Number System for Digital Computers

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Abstract : A number system is the method or system of representing the digits in the computer system. The digital computer represents data in binary forms. The total number of digits used in a number system is called its base or radix. The base is written after the number as subscript; for example

- Binary number system (Base 2), like 10110_2 (10110 base 2).
- Octal number system (Base 8), like 76_8 (76 base 8).
- Decimal number system (Base 10), like 560_{10} (560 to base of 10).
- Hexadecimal number system (Base 16), like $5AC_{16}$ (5AC to base of 16).

This paper discusses the binary Addition, Subtraction, Multiplication and Division that will be useful for digital computers, and the field of computer science.

Keywords: Binary Addition, Binary Subtraction, Binary Multiplication and Binary Division.

1 OBJECTIVE

1. To study a basic arithmetic operation of binary number system.
2. To simplify the calculation for better understanding of students.

2 BINARY ADDITION

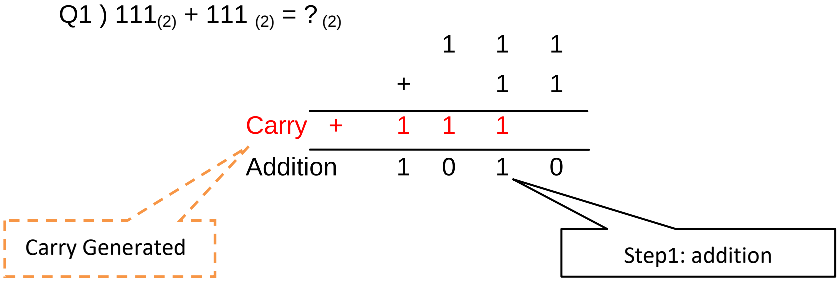
Addition is one of the easy operations of the basic arithmetic operations. The binary addition operation is similar to the decimal system. Rules of binary addition are as follows.

Rules of Addition

Number of Times	Addition	Decimal Equivalent	Binary Equivalent	Carry Generated	Addition value
1 Time 1	1	1	1	0	1
2 Times 1	1+1	2	10	1	0
3 Times 1	1+1+1	3	11	1	1
4 Times 1	1+1+1+1	4	100	10	0
5 Times 1	1+1+1+1+1	5	101	10	1
6 Times 1	1+1+1+1+1+1	6	110	11	0
7 Times 1	1+1+1+1+1+1+1	7	111	11	1
8 Times 1	1+1+1+1+1+1+1+1	8	1000	100	0
9 Times 1	1+1+1+1+1+1+1+1+1	9	1001	100	1
10 Times 1	1+1+1+1+1+1+1+1+1+1	10	1010	101	0

Solved Example of Binary Addition

Q1) $111_{(2)} + 111_{(2)} = ?_{(2)}$



	Binary No	Decimal equivalent
1st Number	111	7
2nd Number	11	3
Addition	1010	10

Answer is : $111_{(2)} + 11_{(2)} = 1010_{(2)}$

Q2) $11001_{(2)} + 111_{(2)} = ?_{(2)}$

		1	1	0	0	1
			+	1	1	1
Carry	1	1	1	1	1	
Addition	1	0	0	0	0	0

Carry Generated

Step1: addition

	Binary No	Decimal equivalent
1st Number	11001	25
2nd Number	111	7
Addition	100000	32

Answer is :

$11001_{(2)} + 111_{(2)} = 100000_{(2)}$

Q3) $11001_{(2)} + 11100_{(2)} + 1000_{(2)} = ?_{(2)}$

		1	1	0	0	1
	+	1	1	1	0	0
	+		1	0	0	0
Carry	1	1				
Addition	1	1	1	1	0	1

Carry Generated

Step1: addition

	Binary No	Decimal equivalent
1st Number	11001	25
2nd Number	11100	28
3rd Number	1000	8
Addition	111101	61

Answer is :

$11001_{(2)} + 11100_{(2)} + 1000_{(2)} = 111101_{(2)}$

Q4) $11011_{(2)} + 11110_{(2)} + 1000_{(2)} = ?_{(2)}$

		1	1	0	1	1
	+ 1		1	1	1	0
	+		1	0	0	0
Carry	10	10	1	1		
Addition	10	0	0	0	0	1

Carry Generated

Step1: addition

	Binary No	Decimal equivalent
1st Number	11011	27
2nd Number	11110	30
3rd Number	1000	8
Addition	1000001	65

