Fully Interval Integer Transportation Problem for Finding Optimal Interval Solution using Row- Column Minima Method

P. Indira, M. Jayalakshmi

Abstract— A new approach namely, row-column minima method is suggested to find an optimal interval solution for fully interval integer transportation problem (FIITP). In the proposed method, the given FIITP is decomposed into two transportation problem as upper (UBITP) and lower (LBITP) and by applying row-column minima method so as to get the optimal interval solution. Fuzzy concept, midpoint, centre, width, interval ordering and multi-objective technique were not used. The proposed method is easier and also, simply because of arithmetic calculation. Using the proposed method, the numerical example is illustrated.

Index Terms— Fully interval integer transportation problem, Optimal Interval Solution, Row- column minima method, Transportation problem

1 INTRODUCTION

In different approaches, solving transportation problems in tackling of specific origin, landing place boundary and penance factors. In a situation of real life problems, parameters are not be satisfied for all conditions. Transportation problem (TP), which is principally developed by Hitchcock [4] and then by Koopmans [7]. Sengupta and Pal [12] developed with fuzzy initiate method to solving TP using intervals of width, midpoint of the integer interval in the given objective function. Pandian and Natarajan [9] developed a new method to solve FIITP in the absence of width and the centre-point of the interval in the profit function. Sudhakar and Navaneetha Kumar [13] developed to find an optimal solution for FIITP in separation by using zero suffix method. Ramesh and Ganesan [11] developed simplex like algorithm to solve interval linear programming problem without changing into classical linear programming problems. Abdul Quddoos et al. [1] developed to find an optimal solution for a wide range of TP, directly by using ASM method. Akilbasha et al. [2] developed a new idea for FIITP, by split and separation idea based on zero point approach. Patel and Dhodiya [5] developed interval TP to convert into classical multi objective transportation by using the conception of upper range, half-width, lower range and centre of an interval. Mohana [8] used zero point idea to find an optimal integer solution for TP to get maximum profit with minimum penalty. Purushoth kumar et.al [10] developed diagonal optimal algorithm to find the optimal for integer interval TP. Akilbasha et.al [3] developed mid-width method for fully interval TP. Keerthana and Ramesh [6] developed a method to solve FIITP without transforming into classical TP. In this paper we recommend a new approach namely, row-column minima method for obtaining an optimal interval solution to the given FIITP. In this method we consider two transportation problem from the given FIITP, one is of maximization (upper) and other one is of minimization (lower) problem. Initially, we find an optimal solution for upper TP, based on this optimal solution, the lower TP is allocated, an optimal interval solution for FIITP is obtained by allocating the maximum possible in minimum number of allocated row (or) column. In row-column minima method, the optimal interval solution is obtained and the solution procedure is illustrated below for better understand. In this method advance to without using fuzzy concept, midpoint, centre, width, interval ordering and multi-objective technique and has been proposed lacking interval parameters in the TP.

- *Indira.P , Research Scholar, Department of Mathematics, School of Advanced Science, VIT University, Vellore 14, Indira.p@vit.ac.in*
- *Dr. M. Jayalakshmi, Assistant Professor(Senior), Department of Mathematics, School of Advanced Science, VIT University, Vellore 14, m.jayalakshmi@vit.ac.in*

2 PRELIMINARIES

Consider T, where T is arrange of all closed bounded interval in horizontal axis R, which is defined as follows:

$$T = \{(c, d), c \le d \; and \; c \,\&\, d \; are \; in \; R$$

We know that the following operator and definitions in closed bounded intervals.

2.1 Definition:

Let P=[r, s] & Q= [t, u] be in D. Then

(i) $P \oplus Q = [r+t, s+u]$

(ii) $P \Theta Q = [r-t, s-u]$

(iii) kP = [kr, ks] if k is the positive real number

(iv) kQ = [ks, kr] if k is the negative real number

(v) $P \otimes Q = [x, y]$ Where $x = \min\{rt, ru, st, su\}$ & $y = \max\{rt, ru, st, su\}$

kQ = [ks, kr] if k is the negative real number

and $P \otimes Q = [x, y]$ where $x = \min\{rt, ru, st, su\}$ & $y = \max\{rt, ru, st, su\}$

2.2 Definition:

Let P=[r, s] & Q= [t, u] be in D. Then

(i) $P \le Q \; if \; r \le t, s \le u$

(ii) $P \ge Q \; if \; Q \le P, that \, is \; r \ge t, s \ge u$

(iii) $P = Q \; if \; P \le Q \; and \; Q \le P, that \, is \; r = t, s = u$

2.3 Definition:
Set of all $[x_{ij}, y_{ij}]\ \forall i = 1 \text{ to } m \ \& \ j = 1 \text{ to } n$, are satisfied the equations (1), (2) and (3) then it is called feasible solution to the FIITP.

