5 \)
   (ii): \( x + 2y = 5 \)  
   (i): \( x + y = 3 \)
   c. 2(i): \( 2x + 2y = 6 \)  
   d. (i): \( x + y = 3 \)  
   (ii): \( x + 2y = 5 \)  
   (ii)+3(i): \( 4x + 5y = 14 \)

In the form of augmented matrix, SLE in the example1 is declared as shown in example 2.

Example 2
Note that in the form of augmented matrix, SLE in Example 1 can be expressed as follows
\[
\begin{pmatrix}
  1 & 1 & 3 \\
  1 & 2 & 5
\end{pmatrix}
\Rightarrow
\begin{pmatrix}
  1 & 2 & 5 \\
  1 & 1 & 3
\end{pmatrix}
\]

From example 1 and example 2 students are expected to make a conclusions in the form of the theorem that the ERO is not changing the solution of an SLE. Next, to find out what types of solutions an SLE, students are asked to work on activity 1,

Activity 1
Determine the solution of SLE below (if any). 
(1). \( x + y = 3 \)  
   \( x + 2y = 5 \)  
   \( 2x + 2y = 6 \)
(2). \( x + y + z = 6 \)  
   \( x + y + 2z = 9 \)  
   \( x + y + 2z = 11 \)
(3). \( x + y + z = 6 \)  
   \( x + y + 2z = 9 \)  
   \( 2x + 2y + 2z = 11 \)

What can you conclude about solution of the SLE?

In a similar manner to that introduced the concept of a solution of an SLE, furthermore was introduced the concept of row echelon matrix, reduced- row echelon matrix, method of Gaussian elimination method, Gauss-Jordan elimination method, determinat, and Cramer’s rule, then students train themselves to find solutions with a wide range of SLE and methods and solve everyday life problem solving that can modeling in the form of SLE. To acquire the characteristics of SLE that does not have a solution, has a single solution, and have many solutions, the student is assigned to determine solution of the following SLE (if any).

Activity 2
1. Determine solution of the following SLE (if any) with Maple software.
   (1). \( x + y + z = 6 \)  
   \( x + y + 2z = 9 \)
   \( x + y + 2z = 11 \)
   (2). \( x + y + z = 6 \)  
   \( x + y + 2z = 9 \)
   \( x + y + 2z = 11 \)
   (3). \( x + y + z = 6 \)  
   \( x + y + 2z = 9 \)
   \( 2x + 2y + 2z = 11 \)
2. Declare SLE a, b, c in the form of augmented matrix and then change into row echelon matrix.
3. Find out the linking between solution of SLE a, b, c with the form of its row echelon matrix.
   What can you conclude about it?

Through activity 2, students are expected to find the characteristics of SLE that has no solution, have a single solution, and have many solutions.

b. Prototype 2
To figure out the alignment of the contents of the student worksheet with the syllabus, the suitability of the APOS theory approach to student worksheet display, the truth
language, and the appeal of the worksheet, prototype 1 was validated by two experts in SLE. The results was presented at Table 2.

**Table 2. Mean Score and Category of the Expert Review on Student Worksheet Based on APOS Theory Approach**

<table>
<thead>
<tr>
<th>No.</th>
<th>Aspects that are assessed</th>
<th>Mean</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>the suitability with APOS theory</td>
<td>3.42</td>
<td>Valid</td>
</tr>
<tr>
<td>2.</td>
<td>the suitability with syllabus</td>
<td>3.35</td>
<td>valid</td>
</tr>
<tr>
<td>3.</td>
<td>the suitability with Indonesian Language</td>
<td>3.25</td>
<td>valid</td>
</tr>
<tr>
<td>4.</td>
<td>the appeal of the worksheet</td>
<td>3.21</td>
<td>valid</td>
</tr>
<tr>
<td></td>
<td>Total mean</td>
<td>3.31</td>
<td>valid</td>
</tr>
</tbody>
</table>

Based on the review of the expert in Table 2, student worksheet not yet complete, it is need a lot of revision, both in terms of content, language, appeal, and the suitability with APOS theory. Prototype 2 was the result of a revision on prototype revision 1 based on the advice given by a validator. Some of the revision are as follows: (1) add the instructions work on worksheet, (2) add a few Example and problem exercises in order to cover all the LO and in accordance with the process of mental construction APOS, (3) add the example program Maplet in determining the solution of an SLE, (4) improve the appeal of the worksheet by varying the size of the letters in the name of definitions and theorems as well as its content.

c. Prototype 3

To find out the practicality of student worksheet (Prototype 2) it, then do one-to-one evaluation against 3 students. One by one the students assigned to observe and do all the activity on the student worksheet. Next, each student interviewed separately associated with ease in understanding the content, readability, and the adequacy of time in working on the student worksheet. Interview results indicate that: (1) 2 of 3 students have not been fully able to directly understand the content of student worksheets, they say that much explanation in student worksheet that can not be understood. Furthermore, the lecturer provides guidance in the form of explanations and questions so that the two students can understand the contents of the student worksheet. Based on the weaknesses found juring one-to-one evaluation, the next student worksheet revised, i.e. with more detailing again the content of the student worksheet. (2) all the students say cannot yet determine the solution of an SLE with Maplet, so necessitating provided Maplet software operation guide. Prototype 3 is the result of the revision of the prototype 2 based on the weaknesses found in the one-to-one evaluation.

d. Prototype 4

To find out the practicality of student worksheet (Prototype 3) in teaching and learning practice, researchers tested its in small group evaluation with 6 students. Implementation of a small group evaluation in the form of a matrix and vector spaces lecture, specifically to the topic of SLE was conducted as many as 4 times. To obtain data about the weakness of the worksheet in the process of teaching and learning, the implementation of the small group was observed by an observer who is in charge of taking down everything that happens in the lectures, especially related to the ease of use the worksheet. At the end of the meeting the students are asked to fill out the questionnaire about students’ response to worksheet and worked on the test about SLE. From the results of students’ response was obtained information about the practicality of student worksheet as stated in Table 3.