Answer is :

$11011_{(2)} + 11110_{(2)} + 1000_{(2)} = 1000001$

Binary Addition Practice Questions

QUESTIONS	ANSWERES
1. $11001101_{(2)} + 1110011_{(2)} = ?_{(2)}$	101000000
2. $1010101_{(2)} + 1111111_{(2)} = ?_{(2)}$	11010100
3. $111000_{(2)} + 101101_{(2)} = ?_{(2)}$	1100101
4. $101011_{(2)} + 101100_{(2)} + 100001_{(2)} = ?_{(2)}$	1111000
5. $111000_{(2)} + 000111_{(2)} + 101010_{(2)} = ?_{(2)}$	1101001

BINARY SUBTRACTION

Binary subtraction is addition of 1'st compliment of subtrahend with minuend, first make 1'st compliment of subtrahend and add with minuend using addition rules, if carry generated, then ignore this carry FROM OBTAINED addition and add with itself which is positive(+ve) final result. And if carry not generated, then make 1'st compliment of obtained result which is negative (-ve) final result.

Steps of Binary Subtraction

Step1: 1st compliment of subtrahend add with the minuend

IF Carry generated

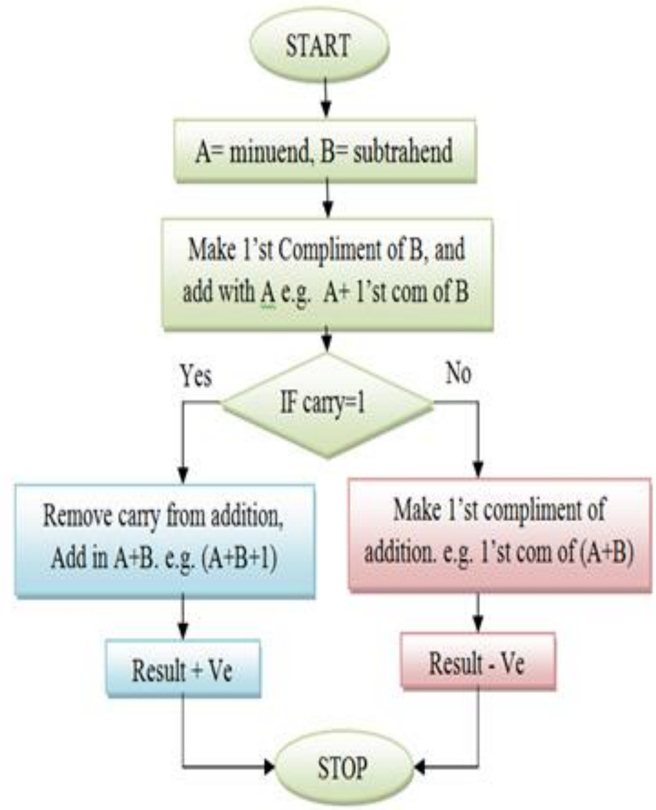
Step2: If carry generated, add in the obtained addition.

Step 3: Ignore carry, Final +V Result.

IF Carry not generated

Step2: If carry not generated, make 1'st compliment of obtained result.

Step 3: Final -V Result.



What is the 1'st compliment?

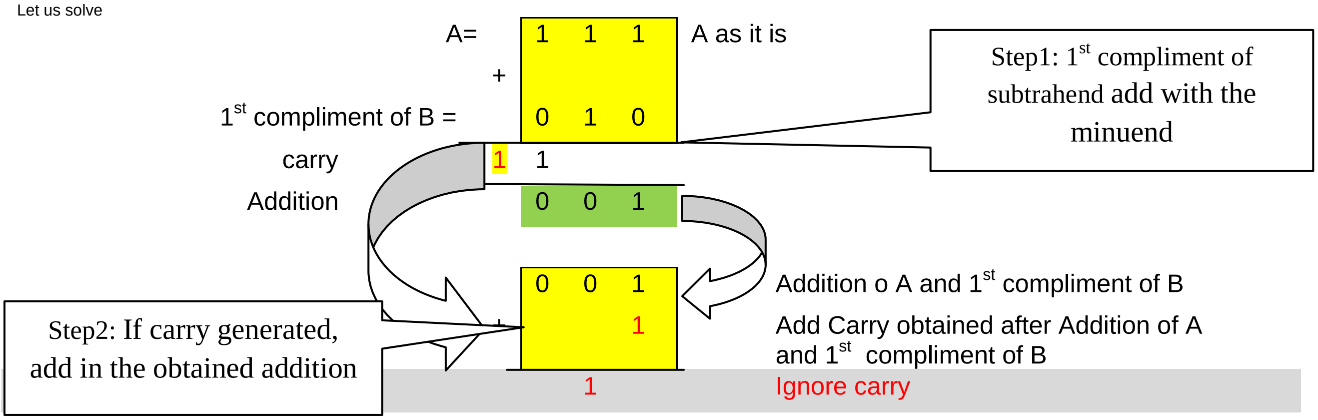
The conversion of 1 into 0 , and 0 into 1 is called as 1'st compliment