2.4 Definition:
Set of all $[x_{ij}, y_{ij}]\ \forall i = 1 \text{ to } m \ \& \ j = 1 \text{ to } n$ is called an optimal solution of FIITP if

$$\sum_{i=1}^{m}\sum_{j=1}^{n} [f_{ij}, g_{ij}] \otimes [x_{ij}, y_{ij}] \le \sum_{i=1}^{m}\sum_{j=1}^{n} [f_{ij}, g_{ij}] \otimes v_{ij}, w_{ij}]$$

$\forall i = 1$ to m & $j = 1$ to n and for all optimal solution $[v_{ij}, w_{ij}]\ \forall i = 1 \text{ to } m \ \& \ j = 1 \text{ to } n$

3 Fully Interval Integer Transportation Problem

Let the FIITP is:

$$Min\ [Z_1, Z_2] = \sum_{i=1}\sum_{j=1} [f_{ij}, g_{ij}] \otimes [x_{ij}, y_{ij}]$$

Subject to constraints,

$$\sum_{j=1}^{n} [x_{ij}, y_{ij}] = [c_i, p_i], \forall i = 1 \text{ to } m \qquad (1)$$

$$\sum_{i=1}^{m} [x_{ij}, y_{ij}] = [d_j, q_j], \forall j = 1 \text{ to } n \qquad (2)$$

$$x_{ij} \ge 0 \ \& \ y_{ij} \ge 0, \forall i = 1 \text{ to } m \ \& \ j = 1 \text{ to } n \ \& \text{ are integer} \qquad (3)$$

where f_{ij} & g_{ij} are non-negative real numbers of minimum and maximum transportation price for all value of i and j, $c_i \ \& \ p_i$ are non-negative real values of minimum and maximum origins for all value of i, $d_i \ \& \ q_i$ are non-negative real values of minimum and maximum requirements for all value of j, $x_{ij} \ \& \ y_{ij}$ are non-negative real number of minimum and maximum transported cost from i^{th} origin to j^{th} requirements.

4 Row-Column Minima Method

To obtain an optimal interval solution for FIITP using Row-Column minima method.
The algorithm proceeds as follows:

Step 1: Construct the Upper bound integer transportation problem (UBITP) from the given problem of FIITP.

Step 2: Solving the UBITP from FIITP using any well-known method. Let $\{v^*_{ii}, \forall \text{ i \& j}\}$ be an optimum solution of the UBITP of the FIITP and obtain maximum interval objective value and is denoted by Z_2^*.

Step 3: Construct the Lower bound integer transportation problem (LBITP) from the particular FIITP.

Step 4: Mark (*) as the optimal solution of UBITP in LBITP.

Step 5: Now we allocate the maximum possible in minimum number of allocation in row (or) column for allotted cell (*).

Step 6: Replicate the procedure of step 5 until the rim requirements are satisfied then we get an optimal solution for LBITP and is denoted by w^*_{ii} with the condition $w^*_{ii} \le v^*_{ii}$ and obtain the minimum interval objective value and is denoted by Z_1^*.

Step 7: The optimal solution to the particular FIITP is $[x_{ij}, y_{ij}], \forall i = 1 \text{ to } m \ \& \ j = 1 \text{ to } n$ and the minimum interval transportation cost is $[Z_1^*, Z_2^*]$.

5 Numerical Illustration

5.1 Example:
A company has 3 machines P1, P2 & P3 with production units of manufacturer. These production unit deliver to 4 places Q1, Q2, Q3 & Q4 with requirements. The given interval are minimum and maximum transport cost. Calculate the minimum transport interval cost from machines manufacturer to deliver places, by using below numerical data, where the minimum origin from P1, P2 & P3 are 7, 17 & 16 respectively and the maximum origin from P1, P2 & P3 are 9, 21 & 18 respectively. The minimum requirement from Q1, Q2, Q3 & Q4 are 10, 2, 13 & 15 respectively and the maximum requirements from Q1, Q2, Q3 & Q4 are 12, 4, 15 & 17 respectively and transport price given in table form of intervals:

	Q_1	Q_2	Q_3	Q_4	Supply
P_1	[1,2]	[1,3]	[5,9]	[4,8]	[7,9]
P_2	[1,2]	[7,10]	[2,6]	[3,5]	[17,21]
P_3	[7,9]	[7,11]	[3,5]	[5,7]	[16,18]
Demand	[10,12]	[2,4]	[13,15]	[15,17]	[40,48]

