A Spline Technique Solution For Vibrations Analysis Of Rectangular Plate Having Parabollically Varying Thickness Considering The Parameters 'Thermally Induced Non-Homogeneity', Damping And Elastic Foundation

Manu Gupta, Ajendra Kumar, Ankit Kumar

Abstract- A mathematical solution to a governing model plate equation is given here in which we study effect of plate parameters namely thermally induced non –homogeneity, damping and elastic foundation on rectangular plate of parabollically varying thickness. The model is solved using quintic spline interpolation technique. Numerical results which consist of three modes of frequency parameter for the rectangular plate are calculated for two assumed boundary conditions (BC's) viz. CL-SS-CL-SS(clamped - clamped) and CL-SS-SS-SS(clamped –simply supported).Comparisons of results obtained by the quintic spline interpolation technique with those that are available in the literature are also presented. Numeric results are obtained and represented in tabular and graphical form.

Key words-Non-homogeneity, Elastic foundation, Damping, Thermal gradient, spline technique, Taper constant

1. INTRODUCTION

Modern engineering structures consist of structural component which should be made up of strong material. Thus stronger materialsare needed in wide range of engineering and industrial application. The applications includes- (i) Automobile industry: In which sports cars require supported beam, stronger bumper structure, strong air pressure vessels; (ii) Aerospace industry: In which aerospace structure and space vehicle require fairings,spoiler, turbine engine, fan blade, landing gear doors, propellers etc.; (iii) Mechanical and civil engineering industry: which require lifting device, agriculture and foresting machine cranes, packaging machines etc. These requirements are full field by anisotropic material which are generally non-homogenous in nature by physical composition, imperfection in material and by design. Plate of variable thickness provides greater efficiency in vibration problems and also helps in material saving, providing high strength and stiffness enhancing.Elastic foundation has applications in pressure vessel technology, And now a day's it's frequently used in engineering structure technology. In the case of vibration, damping is a natural phenomenon to make problem real to working environment.Design structures require accurate determination of natural frequency and mode shape.In real all the vibrations are subjected to damping ascondition of "free vibrations" are ideal and practicallyimpossible. So we cannot study vibration analysis without considering damping. Crandall discussed on role of damping in the theory of vibration [1].

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A large number of researchers dealtwith the analysis of non-uniformrectangularplate with various parameters[2,3, 4].The problem of evaluating the natural frequencies of rectangular plate with variousthicknesseshavelong been recognized [5]. The correlation between the frequency parameters and various types of temperature fields is very useful as a design tool especially in those mechanical structures in which certain parts of a system have to operate under elevated temperature. Problems of uniform and non-uniform rectangular plates under different type of temperature variation have been considered by Chakraverty and Pradhan with general boundary conditions [6]. It is known fact that in presence of constant thermal gradient,elastic co-efficient of homogeneous material becomes the function of coordinates. In continuationRana and Robin [7] presented a research paper on non-homogeneous rectangular plate of parabollically varying thickness. They observed that nodal lines that are shifted towards the boundary [8].Vibration analysis of modes of frequency parameter of Rectangular Plates (Orthotropic)which rest on elastic foundation considering bi-directional parabollically varying thickness have also been discussed by Gupta et al. [9,10], which has applications in aerospace industry and pressure vessels technology. Sharma S., et al.[11] have analysed the vibration of an orthotropic rectangular plate which rest on Winkler type foundation Detailed work on thermally induced vibration on frequency parameter of orthotropic rectangular plate of varying thickness has been done by Tomar and Gupta [12] where the plate thickness varies in two directions. Recently Analysis of Damping & Non-homogeneity through spline interpolation technique of a Rectangular plate of Parabollically varying thickness resting Winklertype of Elastic foundation was discussed by Gupta M., et al [13].Different numerical techniques such as Frobenius method, Galerkinmethod, Gauss quadrature method, F.E.(Finite element) method and some other techniques have been used by researchers to analyze and study the

modes, behavior and nature of vibration of plates with variousparameters and diversegeometrical shapes. Since each method has its advantage and disadvantage. In our problem quintic spline interpolation technique is applied to solve the model for the above discussed problem, and this method provides us highly accurate results with less computational work with two assumed boundary conditions (BC's) CL-SS-CL-SS(clamped - clamped)and CL-SS-SS-SS(clamped –simply supported).

2. EQUATIONS OF MODEL FORMULATION (MATHEMATICAL MODEL)

In the present model we have consider plate which is an isotropic non-homogeneous rectangular plate of dimension 'a'(length), 'b'(width) and thickness h *where* $h = h(x, y)$ respectively .Plate density is assumed $'\rho'$ & plate rests on an elastic foundation material(Winkler-type)occupying the rectangular area $0 \le x \le a, 0 \le y \le b$ in two dimensional coordinates system. The mathematical equation/model which governs the vibrational behavior of damped plate which rest on elasticfoundation is given by a standard equation,

$$\nabla^2 (D\nabla^2 u) - (1-\upsilon)\left[\frac{\partial^2 D}{\partial y^2}\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 D}{\partial x \partial y}\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 D}{\partial x^2}\frac{\partial^2 u}{\partial y^2}\right] +$$

$$\rho h \frac{\partial^2 u}{\partial t^2} + k\frac{\partial u}{\partial t} + k_f u = 0, \qquad (I)$$

Where $D = Eh^3/12(1-\upsilon^2)$ *and* $h = h(x, y)$ D is called 'flexural rigidity' of the plate at a point in the middle plane, k_d is assumed damping parameter, k_f is elastic foundation parameter and $u = u(x, y, t)$ is the deflection function (transverse deflection). Two plate edgesnamely y=0 and y=b which are opposite to each other are considered asSimply Supported (SS) & thickness varies parabollically in direction of x-axis only. So 'h'does not depend on y $i.e.$ we consider h=h(x). For the harmonic solution the above problem the transverse deflection function $'u = u(x, y)'$ is of the form,

$$u = U(x)\, e^{-\kappa t} \sin\frac{n\pi y}{b}\cos ct \text{ At } y = 0 \text{ and } y = b, \quad (II)$$

Here c is 'circular frequency' for vibration ofplate &n ε N. Eq. (I) on substituting Eq. (II) becomes

$$Eh^3\frac{\partial^4 U}{\partial x^4}\cos pt + \left\{2h^3\frac{\partial E}{\partial x} + 6Eh^2\frac{\partial h}{\partial x}\right\}\frac{\partial^3 U}{\partial x^3}\cos pt$$
$$+\left\{h^3\frac{\partial^2 E}{\partial x^2} + 6h^2\frac{\partial E}{\partial x}\frac{\partial h}{\partial x} + 6Eh\left(\frac{\partial h}{\partial x}\right)^2 + 3Eh^2\frac{\partial^2 h}{\partial x^2}\right.$$
$$\left. -2Eh^3\left(\frac{\pi m}{b}\right)^2\right\}\frac{\partial^2 U}{\partial x^2}\cos pt + \left\{-2h^3\frac{\partial E}{\partial x} - 6Eh^2\right\}\left(\frac{\pi m}{b}\right)^2\frac{\partial U}{\partial x}\cos pt$$
$$+\left[\frac{\pi^4 m^4}{b^4}Eh^3 - \upsilon\frac{\pi^2 m^2}{b^2}\left\{h^3\frac{\partial^2 E}{\partial x^2} + 6h^2\frac{\partial h}{\partial x}\frac{\partial E}{\partial x} + 6Eh\left(\frac{\partial h}{\partial x}\right)^2\right.\right.$$
$$\left.\left. +3Eh^2\frac{\partial^2 h}{\partial x^2}\right\}\right]U\cos pt + 12\left(1-\upsilon^2\right)\left[\left\{\rho h\left(\gamma^2 - p^2\right) - k\gamma\right.\right.$$
$$\left.\left. +k_f\right\}U\cos pt + \left\{2\rho hp\gamma - kp\right\}U\sin pt\right] = 0 \qquad (III)$$