e.g. 10010 → 1'st compliment is 01101

Solved Example of Binary subtraction

Q1) $111_{(2)} - 101_{(2)} = ?_{(2)}$

Let us solve



Addition +V Result 0 1 0 Final +V Result

	Binary No	Decimal equivalent
1st Number	111	7
2nd Number	101	5
Subtraction	010	+3

Answer is :

$111_{(2)} - 101_{(2)} = 010_{(2)}$

Q2) $1101_{(2)} - 1010_{(2)} = ?_{(2)}$

Let us Solve

A= 1 1 0 1 A as it is

\+

1st compliment of B = 0 1 1 0

carry 1 1

Addition 0 0 1 1

Step1: 1st compliment of subtrahend add with the minuend

0 0 1 1 Addition o A and 1st compliment of B

\+ 1 Add Carry obtained after Addition of A and 1st compliment of B

carry 1 1 Ignore carry

Addition +V Result 0 1 0 0 Final +V Result

Step2: If carry generated, add in the obtained addition

	Binary No	Decimal equivalent
1st Number	1101	13
2nd Number	1001	9
Subtraction	0100	+4

Answer is :

$1101_{(2)} - 1001_{(2)} = 0100$

Q3) $101_{(2)} - 111_{(2)} = ?_{(2)}$

Let us Solve

A= 1 0 1 A as it is

\+

1st compliment of B = 0 0 0

carry absent .. 1 0 1

Final - V Result - 0 1 0

Step1: 1st compliment of subtrahend add with the minuend

Step2: If carry not generated, 1st compliment of result

	Binary No	Decimal equivalent
1st Number	101	5
2nd Number	111	7
Subtraction	-010	-2

Answer is :

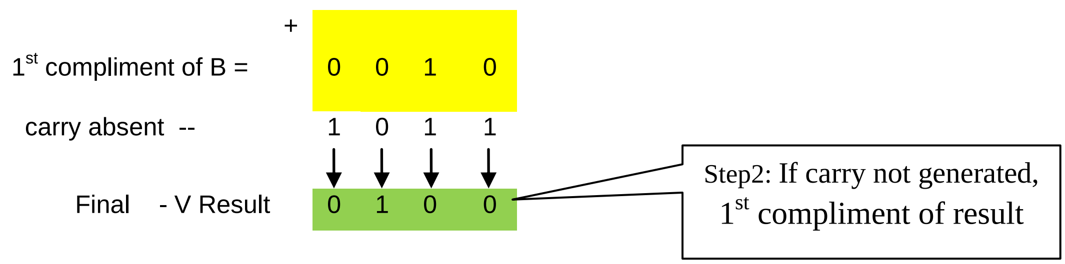
$101_{(2)} - 111_{(2)} = -010_{(2)}$

Q4) $1001_{(2)} - 1101_{(2)} = ?_{(2)}$

Let us Solve

A= 1 0 0 1 A as it is

Step1: 1st compliment of subtrahend add with the minuend



	Binary No	Decimal equivalent
1st Number	101	5
2nd Number	111	7
Subtraction	-010	-2