Solution:
By step (1),
Now, the upper bound of the integer transportation problem (UBITP) from the above problem is given below,

	Q_1	Q_2	Q_3	Q_4	Supply
P_1	2	3	9	8	9
P_2	2	10	6	5	21
P_3	9	11	5	7	18
Demand	12	4	15	17	48

By step (2),
Using existing idea, to find optimal solution for the UBITP is

$$v^*_{11} = 5, v^*_{12} = 4, v^*_{21} = 7, v^*_{24} = 14, v^*_{33} = 15, v^*_{34} = 3 \ \& \ Z^*_2 = 202$$

By step (3), LBITP is

	Q_1	Q_2	Q_3	Q_4	Supply
P_1	1	1	5	4	7
P_2	1	7	2	3	17
P_3	7	7	3	5	16
Demand	10	2	13	15	40

By step (4),

	Q_1	Q_2	Q_3	Q_4	Supply
P_1	1*	1*	5	4	7
P_2	1*	7	2	3*	17
P_3	7	7	3*	5*	16
Demand	10	2	13	15	40

By step (5),

Iteration 1:

We allocate the maximum possible of the minimum number of allocation in row (or) column (ie.) Here no. of allocation in row is (2, 2, 2) and column is (2, 1, 1, 2) then we can allocate the second column as 2.

	Q_1	Q_2	Q_3	Q_4	Supply
P_1	1	1 **2**	5	4	~~7~~ 5
P_2	1	7	2	3	17
P_3	7	7	3	5	16
Demand	10	~~2~~	13	15	40

Iteration 2:

We allocate the maximum possible of the minimum number of allocation in row (or) column (ie.) Here no. of allocation in row is (1, 2, 2) and column is (2, -, 1, 2) then we can allocate the first row as 5.

	Q_1	Q_3	Q_4	Supply
P_1	1 **5**	5	4	~~5~~
P_2	1	2	3	17
P_3	7	3	5	16
Demand	~~10~~ 5	13	15	40

Iteration 3:

We allocate the maximum possible of the minimum number of allocation in row (or) column (ie.) Here no. of allocation in row is (-, 2, 2) and column is (1, -, 1, 2) then we can allocate the first column as 5.

	Q_1	Q_3	Q_4	Supply
P_2	1 **5**	2	3	~~17~~ 12
P_3	7	3	5	16
Demand	~~5~~	13	15	40

Iteration 4:

We allocate the maximum possible of the minimum number of allocation in row (or) column (ie.) Here no. of allocation in row is (-, 1, 2) and column is (-, -, 1, 2) then we can allocate the second row as 12.

	Q_3	Q_4	Supply
P_2	2	3 **12**	~~12~~
P_3	3	5	16
Demand	13	~~15~~ 3	40

Iteration 5:

We allocate the maximum possible of the minimum number of allocation in row (or) column (ie.) Here no. of allocation in row is (-, -, 2) and column is (-, -, 1, 1) then we can allocate the third column as13.

	Q_3	Q_4	Supply
P_3	3 **13**	5	~~16~~ 3
Demand	~~13~~	3	40

Iteration 6:

We allocate the maximum possible of the minimum number of allocation in row (or) column (ie.) Here no. of allocation in row is (-, -, 1) and column is (-, - ,-, 1) then we can allocate the third row as 3.

	Q_4	Supply
P_3	5 **3**	~~3~~
Demand	~~3~~	40

By step (6),

Therefore the optimal solution to the LBITP is

$$\mathbf{w_{11}^* = 5, w_{12}^* = 2, w_{21}^* = 5, w_{24}^* = 12, w_{33}^* = 13, w_{34}^* = 3} \ \& \ \boldsymbol{Z_1^* = 102}$$

By step (7),

Thus an optimal solution to a FIITP is:

$$[v_{11}^*, w_{11}^*] = [5,5],\ [v_{12}^*, w_{12}^*] = [2,4],\ [v_{21}^*, w_{21}^*] = [5,7],$$

$$[v_{24}^*, w_{24}^*] = [12,14],\ [v_{33}^*, w_{33}^*] = [13,15],\ [v_{34}^*, w_{34}^*] = [3,3],$$

and **[102,202]** is the minimum interval transportation cost.

5.2 Example:

For the following FIITP, find the optimal solution and minimum interval transportation cost.

