Role Of End-Fitting On The Fundamental Bending Mode Frequency Of A Structural Support Bracket

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Abstract – In many occasions, a small object like a sensor is placed at a distance from the spacecraft body by using a long bracket. Such brackets have small end fittings to facilitate its mounting on to the spacecraft body. One of the important considerations for the design of such brackets is the need of having the fundamental bending mode frequency to be more than certain specified value. As the end fitting is short and considered to be stiff, the natural frequency is calculated without considering the end fitting. Through several experimental and theoretical investigations, it is shown here that the fundamental bending mode frequency of such support bracket is very much influenced by the stiffness of the end fitting. The end fitting acts like a rotational spring. The rotational stiffness of this specific end fitting is also determined.

Index Terms – natural frequency, beams, fundamental bending mode, end fitting, vibration

1 NTRODUCTION

At several instances it is necessary to mount a sensor or antenna element at some distance from the spacecraft body. To have a feel, the mass of the sensor could be around 400 g and may be placed at a distance of 300 mm. This could be achieved using a support bracket of required length at the end of which the sensor is placed. This could be made out of a hollow circular tube, often a tubular structure made of composite material, with required additional elements at the end to enable the mounting. The sensor is mounted at one end. The other end of the support bracket is attached to the spacecraft body through an end fitting. The end fitting a small (say 30 mm) metallic piece one end of which accommodates the circular support structure and the other end has a rectangular pattern of holes to connect it to the body of the spacecraft. As the spacecraft is subjected to low frequency sinusoidal vibrations, one of the important design driver for this support bracket is to have the fundamental bending mode frequency more than certain specified value. This is normally specified for a fixed condition where the end fitting is connected to the spacecraft body. As the end fitting is very small and expected to be very stiff, it is thought that the drop in the fundamental bending mode frequency of the support structure due to the end fitting is negligible. Therefore, while designing the support bracket for the specified bending mode frequency, the usual practice is to meet the requirement on the natural frequency without considering the end fitting. One such support bracket is shown in Fig. 1. It can be seen that (Fig. 2) the end fitting is very small and stiff. The natural frequency of the bracket without considering the end fitting is estimated as 204 Hz. It is thought that with the end fitting the natural frequency would not change significantly. But the experiment conducted, where the end fitting is also present, showed the natural frequency to be 123 Hz, which is a very significant drop.

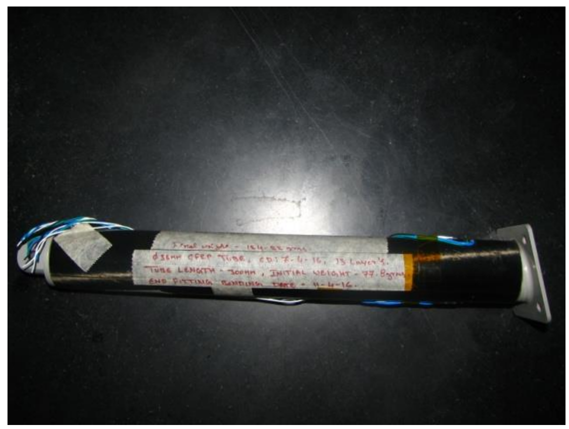

Fig. 1 *The support bracket*

The reasons for the above discussed reduction in the fundamental bending mode frequency are investigated and the results are presented here. It is found that the end fitting has a significant role on the bending mode frequency. These conclusions are arrived at based on systematic experiments and finite element model. Influence of end conditions on the fundamental bending mode frequency of a beam has been a subject of research [1,2]. Once the end stiffness is determined, the natural frequencies can be obtained through analytical method [3] or methods like FEM [4]. The main difficulty is in determining the end stiffness. Several methods are being pursued in identifying the end stiffness [5,6]. Some of them are based on the measured natural frequencies [7] and in some other techniques measured Frequency Response Functions are used [8]. In this work simple static load tests are performed and those results are used in determining the rotational stiffness of the end fitting.

Fig. 2 *The end fitting*

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2. BENDING MODE FRERQUENCY OF THE SUPPORT STRUCTURE

Details of the support structure, its estimated fundamental mode frequency and the experimental results are presented here.

2.1 Details of the support structure

Fig.1 shows the support structure, which consists of a tubular (hollow) structure made of Carbon Fibre Reinforced Plastics (CFRP) bonded and riveted to the end-fitting. The element seen at the other end of the support bracket represents the sensor. The tube has a length of 300 mm. Its inner diameter is 38 mm and the thickness is 1.235 mm. The tube is made of 13 layers of CFRP and the stacking sequence of 30/0/0/0/-30/0/30/0/-30/0/0/0/30. The properties of each layer are given in Table 1.

