Electron Acceleration By The Use Of Segmented Cylindrical Electrodes In An Inverse Free Electron Laser

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Abstract- In this paper we expend a theory of high gradient laser excited electron accelerator by the use of an inverse free-electron laser (IFEL), but with using new structure and design. The wiggler used in our scheme, that is to say Paul wiggler, is obtainedby segmented cylindrical electrodes with applied oscillatory voltages$V_{osc}(t)$over 90-degrees segments. The inverse free-electron laser interaction can be demonstrated by the equations that govern the electron motion in the composed fields of both laser pulse and Paul wiggler field. A numerical research of electron energy and electron trajectories has been made using fourth order Runge-Kutta method. The results show that the electron gains the maximum energy at a short distance for high wiggler amplitude intensities a_{0w}. In addition, it is discovered that the electron energy gains various peaks for different initial axial velocities. It is seen that aappropriate small initial axial velocity of e-beam produces remarkably high energy gain. According to the transverse limitation of the electron beam in a Paul wiggler, there is no applied axial guide magnetic field in this devise.

Keywords: Electron acceleration, Inverse free-electron laser, Paul wiggler, Runge-Kutta method, Oscillatory voltage, cylindrical electrodes

1 INTRODUCTION

Laser-driven accelerators are taken as a potential layout to create experimental facilities that generate high energy particle beams [1, 2]. To lessen the size and expense of the particle accelerators, different laser-based advanced acceleration techniques, like a laser wake-field accelerator [3, 4] and inverse free-electron laser [5-8], have been proponed.In an IFEL (inverse free-electron laser), relativistic electrons copropagate via a laser beam through a wiggler. The effect of wiggler field is to supply a pairing between the e-beam and electromagnetic radiation fields those results in a pondermotive force along the axis of the beam. In addition, the wiggler produces a small oscillatory transverse velocity in a direction parallel to the electric vector of the electromagnetic wave hence energy can be transferred from the wave to the electrons [9, 10]. Recent experiments have demonstrated that inverse-FEL can reach both very high-energy gradient and relatively good output beam quality [11, 12]. Other considerable advantages over different advanced accelerator schemes consist the fact that inverse-FEL does not need any medium such as plasma or dielectric in the interaction region [12, 16]. In this article we expend a theory of inverse-FEL based on an electrostatic wiggler. This wiggler is produced by segmented cylindrical electrodes with applied oscillatory voltages $\mathbf{V_{osc}(t)}$ over 90-degrees segments. The scheme of this paper is as follows. In Section 2, the relativistic equations of motion for an electron in the Paul wiggler inverse-FEL are derived.Eventually, the outcomes of numerical studies of electron orbits and electron energy gain are provided in Section 3.

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2 Theoretical model

Suppose a Paul wiggler with oscillatory voltages $V_{osc}(t)$ with $V_{osc}(t+T) = V_{osc}(t)$ and $\int_0^T V_{osc}(t)dt = 0$ over 90° segments. In lots of applications, the electrodes can be excited sinusoidally $V_{osc}(t) = V_{o\,max}\sin(2\pi ft)$, while$f = 1/T$ is the oscillation frequency. Representing the applied electric field the appropriate boundary conditions at $r = r_w$ is given by[22]

$$\emptyset(r,t) = \frac{4V_{o\,max}}{\pi}\sin(2\pi ft)\sum_{l=1}^{\infty}\frac{\sin(\frac{l\pi}{2})}{l}(\frac{r}{r_w})^{2l}\cos(2l\theta) \quad (1)$$

For $0 \le r \le r_w$and $0 \le \theta \le 2\pi$. Near the cylinder axis $(r \ll r_w)$, the recent equation gives to the lowest order

$$q\emptyset(r,t) = \frac{m}{2}\Gamma\sin(2\pi ft)(x^2 - y^2), \quad (2)$$

while the oscillation quadruple focusing coefficient Γ is specified by $\Gamma = 8qV_{o\,max}/m\pi r_w^2$. The electric field of the periodic quadruple focusing transport system (i.e., Paul wiggler) is given by

$$E_\omega = \frac{8eV_{o\,max}}{m\pi r_\omega^2}\sin(2\pi ft)\,[x\hat{e}_x - y\hat{e}_y] \quad (3)$$