Answer is :

$$101_{(2)} - 111_{(2)} = -010_{(2)}$$

Binary Subtraction Practice Questions

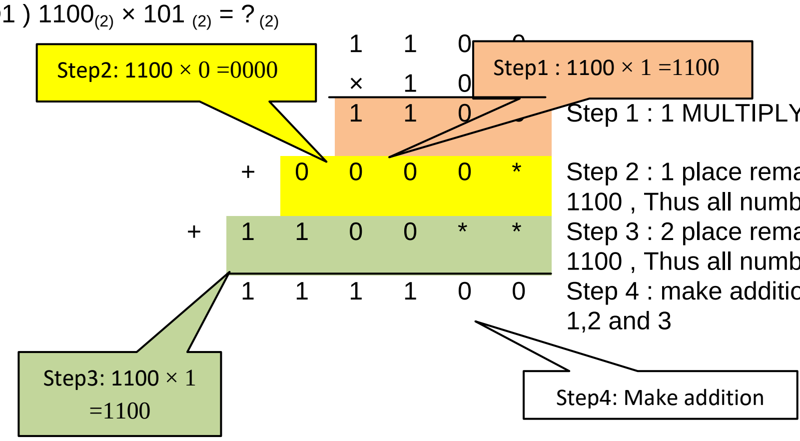
QUESTIONS	ANSWERES
1. $110010_{(2)} - 11001_{(2)} = ?_{(2)}$	+ 11001
2. $10100_{(2)} - 110010_{(2)} = ?_{(2)}$	- 11110
3. $1111_{(2)} - 11001_{(2)} = ?_{(2)}$	-1010
4. $1100_{(2)} - 1001_{(2)} = ?_{(2)}$	- 0011
5. $100000_{(2)} - 101000_{(2)} = ?_{(2)}$	-1000
6. $1010000_{(2)} - 101000_{(2)} = ?_{(2)}$	+ 101000
7. $110000_{(2)} - 110011_{(2)} = ?_{(2)}$	+11
8. $10010_{(2)} - 100000_{(2)} = ?_{(2)}$	-1110
9. $10100_{(2)} - 110000_{(2)} = ?_{(2)}$	-11100

BINARY MUTIPLICATION

Binary Multiplication is same as binary addition. The multiplier contains only 0s and 1s, so each multiplication step produces either zeros or a copy of the multiplicand and finally performs addition of all steps produces after multiplication.

Solved Example of Binary Multiplication

Q1) $1100_{(2)} \times 101_{(2)} = ?_{(2)}$



Step 1 : 1 MULTIPLY WITH 1100 ,THUS AS IT IS 1100

Step 2 : 1 place remain empty i.e.(*), 0 MULTIPLY WITH 1100 , Thus all numbers will be 0000

Step 3 : 2 place remain empty i.e.(*), 1 MULTIPLY WITH 1100 , Thus all numbers will be 1100

Step 4 : make addition of the number which get from Step 1,2 and 3

	Binary No	Decimal equivalent
Multiplicand	1100	12
Multiplayer	101	5
Addition	111100	60

Answer is :

$1100_{(2)} \times 101_{(2)} = 111100_{(2)}$

Q2) $1101_{(2)} \times 110_{(2)} = ?_{(2)}$

```
          1  1  0  1
          ×  1  1  0
          0  0  0  0
   +   1  1  0  1  *
+  1  1  0  1  *  *
1  1
1  0  0  1  1  1  0
```

Step1 : 1101 × 0 =0000

Step2: 1101 × 1 =1101

Step3: 1101 × 1 =1101

Carry Generated

Step4: Make addition

Step 1 : 0 MULTIPLY WITH 1101 , Thus all numbers will be 0000

Step 2 : 1 place remain empty i.e.(*), 1 MULTIPLY WITH 1101 , Thus all numbers will be 1101

Step 3 : 2 place remain empty i.e.(*), 1 MULTIPLY WITH 1101 , Thus all numbers will be 1101

Step 4 : make addition of the number which get from Step 1,2 and 3

	Binary No	Decimal equivalent
Multiplicand	1101	13
Multiplayer	110	6
Addition	1001110	78

Answer is

$1101_{(2)} \times 110_{(2)} = 1001110$

Q3) $1111_{(2)} \times 11_{(2)} = ?_{(2)}$

```
       1  1  1  1
       ×     1  1
       1  1  1  1
+   1  1  1  1  *
1   1  1  1
1   0  1  1  0  1
```

Step1 : 1111 × 1 =1111

Step2: 1111 × 1 =1111

Carry Generated

Step3: Make addition

Step 1 : 1 MULTIPLY WITH 1111 , Thus all numbers will be 1111

Step 2 : 1 place remain empty i.e.(*), 1 MULTIPLY WITH 1111 , Thus all numbers will be 1111

Step 3 : make addition of the number which get from Step 1 and 2

	Binary No	Decimal equivalent
Multiplicand	1111	15
Multiplayer	11	3
Addition	111100	60