Table 1 *Properties of CFRP ply*

Properties	Values
Young's modulus in fibre direction	28×10^{10} N/m^2
Young's modulus in transverse direction	0.66×10^{10} N/m^2
In plane shear modulus	0.39×10^{10} N/m^2
Major Poisson's ratio	0.23
Thickness of a ply	95 microns
Density of ply	1600 kg/m^3
Mass per unit area of a ply	0.15 kg/m^2

The calculated [9] elastic properties of the laminate are as given in Table 2. The suffix 11 stands for the axial direction of the tube and 22 stands for tangential direction.

Table 2 *Elastic Properties of the laminate*

Properties	Values
A_{11}	29×10^7 N/m
A_{22}	1.7×10^7 N/m
A_{12}	2.6×10^7 N/m
A_{66}	2.9×10^7 N/m
A_{16}	0.84×10^7 N/m
A_{26}	0.29×10^7 N/m

The mass of the tubular structure is about 40 g. The end fitting is made of aluminium alloy and it has a length of 30 mm. It accommodates the tube at one and the other end has four through holes to interface with the spacecraft body. The tube is placed outside of the end fitting and it is then bonded and riveted to the tube for a length of 20 mm. Four M6 bolts are used to assemble the support structure with the spacecraft body applying a torque of 7.5 N m. The holes have a pitch of 42 mm. In the experiment the sensor is simulated by a mass of 350 g which is placed at a distance of 20 mm from the tip of the tube.

2.2 Theoretical Prediction

Theoretical prediction is made using Finite Element Method (FEM). As mentioned earlier, it was thought that the end fitting does not significantly affect the bending mode frequency. Therefore, the finite element model is developed without the end fitting. In the theoretical model, the tubular support structure is fixed where the end-fitting ends. The length of the tube in the finite element model is 300 mm (300 +20 – 20). The finite element model is shown in Fig. 3.

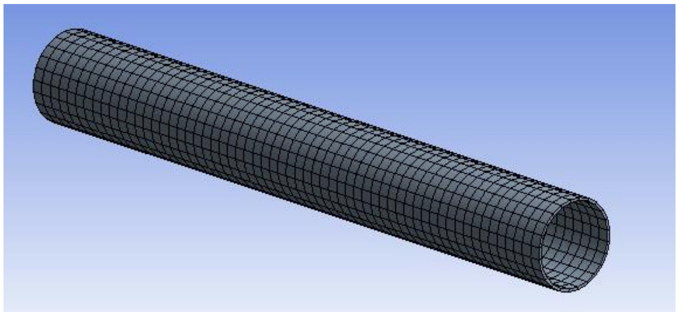

Fig.3 *The finite element model of the support structure*

Table 3 *List of modes and their frequencies*

Mode	Frequency (Hz)	Mode description
1	201	Lateral bending (Y)
2	201	Lateral bending (Z)
3	2596	Axial (X)

Quadrilateral plate elements having four nodes are used to create the model. The size of the element is approximately 5 mm x 5 mm. The first 3 natural modes are estimated using ANSYS software and the results are given in Table 3. A convergence study is also carried out. The fundamental bending mode frequency is 201 Hz if the element size is 3 mm x 3 mm. Therefore, 201 Hz is taken as the frequency of the lateral bending mode. The above results are verified through another software NASTRAN. The corresponding modes are 204 Hz, 204 Hz and 2676 Hz with a FE mesh having element size of 5 mm x 5 mm.

2.3 Experimental Results

The support structure with the simulated mass at the tip is subjected to a resonance search test. The fundamental bending mode frequency is obtained as 123 Hz. It is seen that the experimentally determined natural frequency is significantly lower than the theoretically estimated value.

3. VERIFICATION OF ELASTIC PROPERTIES

As there is a huge difference between the estimated and the observed natural frequencies, investigation is first directed towards the correctness of the elastic properties of the tube. The elastic properties used in the finite element model are the values determined through the tests conducted on laminates. These laminates are made of several layers all layers either along 0° or all along 90° or all along 45°. From the results of these tests, properties of a single layer are determined. They are given in Table 1.

Fig. 4 The support tube under static load test

The properties of the given laminate are then derived [1] from the properties of the lamina and given in Table 2. Though this methodology is normally followed, as the manufacturing process parameters used for the composite tube and the laminate are slightly different, it is thought that whether the elastic properties of the tube are different from those derived from unidirectional laminae. To verify this, the tube is subjected to axial tensile load and the strains are measured. The measured strains are compared with those obtained through finite element model with properties of lamina as given in Table 1. The tube under static load test is shown in Fig. 4. The tensile load applied is 20 kN. The strains are measured at 2 locations (diametrically opposite) in the middle region of the tube. The measured results do not indicate any bending. The measured axial strain is 624 με and the measured transverse strain is -1134 με. The strains are computed using Classical Lamination Plate Theory (CLPT). The computed axial strain is 648 με and the transverse strain is -1000 με. The above results confirm that the elastic properties of the laminate determined from the properties of the laminae are valid.

4. BENDING TESTING OF THE SUPPORT STRUCTURE

As it is concluded that the elastic properties of the tubular structure are properly represented, it is then thought to verify the bending stiffness. For this purpose, the support structure is experimentally subjected to a lateral load. The test setup is shown in Fig. 5.