We now introduce the dimensionless variables as,

$$X = \frac{x}{a}, \qquad \overline{E} = \frac{E}{a} \quad , \quad \overline{U} = \frac{U}{a}, \overline{\rho} = \frac{\rho}{a}, H = \frac{h}{a},$$
$$\eta = n^2\pi^2\left(\frac{a}{b}\right)^2$$

And putting physical quantities of interest to our study which are variation in thickness(taper constant)and non-homogeneity, given by

$H = H_0\left(1-\alpha X^2\right)$, $\overline{E} = \overline{E}_0\left(1-\eta^2(1-X)^2\right)$, $\overline{\rho} = \overline{\rho_0}\left(1-\eta^2(1-X)^2\right)$, where α is a taper constant and η is thermal gradient and performing suitable mathematical calculations the following relation is obtained:

$$V_0\frac{\partial^4\vec{U}}{\partial X^4} + V_1\frac{\partial^3\vec{U}}{\partial X^3} + V_2\frac{\partial^2\vec{U}}{\partial X^2} + V_3\frac{\partial\vec{U}}{\partial X} + V_4\overline{U} = 0, \quad (IV)$$

Where

$$V_0 = (1-\alpha X^2)^4(1-\eta^2(1-X)^2)$$
$$V_1 = -2(2\eta^2(1-X)(1-\alpha X^2)^4 + 6\alpha X(1-\alpha X^2)^3(1-\eta^2(1-X)^2))$$
$$V_2 = 24\alpha^2X^2(1-\alpha X^2)^2(1-\eta^2(1-X)^2)$$
$$+24\alpha X(1-\alpha X^2)^3\eta^2(1-X)$$
$$-6\alpha(1-\alpha X^2)^3(1-\eta^2(1-X)^2)$$
$$+2\eta^2(1-\alpha X^2)^4 - 2Y^2(1-\alpha X^2)^4(1-\eta^2(1-X)^2)$$
$$V_3 = 12\alpha XY^2(1-\alpha X)^3(1-\eta^2(1-X)^2)$$
$$-2Y^2(1-\alpha X^2)^4(1-2\eta^2(1-X)$$
$$V_4 = Y^2(1-\alpha X^2)^4(1-\eta^2(1-X)^2 - Y^2\nu(2\eta^2(1-\alpha X^2)^4$$
$$-12\alpha X(1-\alpha X^2)^3(1-2\eta^2(1-X)$$
$$+24\alpha^2X^2(1-\alpha X^2)^2(1-\eta^2(1-X)^2)$$
$$-6\alpha(1-\alpha X^2)^3(1-\eta^2(1-X)^2)$$
$$-(D_k{}^2I^{*2}/(1-\eta^2(1-X)^2$$
$$+E_fC^*(1-\alpha X^2) - \Omega^2(1-\alpha X^2)^2(1-\eta^2(1-X)^2)I^*$$

Here $E_f, d_k, .\Omega$ are foundation parameter, damping parameter and frequency parameter respectively given as-,

$$D_k = \frac{3(1-v^2)k^2}{a^2\overrightarrow{\rho_0}\overrightarrow{E_0}}, \; E_f = \frac{12(1-v^2)k_f}{\overrightarrow{E_0}},$$
$$\Omega^2 = \frac{12(1-v^2)a^2p^2\vec{\rho}}{\overrightarrow{E_0}}$$

In order to solve equation (III) using assumed BC'sat edges X=0 & X=1, we apply the technique of quintic splines. For we suppose U(x) is continuous differentiable function $\forall x \in [0,1]$ and let interval [0, 1] ispartitioned into 'm' sub intervals .For we have $0 = X_0 < X_1 < X_2 < \ldots < X_m = 1$

is required partition, taking

$$\Delta X = \frac{1}{m}, X_r = r\Delta X, r = 0(1)m$$

Let $\vec{U}(X)$ be a quintic spline which an approximating function for U(x) and the condition given below are satisfied,

(i) $\vec{U}(X)$ is quintic polynomial where X $\in$ $(X_{i,}X_{i+1}), \quad \forall i \in 0,1,2,.....m-1$

(ii) $\vec{U}(X) = U(X_r), r = 0,1,2,...,n.$

(iii) The partial derivatives of $\vec{U}(X)$ i.e. $\frac{\partial \vec{U}}{\partial X}, \frac{\partial^2 \vec{U}}{\partial X^2}, \frac{\partial^3 \vec{U}}{\partial X^3} and \frac{\partial^4 \vec{U}}{\partial X^4}$ are continuous.

Considering the above properties, the quintic spline interpolation technique gives

$$\vec{U}(X) = a_0 + \sum_{i=1}^{4} a_i (X - X_0)^i + \sum_{t=0}^{m-1} b_j (X - X_t)_+^5, \quad (V)$$

Where, $(X - X_t)_+ = \begin{cases} 0, & if\ X \le X_t \\ (X - X_t), & if\ X > X_t \end{cases}$ and $a_i's, b_j's$ are coefficients considered constants.

Thus for m^{th} knot, eq. (IV) reduced to

$$V_4 a_0 + \left[V_4(X_j - X_0) + V_3\right] a_1 + \begin{bmatrix} V_4(X_j - X_0)^2 \\ +2V_3(X_j - X_0) + 2V_2 \end{bmatrix} a_2$$
$$+ \left[V_4(X_j - X_0)^3 + 3V_3(X_j - X_0)^2 + 6V_2(X_j - X_0) + 6V_1\right] a_3$$
$$+ \left[V_4(X_j - X_0)^4 + 4V_3(X_j - X_0)^3 + 12V_2(X_j - X_0)^2 + 24V_1(X_j - X_0) + 24V_0\right] a_4$$
$$+ \sum_{t=0}^{m-1} b_j \begin{bmatrix} V_4(X_j - X_t)_+^5 + 5V_3(X_j - X_t)_+^4 + 20V_2(X_j - X_t)_+^3 \\ +60V_1(X_j - X_t)_+^2 + 120V_0(X_j - X_t)_+ \end{bmatrix} = 0, \quad (VI)$$

For $j = 0(1)m,$ (m+1) homogeneous equation with (m+5) unknowns are obtained from above system, $a_i, i = 0(1)4$ and the above system of equations can be represented in matrix equation form as

$$[P][Q] = O(zero\ matrix), \quad (VII)$$

Where $[P]$ matrix is of order $(m+1)\times(m+5)$, while $[Q]$ is column matrix of order m+5 and $O(zero\ matrix)$ is also column matrix of order (m+5).