The inverse-FEL interaction can be demonstrated by the equations that govern the electron motion in the composed fields of a laser pulse and a PT-wiggler field. The laser pulse is supposewith the following vector potential,

$$A_l = -A_0\ \mathrm{Sin}(\omega t - kz)\ \exp[-(t-(z-z_l))/c)^2/\tau_l{}^2)]\hat{e}_x \quad (4)$$

Where τ_l is the pulse duration,$k = (\omega/c)$ is the laser wavenumber, andz_l is the initial position of the pulse peak. At the first step we introduce the equation corresponding to the vector potential of wiggler field as follows,

$$A_w = \frac{-8V_{0max}}{\omega_w\pi r_\omega^2}\mathrm{Cos}(\omega_w t)(x\hat{e}_x - y\hat{e}_y) \quad (5)$$

Then, the Potential vector components of the combined laser and wiggler fields, and the E and B fields generated by them are as follows,

$$A_x = -A_0 \mathrm{Sin}(\omega t - kz)\exp\left[-\frac{\left(\frac{t-(z-z_l)}{v_g}\right)^2}{\tau_l^2}\right] - \frac{8V_{0max}}{\omega_w \pi r_\omega^2}\mathrm{Cos}(\omega_w t)x \quad (6)$$

$$A_y = \frac{8V_{0max}}{\omega_w \pi r_\omega^2}\mathrm{Cos}(\omega_w t)y \quad , \quad A_z = 0 \quad (7)$$

$$B_x = 0 \;, B_y = \frac{\partial}{\partial z}(A_L + A_{wx}), B_z = 0 \quad (8)$$

$$E_x = -\frac{1}{c}\frac{\partial}{\partial t}(A_L + A_{wx}) \;, E_y = -\frac{1}{c}\frac{\partial}{\partial t}(A_{wy}), E_z = 0 \quad (9)$$

An analysis of relativistic motion of an electron will be based on Lorentz equation,

$$\frac{dP}{dt} = -e[E_{laser} + E_{wiggler} + \frac{V}{c}\times B_{laser}] \quad (10)$$

By considering the equations of electric and magnetic fields, Eqs.(11-14) the scalar equations of momentum and energy of an electron are as follows,

$$\frac{dP_x}{dt} = -e\left[E_x + \left(\frac{V}{c}\times B\right)_x\right] = \frac{e}{c}\left[\frac{\partial}{\partial t}(A_L + A_{wx}) + +V_z\frac{\partial}{\partial z}(A_L)\right] \quad (11)$$

$$\frac{dP_y}{dt} = -e\left[E_y + \left(\frac{V}{c}\times B\right)_y\right] = \frac{e}{c}\frac{\partial}{\partial t}(A_{wy}) \quad (12)$$

$$\frac{dP_z}{dt} = -e\left[E_z + \left(\frac{V}{c}\times B\right)_z\right] = \frac{-e}{c}V_x\frac{\partial}{\partial z}(A_L) \quad (13)$$

$$\frac{d\gamma}{dt} = \frac{e}{m_0c^3}\left[V.\frac{\partial A}{\partial t}\right] = \frac{e}{m_0c^3}\left[V_x\frac{\partial}{\partial t}(A_L + A_{wx}) + V_y\frac{\partial}{\partial t}(A_{wy})\right] \quad (14)$$

$$\frac{dP}{dt} = \frac{d(m_0\gamma V)}{dt} = m_0\gamma\frac{dV}{dt} + m_0V\frac{d\gamma}{dt} \quad (15)$$