Answer is :

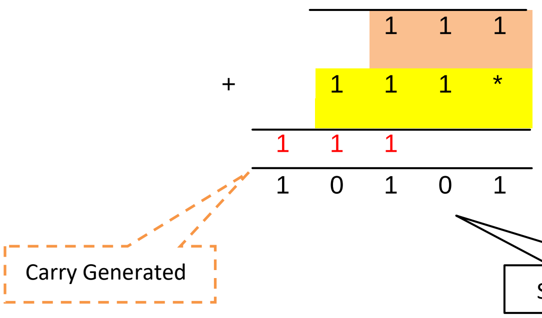
$1101_{(2)} \times 110_{(2)} = 1001110$

Q4) $111_{(2)} \times 111_{(2)} = ?_{(2)}$

```
   1  1  1
×     1  1
```

Step2: 111 ×1 =111

Step1 : 111 × 1 =111



Step 1 : 1 MULTIPLY WITH 111 ,THUS AS IT IS 111

Step 2 : 1 place remain empty i.e.(*), 1 MULTIPLY WITH 111 , Thus all numbers will be 111

Step 3 : make addition of the number which get from Step 1,2 and 3

	Binary No	Decimal equivalent
Multiplicand	111	7
Multiplayer	11	3
Addition	10101	21

Answer is :

$$111_{(2)} \times 11_{(2)} = 10101_{(2)}$$

Binary Multiplication Practice Questions

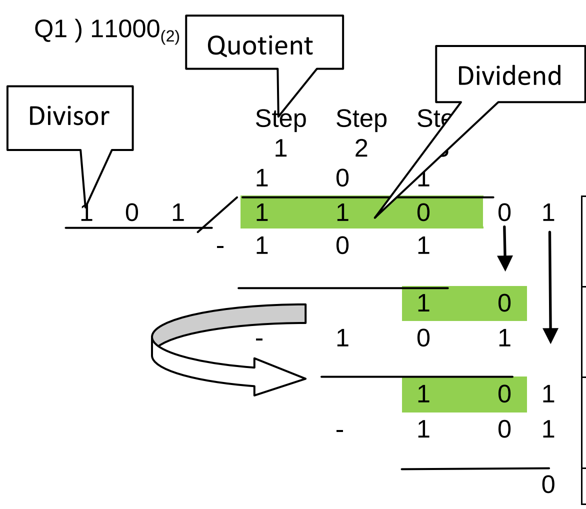
QUESTIONS	ANSWERES
10. $100111_{(2)} \times 1011_{(2)} = ?_{(2)}$	110101101
11. $10101_{(2)} \times 11_{(2)} = ?_{(2)}$	111111
12. $1100_{(2)} \times 100_{(2)} = ?_{(2)}$	110000
13. $1111100_{(2)} \times 10_{(2)} = ?_{(2)}$	11111000
14. $11010_{(2)} \times 101_{(2)} = ?_{(2)}$	10000010
15. $1011110_{(2)} \times 101_{(2)} = ?_{(2)}$	11100110
16. $11110_{(2)} \times 111_{(2)} = ?_{(2)}$	10010110
17. $100000_{(2)} \times 10_{(2)} = ?_{(2)}$	1000000
18. $101101_{(2)} \times 11_{(2)} = ?_{(2)}$	10000111
19. $10110_{(2)} \times 1010_{(2)} = ?_{(2)}$	11011100

BINARY DIVISION

Division is one of the difficult operations of the basic arithmetic operations. The binary division operation is similar to the decimal system, except the base 2 system.