3. BOUNDARY CONDITIONS (BC's)

The following BC's have been assumed in our problem:

(i) (CL-CL) boundary condition i.e. clamped at edges X=0 & X=1.

(ii) (CL-SS)boundary condition i.e. clamped at X=0 &simply supported at edge X=1.

These B.C.'s have the following relations to be satisfied

$$U = \frac{dU}{dX} = 0, \ (CL - CL\ Position)$$

$$U = \frac{d^2U}{dX^2} = 0, (CL - SS\ Position), \quad (VIII)$$

Applying the BC's(CL-CL) to the deflection function given by(eq. V) we obtain a system of four homogeneous eqns.in (m+5) unknown,

$[A^{cc}][Q] = O(zero\ matrix)$ where A^{cc} is a matrix having order 4× (m+5). (IX)

Therefore (VII) together with (IX) provide usa system of (m+5) eqns. which arehomogeneous in (m+5) unknowns. The complete system is represented in matrix form as,

$$\begin{bmatrix} P \\ A^{cc} \end{bmatrix} [Q] = O(zero\ matrix) \ (X)$$

For a non-zero results of (eq. X), the corresponding determinantobtain mustbe zeroi.e.

$$\left|\frac{P}{A^{cc}}\right| = 0 \ (XI)$$

Again applying B.C.'s for (CL-SS) the required determinant is obtained as,

$\left|\frac{P}{A^{ss}}\right| = 0.$ Where A^{ss} is a matrix of order $4\times(m+5), \quad (XII)$

4. RESULTS AND DISCUSSION

On solving the frequency equation XI and XII they provide us the numerical results for frequency parameter (Ω) considering variation in differentcombinations of plate parameters. Here in this model solution first 3 modes of vibration are obtained forthe two assumed boundary conditions (BC's). The programming code is developed using MATLAB. Since method is approximate, therefore to minimize the error optimum size of interval length ΔX is determined. For present problem $\Delta X = \frac{1}{m}$, where $m = 10(10)130$, for $m \ge 130,$ there is no improvement in results.The results obtained are given by table (I) and depicted in fig. 1(a) & 1(b) for the following conditions; $h = 0.03,\ v = 0.3,\ n = 1,\ \eta = 0.4,\ \alpha = 0.4,\ d_k = 0.01,$ $E_f = 0.01$ and $a/b = 0.25$ for both boundary conditions. The value of different plate parameters are considered as (i)taper

constant (α) =0.0(0.1)0.4,(ii)foundation paramete $k_f = 0.0(0.005)0.02,$ (iii)non-homogeneity parameter η=0.0(0.1)0.4and damping k_d =0.0(.0025)0.01respectively.Also calculating the results for different variation in parameters we have considered: $\nu = 0.3, \frac{a}{b} = 0.25, plate\ thickness\ h = 0.03 \& n = 1$.

Table1.*Convergence of Ω for Number of increasing nodes for CL-CL combination and CL-SS combination plates for* $n = 1, = 0.4, \alpha = 0.4, \eta = 0.4, k_d = 0.01, k_f = 0.01, a/b = 0.25, h = 0.03, \nu = 0.3,$

values of M	CL-CL combination in plate Modes			CL-SS combination in plate Modes		
10	.7805	1.7209	3.3022	.6351	1.4038	2.8368
20	.7788	1.6991	3.2074	.6342	1.3868	2.7539
30	.7785	1.6951	3.1898	.6341	1.3837	2.7389
40	.7784	1.6937	3.1837	.634	1.3826	2.7338
50	.7783	1.693	3.1808	.634	1.3821	2.7314
60	.7783	1.6927	3.1793	.634	1.3819	2.7301
70	.7783	1.6924	3.1783	.634	1.3817	2.7293
80	.7782	1.6923	3.1777	.634	1.3816	2.7288
90	.7782	1.6922	3.1773	.634	1.3815	2.7284
100	.7782	1.6921	3.177	.634	1.3815	2.7282
110	.7782	1.6921	3.1768	.634	1.3814	2.728
120	.7782	1.6921	3.1766	.634	1.3814	2.7278
130	.7782	1.692	3.1765	.634	1.3814	2.7277
140	.7782	1.692	3.1765	.634	1.3814	2.7276

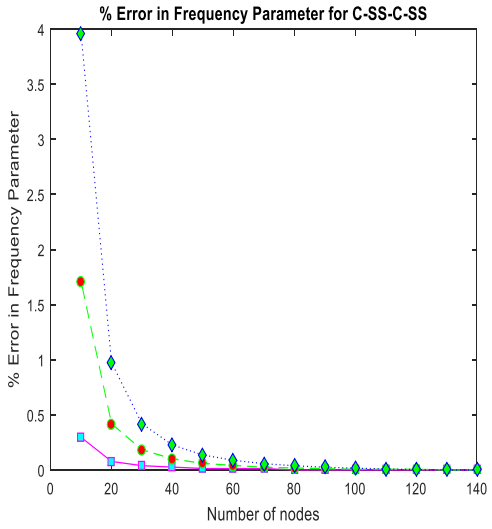

Fig.1(a)

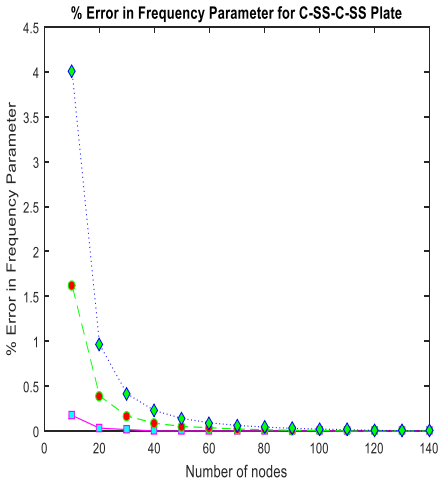

Fig.1(b)

Fig.1 *(a) & (b) % error in Ω for (i) CL-CL combination (ii) CL-SS combination, for α=0.4,* η *=0.4,* $k_f = 0.01, k_d = 0.01$*: □first mode; ○ second mode; ◊ third mode.*

Table 1 express convergence of different modes of frequency parameters Ω with increment in number of nodes for particular parameters combinations given as: α=0.4, $k_f = 0.01$, η =0.04, $k_d = 0.01$ for this a MATLAB coding was developed & calculation were completed for $m = 10(10)140.$ It was further observed that Ω for the 3 modes converges as number of nodes m increases; i.e. convergence of Ω as number of nodes increases is monotonic for both plate combinations. % error in Ω for first 3 modes of vibrations of plates is depicted in fig. 1(a) &1(b). It is calculated and find out that there is no improvement in results if $m \geq 130$ and an error in Ω is greater for clamped (CL-CL) plate than in simply supported (CL-SS) plate, taking different combinations of values of the other plate parameters. Also Verification of work is obtained by allowing varying parameters to be zero, then the problem reduced to the problem of a freely vibrating homogeneous rectangular plate of parabollically varying thickness and results of our problem are then very well compared with the results obtained by Jain and Soni [14] and if values for

parameters are not zero then results compares well with Gupta M. [15], who have studied this combination with Frobenious method.