	D_1	D_2	D_3	D_4	Supply
S_1	[3,5]	[2,6]	[2,4]	[1,5]	[7,9]
S_2	[4,6]	[7,9]	[7,10]	[9,11]	[17,21]
S_3	[4,8]	[1,3]	[3,6]	[1,2]	[16,18]
Demand	[10,12]	[2,4]	[13,15]	[15,17]	[40,48]

Solution:

By step (1),

Now, the upper bound of the integer transportation problem (UBITP) from the above problem is given below,

	D_1	D_2	D_3	D_4	Supply
S_1	5	6	4	5	9
S_2	6	9	10	11	21
S_3	8	3	6	2	18
Demand	12	4	15	17	48

By step (2),

Using existing idea, to find optimal solution for the UBITP is

$$v_{13}^* = 9, v_{21}^* = 12, v_{22}^* = 3, v_{23}^* = 6, v_{32}^* = 1, v_{34}^* = 17 \ \& \ Z_2^* = 232$$

	D_1	D_2	D_3	D_4	Supply
S_1	5	6	4	5	9
S_2	6	9	10	11	21
S_3	8	3	6	2	18
Demand	12	4	15	17	48

	D_1	D_2	D_3	D_4	Supply
S_1	3	2	2*	1	7
S_2	4*	7*	7*	9	17
S_3	4	1*	3	1*	16
Demand	10	2	13	15	40

By step (5),

Iteration 1:
We allocate the maximum possible of the minimum number of allocation in row (or) column (ie.) Here no. of allocation in row is (1, 3, 2) and column is (1, 2, 2, 1) then we can allocate the first row as 7.

	D_1	D_2	D_3	D_4	Supply
S_1	3	2	2 **7**	1	~~7~~
S_2	4	7	7	9	17
S_3	4	1	3	1	16
Demand	10	2	~~13~~ 6	15	40

Iteration 2:
We allocate the maximum possible of the minimum number of allocation in row (or) column (ie.) Here no. of allocation in row is (-, 3, 2) and column is (1, 2, 1, 1) then we can allocate the first column as 7.

	D_1	D_2	D_3	D_4	Supply
S_2	4 **10**	7	7	9	~~17~~ 7
S_3	4	1	3	1	16
Demand	~~10~~	2	6	15	40

Iteration 3:
We allocate the maximum possible of the minimum number of allocation in row (or) column (ie.) Here no. of allocation in row is (-, 2, 2) and column is (-, 2, 1, 1) then we can allocate the third

	D_2	D_3	D_4	Supply
S_2	7	7 **6**	9	~~7~~ 1
S_3	1	3	1	16
Demand	2	~~6~~	15	40

Iteration 4:
We allocate the maximum possible of the minimum number of allocation in row (or) column (ie.) Here no. of allocation in row is (-, 1, 2) and column is (-, 2, -, 1) then we can allocate the second row as 7.

	D_2	D_4	Supply
S_2	7 **1**	9	~~1~~
S_3	1	1	16
Demand	~~2~~ 1	15	40

Iteration 5:
We allocate the maximum possible of the minimum number of allocation in row (or) column (ie.) Here no. of allocation in row is (-, -, 2) and column is (-, 1, -, 1) then we can allocate the second column as 1.

	D_2	D_4	Supply
S_3	1 **1**	1	~~16~~ 15
Demand	~~1~~	15	40

We allocate the maximum possible of the minimum number of allocation in row (or) column (ie.) Here no. of allocation in row is (-, -, 1) and column is (-, -, -, 1) then we can allocate the second column as 1.

	D_4	Supply
S_3	1 **15**	~~15~~
Demand	~~15~~	40

By step (6),

Therefore the optimal solution to the LBITP is

$$\boldsymbol{w_{13}^* = 7, w_{21}^* = 10, w_{22}^* = 1, w_{23}^* = 6, w_{32}^* = 1, w_{34}^* = 15}$$

Thus an optimal solution to a FIITP is:

$$[v_{13}^*, w_{13}^*] = [7,9], [v_{21}^*, w_{21}^*] = [10,12], [v_{22}^*, w_{22}^*] = [1,3],$$

$$[v_{23}^*, w_{23}^*] = [6,6], [v_{32}^*, w_{32}^*] = [1,1], [v_{34}^*, w_{34}^*] = [15,17]$$

and [119,232] is the minimum interval transportation cost.