So that,

$$\frac{dV_x}{dt} = \frac{1}{m_0\gamma}\left(\frac{e}{c}\left[\frac{\partial}{\partial t}(A_L + A_{wx}) + V_z\frac{\partial}{\partial z}(A_L)\right] - m_0V_x\frac{d\gamma}{dt}\right) \quad (16)$$

$$\frac{dV_y}{dt} = \frac{1}{m_0\gamma}\left(\frac{e}{c}\left[\frac{\partial}{\partial t}(A_{wy})\right] - m_0V_y\frac{d\gamma}{dt}\right) \quad (17)$$

$$\frac{dV_z}{dt} = \frac{1}{m_0\gamma}\left(\frac{-e}{c}V_x\frac{\partial}{\partial z}(A_L) - m_0V_z\frac{d\gamma}{dt}\right) \quad (18)$$

$$\frac{d\gamma}{dt} = \frac{e}{m_0c^3}\left[V_x\frac{\partial}{\partial t}(A_L + A_{wx}) + V_y\frac{\partial}{\partial t}(A_{wy})\right] \quad (19)$$

In the following survey we will use the dimensionless variables $t \to \omega t$, $\tau \to \omega\tau$, $\tau_l \to \omega\tau_l x \to kx$, $y \to ky$, $z \to kz$, $z_l \to kz_l$, $\frac{dx}{dt} \to \frac{k}{\omega}\frac{dx}{dt}$, $\frac{dy}{dt} \to \frac{k}{\omega}\frac{dy}{dt}$, $\frac{dz}{dt} \to \frac{k}{\omega}\frac{dz}{dt}$, $k \to \frac{kc}{\omega}$, $\frac{\lambda_l}{\lambda_w} \to \frac{k_w}{k}$. By using these variables, we can write Eqs. (16-19) as follows:

$$\frac{dV_x}{dt} = \frac{k}{\gamma}\left(\frac{\partial}{\partial t}(a_L + a_{wx}) + V_z\frac{\partial}{\partial z}(a_L) - \frac{1}{\gamma}V_x\frac{d\gamma}{dt}\right. \quad (20)$$

$$\frac{dV_y}{dt} = \frac{k}{\gamma}\left(\frac{\partial}{\partial t}a_{wy}\right) - \frac{1}{\gamma}V_y\frac{d\gamma}{dt} \quad (21)$$

$$\frac{dV_z}{dt} = \frac{k}{\gamma}\left(V_x\frac{\partial}{\partial z}a_L\right) - \frac{1}{\gamma}V_z\frac{d\gamma}{dt} \quad (22)$$

$$\frac{d\gamma}{dt} = \frac{1}{k}\left(V_x\frac{\partial}{\partial t}(a_L + a_{wx}) + V_y\frac{\partial}{\partial t}a_{wy}\right) \quad (23)$$

where $a_L = -a_0\ \mathrm{Sin}(\omega t - kz)\ \exp[-(t-(z-z_l))/c)^2/\tau_l{}^2)]\hat{e}_x$, $a_w = -a_{0w}\ \mathrm{Cos}(\omega_w t)(x\hat{e}_x - y\hat{e}_y)$, $a_0 = \frac{eA_0}{mc^2}$ and $a_{0w} = e8V_{0max}/mc^2\omega_w\pi r_\omega^2$.

Equations (20-23) together with the relations $v_x = \frac{dx}{dt}$, $v_y = \frac{dy}{dt}$, $v_z = \frac{dz}{dt}$ are seven coupled equations which are solved numerically using the fourth order Runge-Kutta method.

3 Numerical Results And Discussions

A numerical study of electron energy and electron trajectories in the inverse free electron laser based on an electrostatic wiggler has been made in this section.

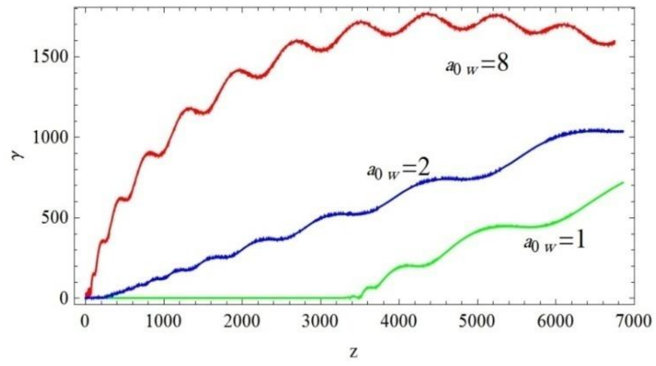

Fig. 1 *Electron energy* γ *as a function of z for* $a_0 = 2$, *k =1,* a_{0w} *=1, 2, and 8, and* v_{0z} *=0.97.*

Figs.1 shows the alteration of the electron energy with the normalized distance z for different wiggler amplitude intensities $a_{0w} = 1, 2$ and 8, respectively. It is seen that the electron attains the maximum energy at a short distance for high a_{0w}(i.e.,$a_{0w} = 8$). In short distances, the electron energy gain augments with the distance z and attains at maximum after sometime.