Table 2(a) and 2(b) shows the numerical results of Ω for different value of taper parameters (α) for non-damped ($k_d = 0.0$) & damped ($k_d = 0.01$) for two BC's CL-CL position and CL-SS position. The results obtained are depicted by fig. 2(a), 2(b) and 2(c) assuming fixed values of non- homogeneity parameter (η) and foundation parameters (F_P) for first 3 mode of vibration of plate combinations (i.e. CL-CL combination and CL-SS combination). Fig. 2(a) represents variation in Ω with represent to taper constant for 1st mode of Ω. We find that the value of Ω decreases as the value of α increase but introduction of foundation parameter $k_f = 0.01$ increase the value of Ω for CL-SS combinations as value of taper constant increase. Fig. 2(b) represents that value of frequency parameters Ω decreases for second mode as the value of taper constant increases. The same is the nature depicted by fig. 2(c) for third mode of Ω when taper constant increases but the changes in C-C plate combination for the combination $k_d = 0.0,\ \eta = 0.4,\ k_f = 0.0$ and $k_d = 0.0,\ \eta = 0.4,\ k_f = 0.01$ when taper constant is increases irregular.

Table 3(a) and 3(b) provided the inference that numerical value of Ω increased with the increase for different values of foundation parameters for non-damped and damped plates. Considering two different value of η and α also from fig. 3(a) observed that there is parabollically increment in frequency parameter for first mode of Ω. From fig. 3(b) we observe that the increment in frequency parameters Ω with respect to increasing foundation parameter is linear. The same is the result for third modes of Ω as for the second mode which is given by fig. 3(c) but the linear change is less in third mode then in second mode.

Table 4(a) and 4(b) provide the inference that the numerical value of Ω decreases when there is increases in the damping parameter for homogeneous and non-homogeneous plate considering two different values α and FP. From fig. 4(a) we observed that there is parabollically decrement in frequency parameters for first mode of Ω. From fig. 4(b) and 4(c) we observed that the value of frequency parameters Ω decreases linearly with increase in the value of damping parameter considering two different combination of α& F_P for homogeneous and non-homogeneous plates. Also it is observed that the linear change in second mode is much greater then third mode of frequency parameters.

Table 5(a) and 5(b) provides the values of Ω as there is increase in values of non-homogeneity parameter Ƞ for $k_f = 0.0$ and $k_f = 0.01$ respectively for both BC's. The results obtained in these tables are plotted in fig. (5a), (5b) and (5c) assuming the constant value of damping parameter (k_d) and taper constant (α) for 3 modes of vibrations of CL-CL combination & C-SS combination. The fig. (5a), (5b) and (5c) represent the behavior of frequency parameters Ω and it decreases as there is increases in value of η for different value, taking $\alpha = 0.0$, 0.4 and $k_d = 0.0,\ 0.01$ & considering k_f as 0.0 and 0.01. Thus we observed that there is linear relationship between Ω and Ƞ for the 3 modes of CL-CL & CL-SS Plates, also the linear change is very small.

Table 2(a): *Value of Ω considering different value of α, when h=0.03, v=0.3, n=1,* $k_d = 0.0,\ a/b = 0.25$

		$\eta = 0.0,\ k_f = 0.0$		$\eta = 0.4,\ k_f = 0.0$		$\eta = 0.0,\ k_f = 0.01$		$\eta = 0.4,\ k_f = 0.01$	
	Mode	CL-CL	SS	CL-CL	SS	CL-CL	SS	CL-CL	SS
	1	0.6778	0.4714	0.8904	0.7454	0.6751	0.4597	0.8969	0.7511
α=0.0	2	1.859	1.509	1.9466	1.6157	1.8553	1.4686	1.9476	1.617
	3	3.6374	3.1385	3.6829	3.1912	3.5377	3.0529	3.5845	3.1896
	1	0.6632	0.4599	0.8848	0.7465	0.6456	0.4494	0.8911	0.7528
α=0.1	2	1.8074	1.4645	1.9003	1.578	1.7585	1.4259	1.9007	1.5796
	3	3.5264	3.0394	3.5749	3.0956	3.5216	2.9572	3.573	3.094
	1	0.6469	0.4451	0.8789	0.748	0.6438	0.4359	0.8852	0.754
α=0.2	2	1.752	1.415	1.8506	1.5367	1.7471	1.3783	1.8061	1.5355
	3	3.4082	2.9326	3.4603	2.9932	3.3162	2.8542	3.3697	2.9916
	1	0.6286	0.4263	0.8727	0.7482	0.6257	0.4182	0.8793	0.7548
α=0.3	2	1.6908	1.3592	1.7973	1.491	1.6863	1.3244	1.7978	1.4931
	3	3.2814	2.8165	3.3378	2.8824	3.276	2.7419	3.3353	2.8095
	1	0.6082	0.4021	0.867	0.7485	0.6057	0.4011	0.8735	0.7556
α=0.4	2	1.624	1.2932	1.7397	1.44	1.6195	1.2914	1.7405	1.4423
	3	3.144	2.6884	3.2058	2.7611	3.1384	2.6833	3.2032	2.7594

Table 2(b): *Value of Ω considering different value of α,when h=0.03, v=0.3, n=1,* $k_d = 0.01,\ a/b = 0.25$

		$\eta = 0.0,\ k_f = 0.0$		$\eta = 0.4,\ k_f = 0.0$		$\eta = 0.0,\ k_f = 0.01$		$\eta = 0.4,\ k_f = 0.01$	
	Mode	CL-CL	SS	CL-CL	SS	CL-CL	SS	CL-CL	SS
	1	0.5901	0.3333	0.8256	0.6667	0.578	0.3156	0.8263	0.667
α=0.0	2	1.8289	1.4718	1.9179	1.581	1.8218	1.4634	1.9157	1.5785
	3	3.6221	3.1206	3.6678	3.1737	3.6162	3.1142	3.6645	3.1702
	1	0.5677	0.3031	0.8156	0.6616	0.5544	0.2846	0.8159	0.6624
α=0.1	2	1.7743	1.4233	1.8688	1.5398	1.7665	1.415	1.866	1.5375
	3	3.5095	3.0197	3.5583	3.0763	3.5029	3.0129	3.5543	3.0726
	1	0.5414	0.2607	0.8044	0.6546	0.5273	0.2405	0.8045	0.6557
α=0.2	2	1.7148	1.3685	1.8155	1.494	1.7065	1.3602	1.8128	1.4919
	3	3.3893	2.9104	3.4417	2.9715	3.3822	2.9034	3.4373	2.9677
	1	0.5103	0.1953	0.7918	0.6451	0.4956	0.1691	0.7921	0.6465
α=0.3	2	1.6493	1.3058	1.7583	1.4424	1.6407	1.2972	1.7552	1.4404
	3	3.2599	2.791	3.317	2.8575	3.2524	2.7837	3.3121	2.8535
	1	0.4726	0.0021	0.7778	0.6325	0.4573	.0012	0.7782	0.634
α=0.4	2	1.576	1.2323	1.695	1.3883	1.5676	1.2233	1.692	1.3814
	3	3.1189	2.6585	3.1812	2.7319	3.1112	2.6508	3.1765	2.7277