6 Results and Discussion

The example (5.1) & (5.2), the proposed idea, divides the given FIITP into two TP as UBITP and LBITP. In this approach, zero point method [8] & [9], mid-value and half-width [3], Zero suffix method and fuzzy [13] were not used. In existing method [8], [9], [13] & [3] and proposed method, we got the same results. The suggested method, solving the FIITP and got the best minimum interval transportation cost than the existing method [10]. The proposed method, first obtain the optimal solution for UBITP and then the primary allocation of LBITP as the optimal solution of UBITP, afterwards solving LBITP using proposed method we got the optimal solution of FIITP and also minimum interval integer transportation cost. Also, we have listed the comparison table for the example 5.1 and 5.2 as below:

Comparisons table for example 5.1

Method	Authors	Allocation	Optimal Solution
Zero point method	P. Pandian & G. Natarajan [4]	$[x_{11}^*, y_{11}^*]$=[5,5] ; $[x_{12}^*, y_{12}^*]$=[4,2] $[x_{21}^*, y_{21}^*]$=[7,5] ; $[x_{24}^*, y_{24}^*]$=[14,12] $[x_{33}^*, y_{33}^*]$=[15,13] ; $[x_{34}^*, y_{34}^*]$=[3,3]	**[102,202]**
Separation Method	V. J. Sudhakar et. al [5]	$[x_{11}^*, y_{11}^*]$=[5,5] ; $[x_{12}^*, y_{12}^*]$=[4,2] $[x_{21}^*, y_{21}^*]$=[7,5] ; $[x_{24}^*, y_{24}^*]$=[14,12] $[x_{33}^*, y_{33}^*]$=[15,13] ; $[x_{34}^*, y_{34}^*]$=[3,3]	**[102,202]**
Zero Point Method	V. Mohana [10]	$[x_{11}^*, y_{11}^*]$=[5,5] ; $[x_{12}^*, y_{12}^*]$=[4,2] $[x_{21}^*, y_{21}^*]$=[7,5] ; $[x_{24}^*, y_{24}^*]$=[14,12] $[x_{33}^*, y_{33}^*]$=[15,13] ; $[x_{34}^*, y_{34}^*]$=[3,3]	**[102,202]**
Exact Method	A. Akilbasha et.al [12]	$[x_{11}^*, y_{11}^*]$=[5,5] ; $[x_{12}^*, y_{12}^*]$=[4,2] $[x_{21}^*, y_{21}^*]$=[7,5] ; $[x_{24}^*, y_{24}^*]$=[14,12] $[x_{33}^*, y_{33}^*]$=[15,13] ; $[x_{34}^*, y_{34}^*]$=[3,3]	**[102,202]**
Proposed Method	P. Indira & M. Jayalakshmi	$[v_{11}^*, w_{11}^*]$=[5,5] ; $[v_{12}^*, w_{12}^*]$=[4,2] $[v_{21}^*, w_{21}^*]$=[7,5] ; $[v_{24}^*, w_{24}^*]$=[14,12] $[v_{33}^*, w_{33}^*]$=[15,13] ; $[v_{34}^*, w_{34}^*]$=[3,3]	**[102,202]**

Method	Authors	Allocation	Optimal Solution
Zero Point Method	P. Pandian & G. Natarajan [4]	$[x_{13}^*, y_{13}^*]$=[7,9] ; $[x_{21}^*, y_{21}^*]$=[10,12] $[x_{22}^*, y_{22}^*]$=[1,3] ; $[x_{23}^*, y_{23}^*]$=[6,6] $[x_{32}^*, y_{32}^*]$=[1,1] ; $[x_{34}^*, y_{34}^*]$=[15,17]	**[119,232]**
Diagonal Optimal Algorithm	M.K. Purushothkumar et.al [11]	$[x_{14}^*, y_{14}^*]$=[7,9] ; $[x_{21}^*, y_{21}^*]$=[10,12] $[x_{23}^*, y_{23}^*]$=[3,13] ; $[x_{32}^*, y_{32}^*]$=[2,4] $[x_{33}^*, y_{33}^*]$=[2,10] ; $[x_{34}^*, y_{34}^*]$=[6,10]	**[82,349]**
Proposed Method	P. Indira & M. Jayalakshmi	$[v_{13}^*, y_{13}^*]$=[7,9] ; $[v_{21}^*, w_{21}^*]$=[10,12] $[v_{22}^*, w_{22}^*]$=[1,3] ; $[v_{23}^*, w_{23}^*]$=[6,6] $[v_{32}^*, w_{32}^*]$=[1,1] ; $[v_{34}^*, w_{34}^*]$=[15,17]	**[119,232]**

7 Conclusion

A new process namely, row-column minima method for computing FIITP is established. It is easy to interpret because we used classical transportation technique and apply a well ordered procedure. Never using fuzzy, zero point method, mid-point, half-width, zero suffix method and diagonal method. Its idea can be given an essential technique for the decision makers to more options when they operate distinct planning problems and factors as intervals.

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