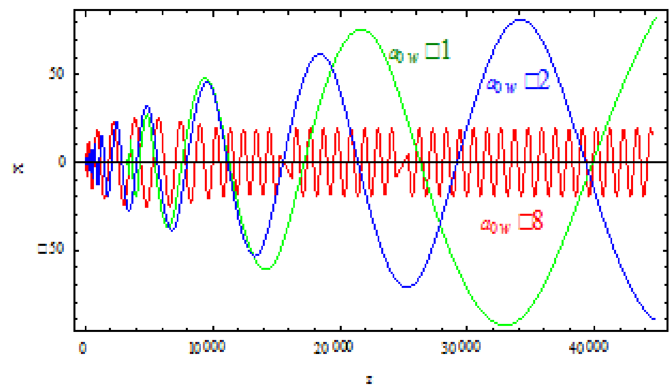

Fig. 3 *Electron trajectories in the x – z plane for* $a_0 = 2$,*k =1,* a_{0w} *=1, 2, and 8*

Figs. 2 shows the transverse component of electron trajectory in (x,z)-plane.In this figures, the electron tolerates a ponderomotive force due to laser pulse in the presence of the Paul wiggler field. So, it oscillates as it drives in the wiggler. At longer z, these oscillations become larger because of lower a_{0w}.

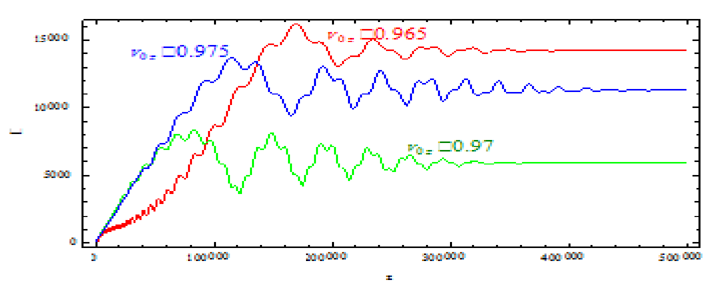

Fig. 6 *Electron energy* γ *as a function of z for* $a_0 = 2$, *k =1,*

a_{0w} = *8, and*
v_{0z}= *0.965 and 0.97, respectively.*

To represent the effect of V_{0z} on electron acceleration, we have plotted the energy gain with z for different V_{0z}in Fig. 3. It is obvious from this figure that the electron energy oscillates and becomes greater in a stepwise manner with z. The electron energy gains various peaks for different initial axial velocities. It is seen that the electrons gain energy about 4 GeV (for $V_{0z} = 0.97\,c$) and 8 GeV (for $V_{0z} = 0.965\,c$), respectively.One may note from this figure that small initial velocity causessignificantly high energy gain.

4 Conclusion

In summary, in this paper we have demonstrated the electron acceleration in an inverse free electron laser by an electrostatic wiggler which named Paul wiggler. A Paul wiggler makes practical and effective uses of oscillatory voltages applied to the external electrodes to supply transverse confinement of the e-beam. Rather, segmented cylindrical electrodes have utilized oscillatory voltages $V_{osc}(t)$ over 90° segments. In this article, the electron energy gain and electron trajectories were considered by numerical simulations. It was found that the electron experiences a ponderomotive force caused by laser pulse in the presence of the Paul wiggler field. accordingly, it oscillates as it drives in the wiggler. It has been discovered that the electron achieves the maximum energy at a short distance for high wiggler amplitude intensities a_{0w} but cannot retain it sufficiently after passing the laser pulse.it was perceived that the electron energy obtains different peaks for different initial axial velocities. It was also found that anappropriate small initial axial velocity of e-beam produces considerably high energy gain.

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