Table3 (a): *Value of Ω considering different value of* k_f *, when h=0.03, v=0.3, n=1,* $k_d = 0.0,\ a/b = 0.25$

		$\eta = 0.0,\ \alpha = 0.0$		$\eta = 0.4,\ \alpha = 0.0$		$\eta = 0.0,\ \alpha = 0.4$		$\eta = 0.4,\ \alpha = 0.4$	
	Mode	CL-CL	SS	CL-CL	SS	CL-CL	SS	CL-CL	SS
	1	0.6778	0.4714	0.6751	0.4679	0.6082	0.4021	0.6057	0.4011
$k_f = 0.0$	2	1.859	1.509	1.8553	1.5048	1.624	1.2952	1.6195	1.2914
	3	3.6374	3.1385	3.6333	3.134	3.144	2.6884	3.1384	2.6833
	1	0.7912	0.6236	0.7938	0.6258	0.7487	0.601	0.7517	0.605
$k_f = 0.005$	2	1.9033	1.5633	1.902	1.5619	1.6829	1.3694	1.6811	1.3689
	3	3.6602	3.165	3.6574	3.1619	3.175	2.725	3.171	2.7216
	1	0.8904	0.7454	0.8969	0.7511	0.8666	0.7485	0.8735	0.7556
$k_f = 0.01$	2	1.9466	1.6157	1.9476	1.617	1.7397	1.44	1.7405	1.4423
	3	3.6829	3.1912	3.6814	3.1896	3.2058	2.7611	3.2032	2.7594
	1	0.9795	0.8499	0.9893	0.8584	0.9702	0.8711	0.9803	0.8806
$k_f = 0.015$	2	1.989	1.6665	1.9921	1.6702	1.7949	1.5073	1.7979	1.5122
	3	3.7055	3.2172	3.7052	3.2171	3.2363	2.7968	3.2351	2.7966
	1	1.0612	0.9428	1.0738	1.0738	1.0637	0.9783	1.0765	0.9899
$k_f = 0.02$	2	2.0304	1.7158	2.0357	1.7217	1.8484	1.5718	1.8536	1.579
	3	3.7279	3.243	3.7289	3.2443	3.2665	2.832	3.2667	2.9334

Table3 (b): *Value of Ω consideringdifferent value of* k_f *, whenh=0.03, v=0.3, n=1,* $k_d = 0.01$, $a/b = 0.25$

		$\eta = 0.0,\ k_f = 0.0$		$\eta = 0.4,\ k_f = 0.0$		$\eta = 0.0,\ k_f = 0.01$		$\eta = 0.4,\ k_f = 0.01$	
	Mode	CL-CL	SS	CL-CL	SS	CL-CL	SS	CL-CL	SS
$k_f = 0.0$	1	0.5901	0.3333	0.578	0.3156	0.4726	0.0021	0.4573	0.0012
	2	1.8289	1.4718	1.8218	1.4634	1.576	1.2323	1.5674	1.2233
	3	3.6221	3.1208	3.6162	3.1142	3.1189	2.6585	3.1112	2.6508
$k_f = 0.005$	1	0.7176	0.5271	0.713	0.5218	0.6436	0.4475	0.6383	0.4434
	2	1.8739	1.5273	1.8693	1.5221	1.6366	1.3099	1.6309	1.3047
	3	3.645	3.1474	3.6404	3.1423	3.1502	2.6954	3.144	2.6895
$k_f = 0.01$	1	0.8256	0.6667	0.8263	0.667	0.7778	0.6325	0.7782	0.634
	2	1.9179	1.581	1.9157	1.5785	1.695	1.3833	1.692	1.3814
	3	3.6678	3.1737	3.6645	3.1702	3.1812	2.7319	3.1765	2.7277
$k_f = 0.015$	1	0.921	0.7818	0.9258	0.7858	0.8919	0.7742	0.8965	0.7791
	2	1.9608	1.6328	1.9609	1.633	1.7515	1.4531	1.751	1.4541
	3	3.6905	3.1999	3.6884	3.1978	3.2119	2.7679	3.2087	2.7654
$k_f = 0.02$	1	1.0075	0.8819	1.0156	0.8889	0.9929	0.8935	1.0009	0.901
	2	2.0029	1.6831	2.0052	1.6858	1.8062	1.5198	1.8082	1.5234
	3	3.713	3.2258	3.7122	3.2251	3.2424	2.8035	3.2405	2.8026

Table4 (a): *Value of Ω considering different value of* $k_{d,}$ *when h=0.03, v=0.3, n=1,* $\eta = 0.0,\ a/b = 0.25$

		$k_f = 0.0\ ,\ \alpha = 0.0$		$k_f = 0.01\ ,\ \alpha = 0.0$		$k_f = 0.0\ ,\ \alpha = 0.4$		$k_f = 0.01\ ,\ \alpha = 0.4$	
	Mode	CL-CL	SS	CL-CL	SS	CL-CL	SS	CL-CL	SS
	1	0.6778	0.4714	0.8904	0.7454	0.6082	0.4021	0.8666	0.7485
k_d=0.0	2	1.859	1.509	1.9466	1.6157	1.624	1.2952	1.7397	1.44
	3	3.6374	3.1385	3.6829	3.1912	3.144	2.6884	3.2058	2.7611
	1	0.6726	0.464	0.8864	0.7407	0.6006	0.3893	0.8613	0.7418
k_d=0.0025	2	1.8572	1.5067	1.9448	1.6136	1.621	1.2913	1.737	1.4365
	3	3.6364	3.1374	3.682	3.1901	3.1424	2.6866	3.2043	2.7593
	1	0.657	0.441	0.8746	0.7265	0.5773	0.3484	0.8453	0.7213
k_d=0.005	2	1.8516	1.4998	1.9395	1.6071	1.6121	1.2797	1.7287	1.426
	3	3.6336	3.1341	3.6792	3.1868	3.1377	2.681	3.1997	2.7539
	1	0.63	0.3997	0.8545	0.7022	0.5362	0.2662	0.8179	0.6858
k_d=0.0075	2	1.8421	1.4882	1.9305	1.5963	1.5972	1.2601	1.7147	1.4084
	3	3.6288	3.1286	3.6744	3.1814	3.1299	2.6716	3.192	2.7447
	1	0.5901	0.3333	0.8256	0.6667	0.4726	0.0021	0.7778	0.6325
k_d=0.01	2	1.8289	1.4718	1.9179	1.581	1.576	1.2323	1.695	1.3833
	3	3.6221	3.1208	3.6678	3.1737	3.1189	2.6585	3.1812	2.7319

