Axial Plate Buckling Coefficient Of Non-Prismatic Steel Plates

Mohamed Mostafa Ibrahim, Ihab Mohamed El Aghoury, Sherif Abdel-Basset Ibrahim

Abstract: An extensive finite element analysis study was conducted to estimate the critical axial buckling coefficient for non-prismatic steel plates simply supported on four sides. The studied geometric parameters are: the tapering ratio of the plate and the plate’s normalized length. A finite element model was constructed utilizing four-noded shell elements with six degrees of freedom at each node to perform the parametric study. A discretization study was performed to determine the suitable mesh size. Linear buckling analysis was performed to determine the elastic critical buckling load for each model. The plate tapering increases its resistance to buckle due to the stiffness provided by the smaller plate width zone to the larger width zone. The coefficient of critical axial buckling was calculated from the critical load. New formulas are proposed for the non-prismatic plate axial buckling coefficient which satisfies the classical value for rectangular plate.

Index Terms: Axial buckling, Non-prismatic plate, Plate Buckling, Tapered plates.

1 INTRODUCTION

Historically, both experimental and theoretical research about local instabilities of steel plates focused on prismatic, rectangular, steel plates. A very little attention was given to non-prismatic, tapered, steel plates. Non-prismatic steel plates are usually used in web plates of steel beam-columns and girders to optimize the material usage by using larger web depths at higher moments. The AISC “AISC Design Guide 25-Frame design using web tapered members” [1] is dedicated to the design of non-prismatic steel members. This design guide uses the same approach of the AISC 2010 specifications [2] for prismatic members assuming the axial plate buckling coefficient of rectangular plates. The EN 1993-1-5 Eurocode 3: Design of steel structures - Part 1-5: General rules - Plated structural elements [3] allows the use of the rules of rectangular panels with non-rectangular panels of an inclination angle not greater than 10 degrees. Otherwise, panels can be evaluated assuming it to be prismatic based on the larger panel width or using finite element analysis methods. Despite that non-prismatic steel members have been used for long time, the available design practices utilize the coefficient of plate buckling for prismatic plates for non-prismatic plates. This is due to the absence of enough research data. Herein, this study targets to utilize the finite element method to numerically evaluate the elastic axial plate buckling coefficient for non-prismatic steel plates. This is the subject of an on-going research program that explores the validity, both numerically and experimentally, of design codes’ equations applicable to web-tapered members.

2 FINITE ELEMENT MODEL

2.1 General

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- Sherif Abdel-Basset Ibrahim: Professor of Steel Structures and Bridges - Department of Structural Engineering - Ain Shams

\[ \text{Material with Young's modulus of elasticity } E = 200 \text{ GPa.} \]

2.2 Material Model

The performed eigenvalue buckling analysis is a liner analysis type. Therefore, the material is defined as an elastic linear material with Young’s modulus of elasticity \( E = 200 \text{ GPa.} \)

2.3 Boundary Conditions

All plate edges are considered as simply supported. All sides are restrained against out-of-plane and in-plane horizontal translations. Only one side is prevented from in-plane vertical translation. Rotations are unrestrained on all edges. A unit load is applied uniformly through a multi-point constraint on the top edge of the plate.

2.4 Finite element model mesh size refinement

Fig. 2 shows a convergence study performed for axially loaded panel as shown in Fig. 3. The number of elements in the panel as shown in Fig. 3.
transverse direction ranges from 10 to 50 and the elements have an aspect ratio of approximately 1.00 to avoid the problem of convergence. When the number of elements was 25 the percentage of error is 0.42%. It is considered that this mesh refinement is acceptable and the same mesh size to be utilized for this study.

2.5 Axial plate buckling coefficient calculation

Each model is solved for the elastic critical buckling load through an eigenvalue buckling analysis then the critical buckling stress \( \sigma_c \) is calculated and the corresponding plate buckling coefficient is calculated using equation (1) as presented by Timoshenko and Gere [5].

\[
\sigma_c = k \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{h}{t}\right) \]

(1)

3 FINITE ELEMENT PARAMETRIC STUDY AND RESULTS

3.1 Overall matrix of numerical models of plates

The previous section discussed the finite element modeling technique used for the parametric study conducted in this section. The parameters considered for this study are the tapering ratio \( T_r \), which is the ratio of the larger plate width to the smaller plate width as given in equation (2). The normalized plate length \( \alpha \) is the ratio of plate length to the plate’s larger width as given in equation (3).

\[
T_r = \frac{h}{h_t} \]

(2)

\[
\alpha = \frac{l}{h} \]

(3)

Table 1 illustrates the ranges and increments used for each parameter. The tapering ratio \( T_r \) ranges from 1 to 5 to cover the cases from prismatic rectangular plate \( (T_r = 1.00) \) to highly tapered web plates \( (T_r = 5.00) \). The normalized plate length \( \alpha \) ranges from 0.25 up to 30.00 to resemble longer columns.