Solved Example of Binary Division

Q1) $11000_{(2)}$



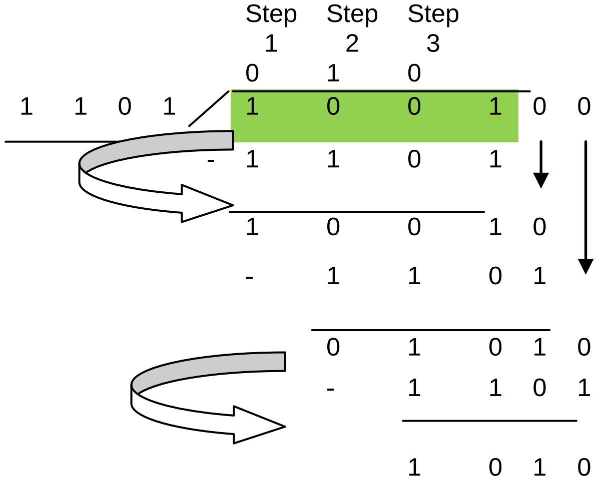
Step 1 : First 3 Digit of dividend i.e.110 $_{(2)}$ =6$_{(10)}$ is greater than divisor i.e.101$_{(2)}$ =5$_{(10)}$ Thus quotient is 1, subtract (6-5=1) Binary equivalent of 1 is 1
Step 2 : Take next number (0) from dividend ,10$_{(2)}$ =2$_{(10)}$ is less than divisor i.e. 101$_{(2)}$ =5$_{(10)}$. Thus quotient is 0, get the (10) as it is without subtraction
Step 3 : Take next number (1) from dividend, 101$_{(2)}$ =5$_{(10)}$ is equal to divisor i.e.101$_{(2)}$ =5$_{(10)}$. Thus quotient is 1, after subtraction, reminder is 0
Step 4 : Remainder is 0 . Thus stop.

	Binary No	Decimal equivalent
Dividend	11001	25
Divisor	101	5
Quotient	101	5
Remainder	0	0

Answer is :

$11000_{(2)} \div 101_{(2)} = 101_{(2)}$

Q2) $11000_{(2)} \div 101_{(2)} = ?_{(2)}$



Step 1 : First 4 Digit of dividend i.e.$1001_{(2)} = 9_{(10)}$ is less than divisor i.e.$1101_{(2)} = 13_{(10)}$ Thus quotient is 0, get the (1001) as it is without subtraction

Step 2 : Take next number (0) from dividend ,$10010_{(2)} = 18_{(10)}$ is greater than divisor i.e. $1101_{(2)} = 13_{(10)}$. Thus quotient is 1, subtract (18-13=5) Binary equivalent of 5 is $101_{(2)}$

Step 3 : Take next number (0) from dividend, $1010_{(2)} = 10_{(10)}$ is less than to divisor i.e.$1101_{(2)} = 13_{(10)}$. Thus quotient is 0, get the (1010) as it is without subtraction

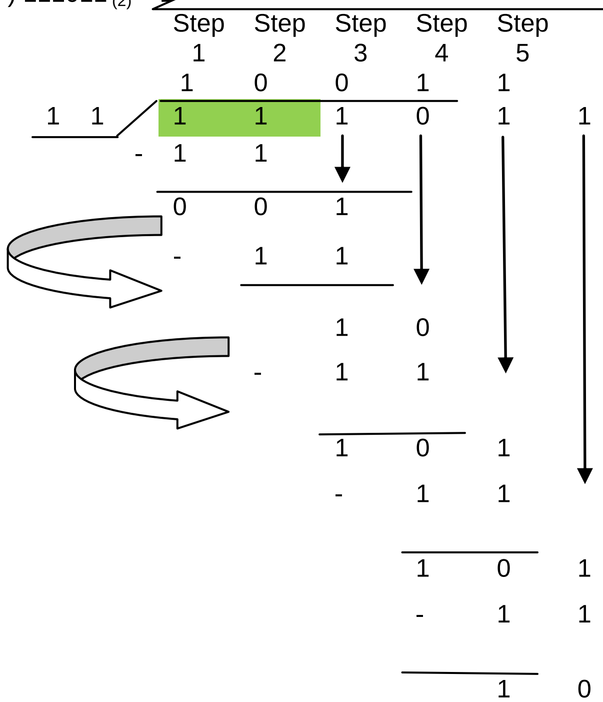
Step 4 : Remainder $1010_{(2)} = 10_{(10)}$ is less than divisor i.e. $1101_{(2)} = 13_{(10)}$. Thus stop.

	Binary No	Decimal equivalent
Dividend	100100	36
Divisor	1101	13
Quotient	010	2
Remainder	1010	10

Answer is :

$100100_{(2)} \div 1101_{(2)} = 010_{(2)}$

Q3) $111011_{(2)} \div 11$



Step 1 : First 2 Digit of dividend i.e.$11_{(2)} = 3_{(10)}$ equal to divisor i.e.$11_{(2)} = 3_{(10)}$. Thus quotient is 1, after subtraction, reminder is 0

Step 2 : Take next number (1) from dividend , $1_{(2)} = 1_{(10)}$ is less than divisor i.e. $11_{(2)} = 3_{(10)}$. Thus quotient is 0, get the (1) as it is without subtraction