***Table 4(b)**: Value of Ω considering different value of k_d, when h=0.03, v=0.3, n=1, $\eta = 0.4$, $a/b = 0.25$.*

		$k_f = 0.0$, $\alpha = 0.0$		$k_f = 0.01$, $\alpha = 0.0$		$k_f = 0.0$, $\alpha = 0.4$		$k_f = 0.01$, $\alpha = 0.4$	
	Mode	CL-CL	SS	CL-CL	SS	CL-CL	SS	CL-CL	SS
	1	0.6751	0.4679	0.8969	0.7511	0.6057	0.4011	0.8735	0.7556
k_d=0.00	2	1.8553	1.5048	1.9476	1.617	1.6195	1.2924	1.7405	1.4423
	3	3.6333	3.134	3.6814	3.1896	3.1384	2.6833	3.2032	2.7594
	1	0.6694	0.4599	0.8927	0.7461	0.5975	0.3877	0.8679	0.7486
k_d=0.0025	2	1.8532	1.5022	1.9456	1.6146	1.6163	1.2873	1.7375	1.4385
	3	3.6322	3.1328	3.6803	3.1884	3.1367	2.6813	3.2015	2.7574
	1	0.6522	0.4348	0.8798	0.731	0.5722	0.3442	0.8507	0.7271
k_d=0.005	2	1.847	1.4946	1.9396	1.6074	1.6066	1.2747	1.7285	1.4273
	3	3.629	3.1291	3.6772	3.1848	3.1316	2.6752	3.1965	2.7515
	1	0.6232	0.3896	0.8579	0.7051	0.5274	0.2557	0.7904	0.6899
k_d=0.0075	2	1.8365	1.4817	1.9297	1.5955	1.5904	1.2536	1.6988	1.4083
	3	3.6237	3.1229	3.6719	3.1787	3.1231	2.665	3.1804	2.7416
	1	0.578	0.3156	0.8263	0.667	0.4573	0.0012	0.7117	0.634
k_d=0.01	2	1.8218	1.4634	1.9197	1.5785	1.5674	1.2233	1.662	1.3824
	3	3.6162	3.1142	3.6645	3.1702	3.1112	2.6508	3.1603	2.7277

***Table 5(a)**: Value of Ω considering different value of η, when h=0.03, v=0.3, n=1, $k_f = 0.0$, a/b=0.25*

		$k_d = 0.0$, $\alpha = 0.0$		$k_d = 0.0$, $\alpha = 0.4$		$k_d = 0.01$, $\alpha = 0.0$		$k_d = 0.01$, $\alpha = 0.4$	
	Mode	CL-CL	SS	CL-CL	SS	CL-CL	SS	CL-CL	SS
$\eta = 0.0$	1	0.6778	0.4714	0.6082	0.4021	0.5901	0.3333	0.4726	0.0021
	2	1.859	1.509	1.624	1.2932	1.8289	1.4718	1.576	1.2323
	3	3.6374	3.1385	3.144	2.6884	3.6221	3.1206	3.1189	2.6585
$\eta = 0.1$	1	0.6776	0.4712	0.608	0.402	0.5894	0.3323	0.4717	1.2318
	2	1.8588	1.5088	1.6237	1.295	1.8285	1.4713	1.5755	2.658
	3	3.6372	3.1383	3.1436	2.688	3.6218	3.1204	3.1184	4.5682
$\eta = 0.2$	1	0.6771	0.4706	0.6076	0.4018	0.5873	0.3292	0.469	1.2301
	2	1.8581	1.508	1.6229	1.2943	1.8272	1.4698	1.574	2.6566
	3	3.6364	3.1374	3.1426	2.6872	3.6207	3.1199	3.1171	4.5669
$\eta = 0.3$	1	0.6763	0.4695	0.6068	0.4015	0.5836	0.3238	0.4643	1.2274
	2	1.857	1.5067	1.6215	1.2931	1.825	1.4672	1.5713	2.6543
	3	3.6351	3.1361	3.1409	2.6856	3.6189	3.118	3.1147	4.5646
$\eta = 0.4$	1	0.6751	0.4679	0.6057	0.4011	0.578	0.3156	0.4573	0.0818
	2	1.8553	1.5048	1.6195	1.2924	1.8218	1.4634	1.5674	1.2233
	3	3.6333	3.134	3.1384	2.6833	3.6162	3.1142	3.1112	2.6508

.

***Table 5(b)**: Value of Ω considering different value of η, when h=0.03, v=0.3, n=1, $k_f = 0.01$, $a/b = 0.25$,*

		$k_d = 0.0,\ \alpha = 0.0$		$k_d = 0.0,\ \alpha = 0.4$		$k_d = 0.01,\ \alpha = 0.0$		$k_d = 0.01,\ \alpha = 0.4$	
	Mode	CL-CL	SS	CL-CL	SS	CL-CL	SS	CL-CL	SS
$\eta = 0.0$	1	0.8904	0.7454	0.8666	0.7485	0.8256	0.6667	0.7778	0.6325
	2	1.9466	1.6157	1.7397	1.44	1.9179	1.581	1.695	1.3833
	3	3.6829	3.1912	3.2058	2.7611	3.6678	3.1737	3.1812	2.7319
$\eta = 0.1$	1	0.8907	0.7457	0.867	0.7489	0.8256	0.6667	0.7778	0.6326
	2	1.9467	1.6158	1.7398	1.4401	1.9177	1.5808	1.6958	1.3832
	3	3.6828	3.1911	3.2056	2.761	3.666	3.1735	3.1809	2.7317
$\eta = 0.2$	1	0.8919	0.7468	0.8682	0.7502	0.8258	0.6668	0.7779	0.6329
	2	1.9468	1.616	1.7399	1.441	1.9174	1.5804	1.6943	1.3829
	3	3.6825	3.1908	3.2051	2.7607	3.667	3.1729	3.1801	2.7309
$\eta = 0.3$	1	0.8939	0.7485	0.8704	0.7524	0.826	0.6669	0.7781	0.6333
	2	1.9471	1.6163	1.7401	1.4412	1.9167	1.5796	1.6934	1.3823
	3	3.6821	3.1903	3.2043	2.7601	3.666	3.1718	3.1786	2.7296
$\eta = 0.4$	1	0.8969	0.7511	0.8735	0.7556	0.8263	0.667	0.7117	0.634
	2	1.9476	1.617	1.7405	1.4423	1.9197	1.5785	1.662	1.3824
	3	3.6814	3.1896	3.2032	2.7594	3.6645	3.1702	3.1603	2.7277

*Fig.2: Graphic representation of Ω considering CL-CL and CL-SS combinations in plates: (2a) I-mode (2b) II-mode (2c) III-mode, for a/b=0.25. —, C-C, ---, C-SS, *,* $k_d = 0.0,\ k_f = 0.0,\ \eta = 0.0$; ◊, $k_d = 0.0,\ k_f = 0.01,\ \eta = 0.0$; v, $k_d = 0.0,\ k_f = 0.0,\ \eta = 0.4$; ∧, $k_d = 0.0,\ k_f = 0.01,\ \eta = 0.4$; ⌂, $k_d = 0.01,\ k_f = 0.0,\ \eta = 0.0$; □, $k_d = 0.01,\ k_f = 0.01,\ \eta = 0.0$; ○, $k_d = 0.01,\ k_f = 0.0,\ \eta = 0.4$; □, $k_d = 0.01,\ k_f = 0.01,\ \eta = 0.4$*