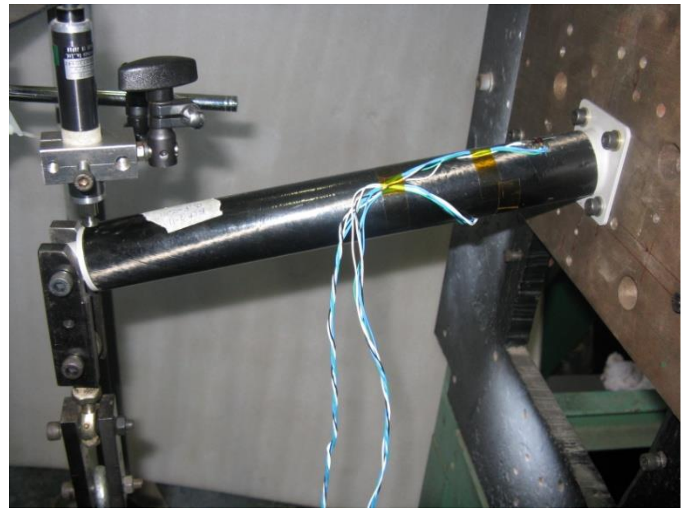

Fig. 5 The support structure under bending test

A lateral load of 1010 N is applied. As an available end fitting was used in the experiment, the tubular structure is at an angle of 13° with respect to the horizontal. Therefore, in the experiment an axial load of 227 N (1010 sin 13) acts and the lateral load causing bending is 984 N (1010 cos 13). The strains measured near the base of the tube along the longitudinal direction are 769 με and -952 με (at the two diametrically opposite locations). This implies an axial strain of -91 με and bending strain of 860 με. The strains are theoretically estimated using CLPT. The computed axial strain is negligible. The computed bending strain is 980 με. This once again proves the correctness of the properties and the finite element model. During the above experiment, the deflection at the tip of the bracket is also measured. The measured deflection for the lateral load of 984 N is 4.6 mm. The displacement in the lateral direction is 4.72 mm (4.6 / cos 13). The same is estimated through the finite element model and the deflection is 1.78 mm. The above results were verified through another software NASTRAN. The corresponding deflection is 1.73 mm.

5. DISCUSSION OF THE RESULTS

As the measured strains are close to the theoretically determined strains, the deflection is also expected to be close to what is theoretically estimated. It can be seen that the measured deflection is very much higher than the estimated bending deflection. The large difference seen is thus attributed to the rotation of the tubular structure at the base. This is due to the rotation of the end fitting. It is to be noted that the end fitting was absent in the theoretical model. The additional deflection due to this rotation is 2.94 mm. The angle of rotation is 0.0098 radian which is equal to 0.56 degrees. This translates into a rotational stiffness of 30 kN m/ rad (525 Nm /degree). To confirm that the difference in the natural frequency is due to the rotational stiffness of the end fitting not being considered, the frequency of the lateral mode incorporating the rotational stiffness needs to be determined. This is done in two ways, directly from the measured deflection and through FEM incorporating the torsional stiffness. It is seen during the bending test that a lateral load of 984 N gives a lateral deflection of 4.72 mm. This amounts to a stiffness of 208 kN/m. Therefore, the natural frequency of the bending mode with a mass of 350 g is 122.7 Hz. It is to me noted that the uniformly distributed mass of 40 g (mass of the tubular structure) is not considered in the above calculation. Using Dunkerley's formula [10,11] this effect can be included in the estimation of the fundamental natural frequency and it is estimated as 121 Hz which is very close to the frequency obtained through the experiment. Thus it is seen that if we use the measured deflection which includes the rotation due the end fitting, the natural frequency estimated is very close to the experimental value, thus confirming the role of the end fitting.The above result is verified through FEM. A torsion spring having stiffness of 30 kN m/ rad is attached at the end of the bracket representing the end fitting. The frequency of the fundamental bending mode is computed as 122 Hz, the experimentally determined frequency being 123 Hz. Thus it is seen that the end fitting, even if it is very small, can influence the bending modes of the support structure. Considering this as a rotational stiffness one can estimate the fundamental bending mode frequency very accurately. Not incorporating it can result in a large error. The rotational stiffness is obtained through a static load bending test.

6. SUMMARY AND CONCLUSIONS

It is shown here that the fundamental bending mode frequency of a bracket is very much influenced by the stiffness of the end fitting. The end fitting acts like a rotational spring. The rotational stiffness of this specific end fitting is 30 kN m / rad. These conclusions are made through several experimental and theoretical investigations. Though the end fitting is very small, when a long bracket is attached to the end fitting, the rotational stiffness of the end fitting plays a very important role in the fundamental bending mode frequency of the bracket. The results point out that the bending mode natural frequencies of such brackets cannot be improved merely by stiffening the tube but needs stiffening of the end fitting.

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