Step 3 : Take next number (0) from dividend, $10_{(2)} = 2_{(10)}$ is less than to divisor i.e.$11_{(2)} = 3_{(10)}$. Thus quotient is 0, get the (10) as it is without subtraction

Step 4 : Take next number (1) from dividend , $101_{(2)} = 5_{(10)}$ is greater than divisor i.e. $11_{(2)} = 3_{(10)}$. Thus quotient is 1, subtract (5-3=2) Binary equivalent of 2 is $10_{(2)}$

Step 5 : Take next number (1) from dividend , $101_{(2)} = 5_{(10)}$ is greater than divisor i.e. $11_{(2)} = 3_{(10)}$. Thus quotient is 1, subtract (5-3=2) Binary equivalent of 2 is $10_{(2)}$

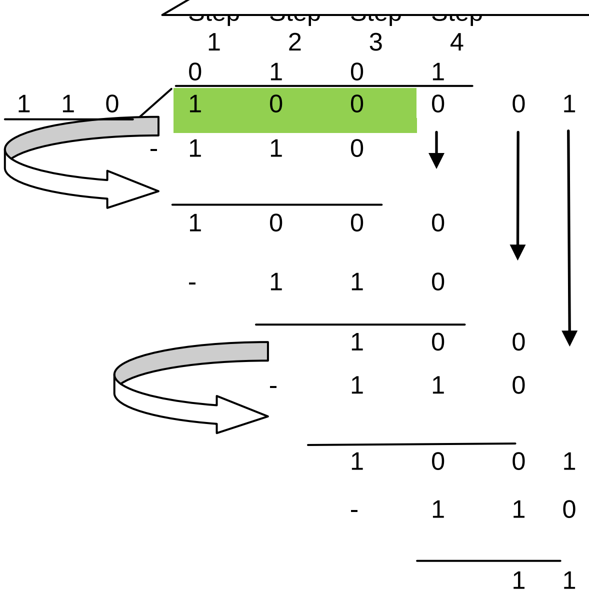
Step 6 : Remainder $10_{(2)} = 2_{(10)}$ is less than divisor i.e. $11_{(2)} = 3_{(10)}$. Thus stop.

	Binary No	Decimal equivalent
Dividend	111011	59
Divisor	11	3
Quotient	10011	19
Remainder	10	2

Answer is

$111011_{(2)} \div 11_{(2)} = 10011_{(2)}$

Q4) $100001_{(2)} \div 110$



Step 1 : First 3 Digit of dividend i.e.100 $_{(2)}$ $=4_{(10)}$ is less than to divisor i.e.$110_{(2)}$ $=6_{(10)}$. Thus quotient is 0, get the (100) as it is without subtraction

Step 2 : Take next number (0) from dividend ,$1000_{(2)}$ $=8_{(10)}$ is greater than divisor i.e. $110_{(2)}$ $=6_{(10)}$. Thus quotient is 1, subtract (8-6=2) Binary equivalent of 2 is $10_{(2)}$

Step 3 : Take next number (0) from dividend, $100_{(2)}$ $=4_{(10))}$ is less than to divisor i.e.$110_{(2)}$ $=6_{(10)}$. Thus quotient is 0, get the (100) as it is without subtraction

Step 4 : Take next number (1) from dividend , $1001_{(2)}$ $=9_{(10)}$ is greater than divisor i.e. $110_{(2)}$ $=6_{(10)}$. Thus quotient is 1, subtract (9-6=3) Binary equivalent of 3 is $11_{(2)}$

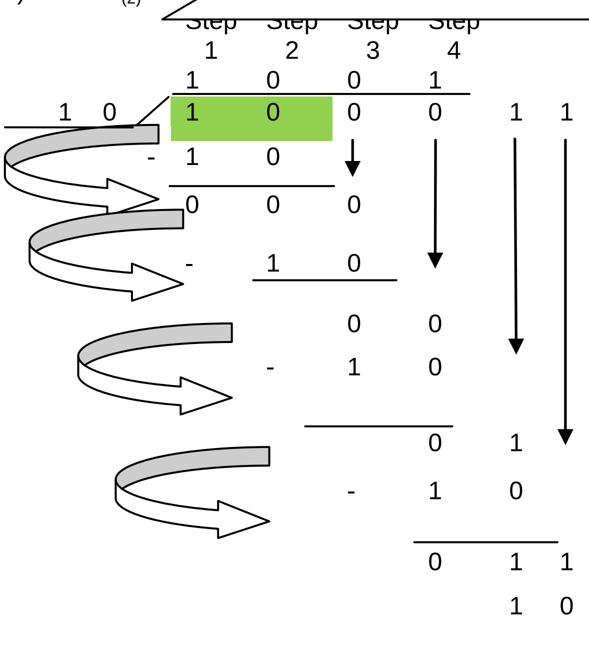
Step 5 : Remainder $11_{(2)}$ $=3_{(10)}$ is less than divisor i.e. $110_{(2)}$ $=6_{(10)}$. Thus stop.