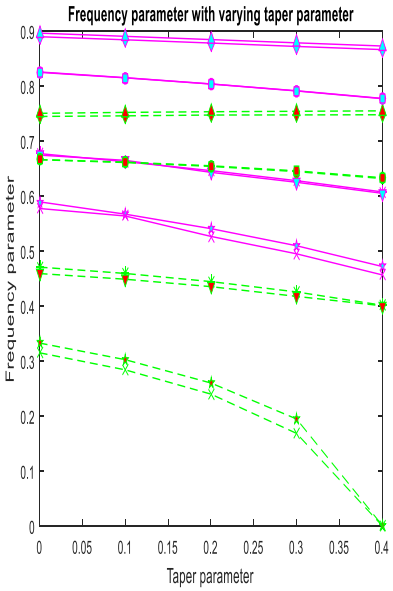

Fig.(2a)

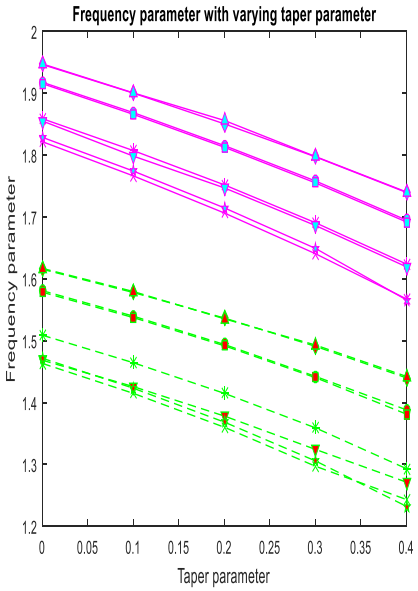

Fig.(2b)

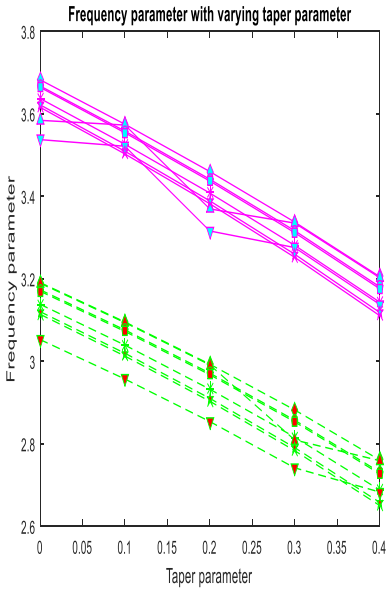

Fig. (2c)

Fig.3: Graphic representation of Ω considering CL-CL and CL-SS combination of plates: (3a) I-mode (3b) II-mode (3c) III-mode, $a/b=0.25$ *–, C-C, ---, C-SS, *,* $k_d=0.0,\ \alpha=0.0,\ \eta=0.0$; ◊, $k_d=0.0,\ \alpha=0.0,\ \eta=0.4$; v, $k_d=0.0,\ \alpha=0.4,\ \eta=0.0$; ∧, $k_d=0.0,\ \alpha=0.4,\ \eta=0.4$; ⌂, $k_d=0.01,\ \alpha=0.0,\ \eta=0.0$; □, $k_d=0.01,\ \alpha=0.0,\ \eta=0.4$; ○, $k_d=0.01,\ \alpha=0.4,\ \eta=0.0$; □, $\alpha=0.4,\ k_d=0.01,\ \eta=0.4$

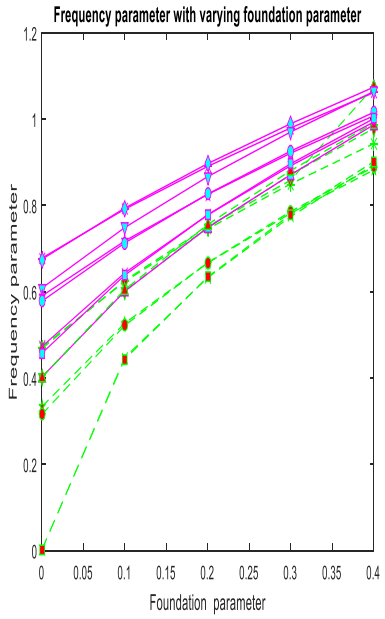

Fig.(3a)

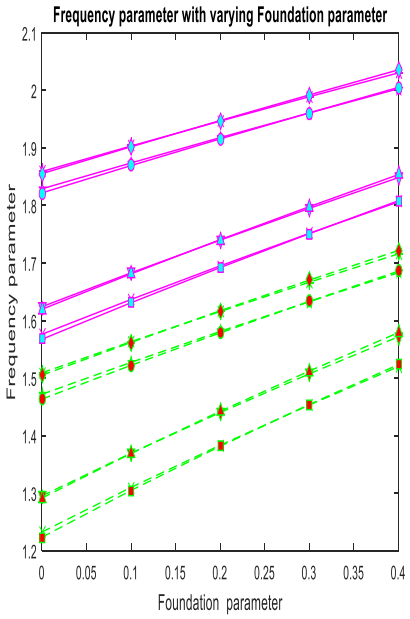

Fig.(3b)

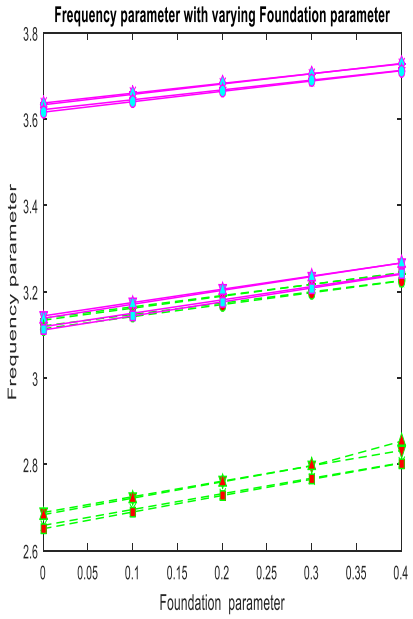

Fig. (3c)

Fig.4: Graphic representation of Ω considering CL-CL and CL-SS combinations inplates: (4a) I- mode (4b) II- mode (4c) III-mode, $a/b=0.25$ *–, CL-CL, ---, CL-SS, *,* $k_d=0.0,\ k_f=0.0,\ \alpha=0.0$; ◊, $k_d=0.0,\ k_f=0.01,\ \alpha=0.0$; v, $k_d=0.0,\ k_f=0.0,\ \alpha=0.4$; ∧, $k_d=0.0,\ k_f=0.01,\ \alpha=0.4$; ⌂, $k_d=0.01,\ k_f=0.0,\ \alpha=0.0$; □, $k_d=0.01,\ k_f=0.01,\ \alpha=0.0$; ○, $k_d=0.01,\ k_f=0.0,\ \alpha=0.4$; □, $k_d=0.01,\ k_f=0.01,\ \alpha=0.4$