**Table 3. Mean Score of Students’ Response on Students’ Worksheet**

<table>
<thead>
<tr>
<th>No.</th>
<th>Indicators</th>
<th>Mean</th>
<th>Perentage Of Practicality (%)</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Ease of use</td>
<td>3.33</td>
<td>82.50</td>
<td>Practice</td>
</tr>
<tr>
<td>2.</td>
<td>Appeal</td>
<td>3.21</td>
<td>80.25</td>
<td>Practice</td>
</tr>
<tr>
<td>3.</td>
<td>Readability</td>
<td>3.35</td>
<td>83.75</td>
<td>Practice</td>
</tr>
<tr>
<td>4.</td>
<td>Allocation of time</td>
<td>3.13</td>
<td>78.25</td>
<td>Practice</td>
</tr>
<tr>
<td></td>
<td>Total mean</td>
<td>3.26</td>
<td>81.38</td>
<td>Practice</td>
</tr>
</tbody>
</table>

From the test results was obtained information about the achievement in SLE as presented in Table 4.

**Table 4. Student Test Result on SLE**

<table>
<thead>
<tr>
<th>Respondent code</th>
<th>Score</th>
<th>Value</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mhs1</td>
<td>70</td>
<td>B</td>
<td>Good</td>
</tr>
<tr>
<td>Mhs2</td>
<td>71</td>
<td>B</td>
<td>Good</td>
</tr>
<tr>
<td>Mhs3</td>
<td>88</td>
<td>A</td>
<td>Excellent</td>
</tr>
<tr>
<td>Mhs4</td>
<td>75</td>
<td>B+</td>
<td>Very good</td>
</tr>
<tr>
<td>Mhs5</td>
<td>74</td>
<td>B</td>
<td>Good</td>
</tr>
<tr>
<td>Mhs6</td>
<td>60</td>
<td>B</td>
<td>Good enough</td>
</tr>
</tbody>
</table>

Table 3 stated total mean score was 3.26 (81.38% percentage of practicality). This means the student worksheet already practical, however still need to be revised. Based on observations by the observer was noted that most students still difficulties in using the Maplet software, so still need to break down the way to determine the SLE solution with Maplet software solution in the worksheet. Next, table 4 stated that 5 out of 6 students (83.33%) have score test on SLE 70 or over. This means that the student worksheet was already effective.

3.2 Discussion

In research and development, initial product must be have expert judgment through validation process. Validate the design of a product is an activity to assess whether the design of a product rationally will be more effective than the old ones or not. It is said to be rationally because in this process not yet including the fact field, but these products assessment are still based on rational thinking. This assessment can be done by bringing the product to experts discussion forum. For example, in research and development of learning material, expert team is learning technology experts, expert field studies on the same subjects, and expert evaluation of learning. Based on the experts review we will know the weakness and strength of our product. The results of the assessment of the experts indicates that the mean total score validation student worksheet was 3.31, it belongs in the category is valid. This
means that the contents of the student worksheet already in accordance with the syllabus, the delights of the material in the worksheet is in compliance with the approach of APOS theory. Its language is in compliance with the rules of Indonesian language, and nice enough in display. Validity is one measure of the appropriateness of a products before it can be used in teaching and learning process ([18]). Products’ testing is done gradually, the early stages of the simulation is done with the use of these products in limited number of student (about 3 student). After the simulated then it can be tested on a limited group (about 6 student), before it was tested on the actual circumstances. Testing conducted with the aim to obtain information on whether a new product can be used as a replacement for the old product. The results of student response shows that total mean score practicality was 3.26, it includes practical categories. This means that the contents of the student worksheet can be well understood by the majority of students, it looks interesting, and belongs to the choice of words, sentences, as well as mathematical symbols quite simple so that the meaning is easily understood by students. Practicality is one measure of the appropriateness of a produkts before it can be used in teaching and learning process ([18]). One of the purposes of tests of the produkts is to obtain information on whether new products that give a more positive impact than the old product. Students’ score test on SLE showed that 83.33% of the students have score above 70 or belongs to the category of effective. This means that student worksheet gives a positive impact towards the enhancement of the learning outcome in SLE. Effectiveness is one measure of the appropriateness of a produkts before it can be used in learning ([18]). Why teaching and learning process with APOS theory approach can improve learning outcomes in SLE? First, the APOS theory embraced constructivism philosophy, students are very enthusiastic in teaching and learning based on constructivism ([20]), so that in the end can improve learning outcomes ([21],[22]). Second, the APOS theory integrated the use of computer, computer in mathematics teaching and learning provides a better educational experience and then be more beneficial for students' achievements ([23]). Third, the APOS theory learning using cooperative learning settings. Cooperative learning group promotes interactive learning experience, enabled learners to receive positive feedback from the process of thinking, enhances students' academic achievement more better ([24]).

4. Conclusion
Before being used in the teaching and learning process, each draft of the learning material must go through the review experts and tested to a limited extent, the goal is to ensure that the learning material is already valid, practical, and effective. Based on the results of this study, it can be concluded that the student worksheets for learning system of linear equations based on APOS theory approach is declared valid, practical and effective.

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