	Binary No	Decimal equivalent
Dividend	100001	33
Divisor	110	6
Quotient	0101	5
Remainder	11	3

Answer is :

$100001_{(2)} \div 110_{(2)} = 101_{(2)}$

Q5) $100011_{(2)} \div 10$



Step 1 : First 2 Digit of dividend i.e.10 $_{(2)}$ $=2_{(10)}$ equal to divisor i.e.$10_{(2)}$ $=2_{(10)}$. Thus quotient is 1, after subtraction, reminder is 0

Step 2 : Take next number (0) from dividend ,$0_{(2)}$ $=0_{(10)}$ is less than to divisor i.e.$10_{(2)}$ $=2_{(10)}$. Thus quotient is 0, get the (0) as it is without subtraction

Step 3 : Take next number (0) from dividend, $0_{(2)}$ $=0_{(10)}$ is less than to divisor i.e.$10_{(2)}$ $=2_{(10)}$. Thus quotient is 0, get the (0) as it is without subtraction

Step 4 : Take next number (1) from dividend , $01_{(2)}$ $=1_{(10)}$ is less than to divisor i.e.$10_{(2)}$ $=2_{(10)}$. Thus quotient is 0, get the (01) as it is without subtraction

Step 5 : Take next number (1) from dividend , $11_{(2)}$ $=3_{(10)}$ is greater than divisor i.e. $10_{(2)}$ $=2_{(10)}$. Thus quotient is 1, subtract (3-2=1) Binary equivalent of is $1_{(2)}$

Step 6 : Remainder $1_{(2)}$ $=1_{(10)}$ is less than divisor i.e. $10_{(2)}$ $=2_{(10)}$. Thus stop

	Binary No	Decimal equivalent
Dividend	100011	35
Divisor	10	2
Quotient	1001	17
Remainder	1	1

Answer is :

$$100011_{(2)} \div 10_{(2)} = 101_{(2)}$$

Binary Division Practice Questions

QUESTIONS	ANSWERES	
	Quotient	Remainder
1. $10010011_{(2)} \div 1011_{(2)} = ?_{(2)}$	01101	100
2. $10101_{(2)} \div 11_{(2)} = ?_{(2)}$	0111	00
3. $1100_{(2)} \div 100_{(2)} = ?_{(2)}$	11	00
4. $1111100_{(2)} \div 10_{(2)} = ?_{(2)}$	111110	00
5. $11010_{(2)} \div 101_{(2)} = ?_{(2)}$	101	01
6. $1011110_{(2)} \div 101_{(2)} = ?_{(2)}$	10010	100
7) $11110_{(2)} \div 111_{(2)} = ?_{(2)}$	100	10
8) $100000_{(2)} \div 10_{(2)} = ?_{(2)}$	10000	00
9) $101101_{(2)} \div 11_{(2)} = ?_{(2)}$	1111	00
10) $10110_{(2)} \div 1010_{(2)} = ?_{(2)}$	10	10

CONCLUSION

Computer is a machine which is based on the principles of mathematics. Binary number system is primarily representation of state of each circuit in a computer where 1 means the state is on and 0 means the state is off. When we type some letters or words, the computer translates them in numbers as computers can understand only numbers. The number based conversions are essential in digital electronics, we have the input in decimal format but it takes as binary number for the computation. Understanding arithmetic operation of binary number systems is extremely useful in many computer-related fields. I encourage you to become very familiar with binary addition, subtraction, multiplication and division. I hope you've learned a lot from this article with easy and graphical presentation.

REFERENCES

[1]. https://www.researchgate.net/publication/320677641_Number_System

[2]. https://www.tutorialspoint.com

[3]. https://swayam.gov.in/nc_details/NPTEL