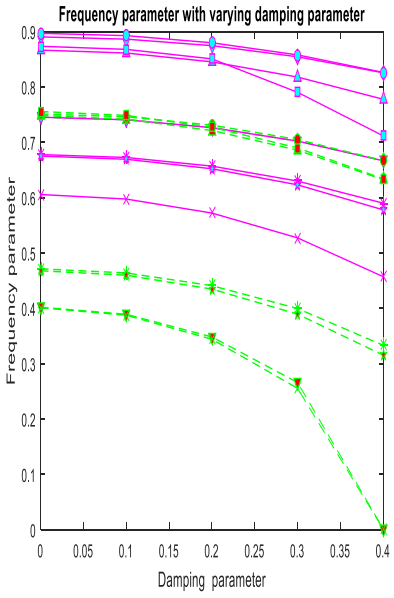

Fig.(4a)

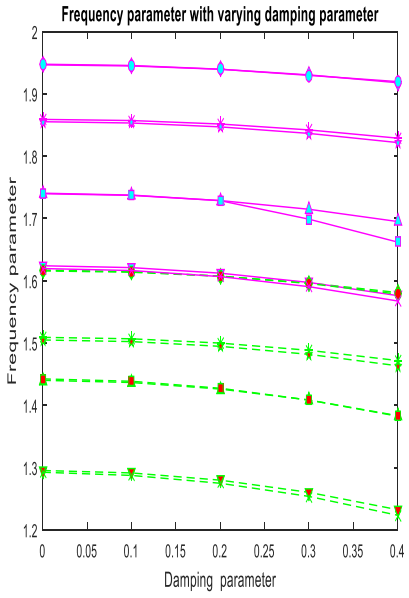

Fig.(4b)

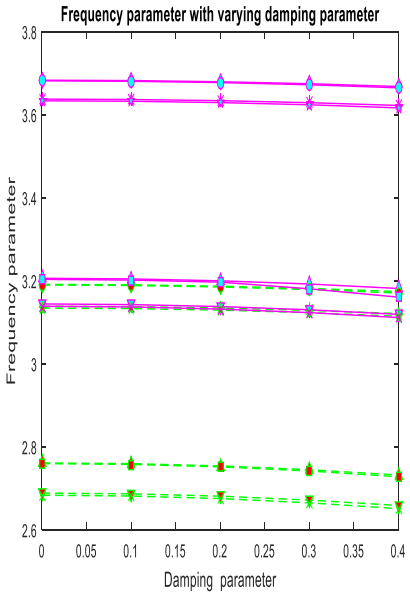

Fig. (4c)

Fig.5: Graphicrepresentation of Ω considering CL-CL and CL-SS combinations in plates: (5a) I- mode (5b) II- mode (5c) III- mode, for $a/b = 0.25$. *–, CL-CL plates , ---, CL-SS, *,* $k_d = 0.0,\ k_f = 0.0, \alpha = 0.0$; ◊, $k_d = 0.0, k_f = 0.01, \alpha = 0.0$; v, $k_d = 0.0, k_f = 0.0, \alpha = 0.4$; ∧, $k_d = 0.0, k_f = 0.01, \alpha = 0.4$; ⌂, $k_d = 0.01,\ k_f = 0.0, \alpha = 0.0$; □, $k_d = 0.01, k_f = 0.01, \alpha = 0.0$; ○, $k_d = 0.01,\ k_f = 0.0, \alpha = 0.4$; □, $k_d = 0.01,\ k_f = 0.01, \alpha = 0.4$

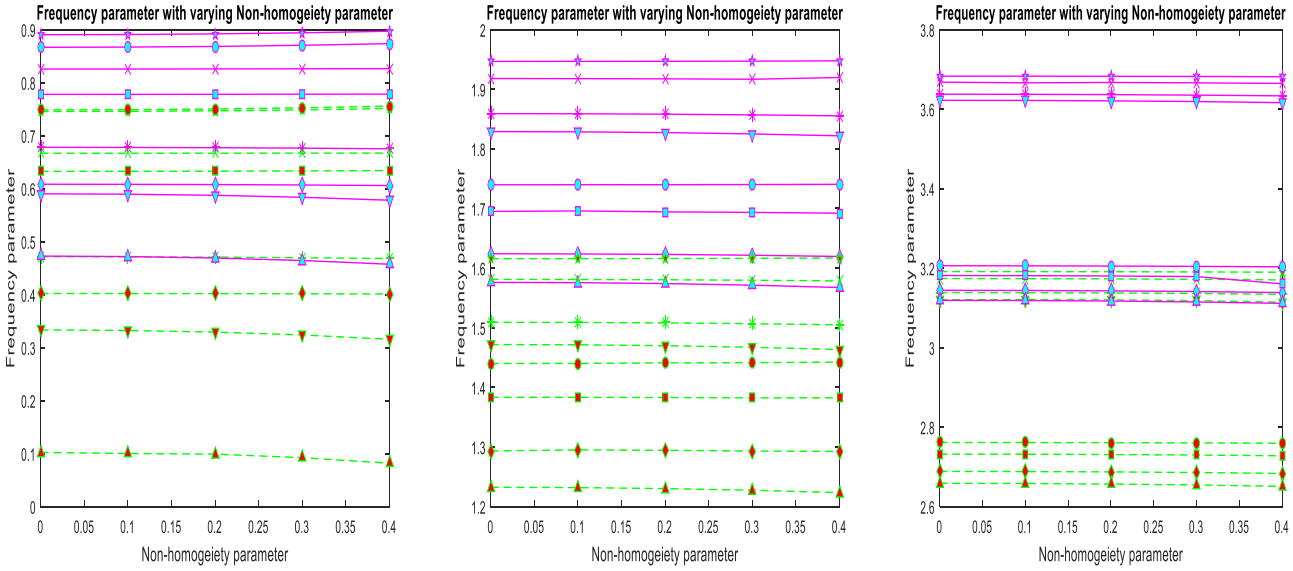

Fig.(5a) *Fig.(5b)* *Fig. (5c)*

5. CONCLUSION

For the computation of results for present problem MATLAB is used within allowed range of plate parameters as per desired accuracy $\left(10^{-8}\right)$, it helps in validation of actual vibrational problem. The present model results deals with the effect of damping on rectangular plate considering varying thickness, thermally induced non-homogeneity and plate which rest on wrinkle type elastic foundation material using the method quintic spline interpolation technique. Determination of natural frequency of plates is of great importance in industry because plate failure can results in loss of money and life. Thus assessment of failure life of a plate structure with effect of different parameters is of most importance. Calculation of natural frequency considering the effect of different parameters helps in assessment of design of plate to engineering who can modified plate structures as per the structural need to prevent cyclic fatigue and complete failure. Thus the present model results will be greatly useful for design to civil and mechanical engineers in various industries to obtain desired frequency by controlling different parameters which in turnalso helps in robustness of the design structures.

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