<table>
<thead>
<tr>
<th>Studied Parameter</th>
<th>Used Parameter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tapering ratio ( T_r )</td>
<td>1.00, 1.25, 1.50, 1.75, 2.00, 3.00, 4.00, 5.00</td>
</tr>
<tr>
<td>Normalized plate length ( \alpha )</td>
<td>0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00, 2.25, 2.50, 2.75, 3.00, 3.25, 3.50, 4.00, 5.00, 10.00, 30.00</td>
</tr>
</tbody>
</table>

3.2 Finite element results

Fig. 4 shows the typical deformed shapes of the lowest three modes of buckling for a non-prismatic steel plate axially loaded. The lowest critical load from MODE 1 is considered the critical elastic buckling mode and used for the calculation of the coefficient of buckling.

3.3 Calculated axial plate buckling coefficient

A total of 144 models are generated and analyzed under uniform compression load. The calculated coefficient of axial plate buckling \( k \), according to equation (1), is plotted against the normalized plate length for different tapering ratios and different boundary conditions as shown in Fig. 5. The calculated coefficient of buckling for prismatic plates \( (T_r = 1.00) \) matches very well with the closed form solution available, 4.00 [6]. Fig. 5 shows that \( k \) decreases with the increase of \( \alpha \) up to \( \alpha \approx 5.00 \) and then the rate of decrease is significantly lower. Fig. 6 shows the calculated coefficient of axial plate buckling against the normalized plate length for different tapering ratios and that \( k \) increases by the increase of
the tapering ratio $T_r$ for all values of normalized plate length. The plate tapering is shown to increases its resistance to buckle due to the stiffness provided by the smaller plate width zone to the larger width zone.

4 PROPOSED FORMULAS FOR AXIAL PLATE BUCKLING COEFFICIENT

For the case of axially loaded rectangular plate simply supported from four sides, the value of $k$ varies as a function of normalized plate length [6]. Full analytical solutions for $k$ as a function of normalized plate length and the number of buckled half-waves along the plate length may be found in Timoshenko and Gere [5]. For design practice, the value of $k = 4.0$ is used as a lower bound value of the exact solution regardless of the number of buckled half waves as shown in Fig. 7.

$$k_{NP} = 4.0 + \frac{a}{b}$$

(4)

Where $a$ and $b$ are constants given in Table 2.

![Fig. 5. Calculated axial plate buckling coefficient versus normalized plate length for different tapering ratios.](image)

![Fig. 6. Calculated axial plate buckling coefficient versus normalized plate length for different tapering ratios.](image)

The same approach is used to find a lower bound line for the value of $k$ for each tapering ratio $T_r$, as show in Figure 8. The lower bound line can be expressed as a function of normalized plate length as given in equation (4) at different tapering ratios.

![Fig. 8. Lower bound line of $k$ values for non-parametric plate under axial compression for different $T_r$ values.](image)

**TABLE 2**

<table>
<thead>
<tr>
<th>$T_r$</th>
<th>$a$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.00</td>
<td>420.00</td>
</tr>
<tr>
<td>1.25</td>
<td>0.275</td>
<td>191.77</td>
</tr>
<tr>
<td>1.50</td>
<td>0.373</td>
<td>137.52</td>
</tr>
<tr>
<td>1.75</td>
<td>0.440</td>
<td>116.10</td>
</tr>
<tr>
<td>2.00</td>
<td>0.481</td>
<td>104.78</td>
</tr>
<tr>
<td>3.00</td>
<td>0.587</td>
<td>85.09</td>
</tr>
<tr>
<td>4.00</td>
<td>0.638</td>
<td>77.96</td>
</tr>
<tr>
<td>5.00</td>
<td>0.666</td>
<td>74.57</td>
</tr>
</tbody>
</table>

A regression analysis is performed for the constants $a$ and $b$ as a function of $T_r$, where a minimum coefficient of determination ($R^2$) of 0.98 is adopted and the results are given by equations (5) and (6).

$$a = \frac{2}{T_r} - \frac{2}{3T_r^2}$$

(5)

$$b = \frac{10T_r}{T_r/6 - 1/7}$$

(6)
By substitution in equation (4), the final formula of the coefficient of axial buckling for non-prismatic plate is given by equation (7) which satisfies the classical $k = 4.0$ value for the simply supported rectangular plate.

$$k_{NP} = 4.67 - \frac{2}{3T_f^2} - \frac{\alpha}{60} - \frac{\alpha}{70T_f}$$

(7)

The coefficient of axial plate buckling for non-prismatic plates $k_{NP}$ calculated by the proposed formula in equation (7) is plotted along with that calculated from finite element analysis results using equation (1) against both normalized plate length ($\alpha$) and the tapering ratio ($T_f$) in Fig. 9.

![Coefficient of axial plate buckling for non-prismatic plate](image)

**Fig. 9.** Coefficient of axial plate buckling for non-prismatic plate ($k_{NP}$) calculated by proposed formula and by finite element results.

5 CONCLUSIONS

An extensive finite element study is presented in this research to analyze the effect of different geometric parameters on the critical buckling stresses of non-prismatic simply supported steel plates. The parameters included in this study are: the tapering ratio ($T_f$) and the normalized plate length ($\alpha$). The parameters’ ranges are selected to represent the extreme values for each parameter, yet keep it within the practical range of use. New formulas are introduced for the axial buckling coefficient for non-prismatic plates which satisfies the classical $k$ value for the simply supported rectangular plate.

6 REFERENCES