The Structure Of Mathematical Reasoning In Mathematical Problems

Hery Suharna, Idrus Alhaddad

ABSTRACT: The mathematic students (prospective teachers) often experience constraints when understanding math problems, one of them that they did mistake of constructing mathematical concepts. This thing was the causal factor when students solve math problems. These errors should not happen for a math student who later becomes a math teacher. Researchers wanted to reveal the problems in order to get a description of errors of constructing concept in overcoming the problems experienced by the student. Therefore, it is necessary knowledge of mathematical reasoning structure of student in solving math problem. Based on the problems and objectives of the study, this type of research is descriptive qualitative which aimed at describing the mathematical reasoning structure of students when solving math problems. The results of the study found some descriptions (1) students' mathematical reasoning that was connective, (2) inductive mathematical reasoning, and (3) deductive mathematical reasoning.

Keywords: Reasoning structures, mathematical problems, connective reasoning, inductive reasoning, and deductive reasoning.

1. Introduction

Students often make mistakes when solving math problems is wrong mathematical concepts, math procedures mistakes, errors in constructing concepts, etc. This results in an improper math problem solving when working on the problem. Student misconduct when solving the math problem needs to get the attention of all circles. These mistakes have an impact on the understanding of students on the next mathematical concept. Brodie (2010) in the results of his research explained that students' errors in building mathematical reasoning include: basic error, appropriate error, missing information, partial insight. Gal & Linchevski (2010) found that student difficulties in geometric representation included: (1) perceptual organization: Gestalt principles, (2) recognition: bottom-up and top-down processing; and (3) representation of perception-based knowledge: verbal vs. pictorial representation, mental images and hierarchical structure of images. Bingobali, dkk (2010) explores the causes of student math difficulties based on lecturers' views, which include: Epistemological causes, Psychological causes, Pedagogical cause. It was further discovered that students had difficulty in understanding concepts, abstracting concepts, and relating mathematics to everyday life. Therefore reasoning has a very important role in overcoming student difficulties. This study examines more in depth the mistakes made by mathematics students when solving mathematical problems by looking at the mathematical reasoning structure of students in solving mathematical problems. The study was conducted to see in more detail the types of student errors, especially from the aspects of thinking when constructing mathematical concepts. By knowing the types of errors made with the structure of reasoning, latter can be designed a learning through scaffolding or remedial scheme that will be used to restructure students' thinking.

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2. Theory of Review

Krulik (2003:89) divide the phases of thinking into four categories namely: (1) recall thinking, (2) basic thinking, (3) critical thinking, dan (4) creative thinking. Explanation of the phases of thinking according to Krulik (2003: 89) can be presented in Figure 1. as follows.

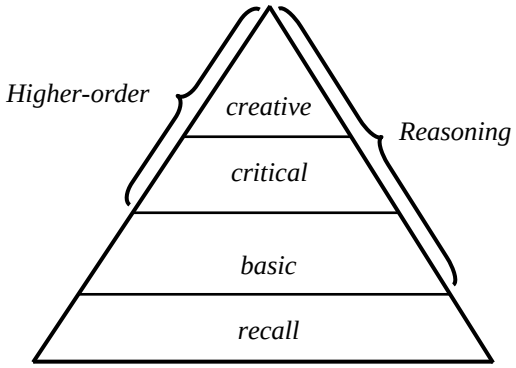

Figure 1. *Thinking Stages Adopted from Krulik (2003)*

The lowest stage of thinking is remembering. At the remembering stage, one's thinking process does not use logical processes or analytic processes. The thinking process at the level of remembering takes place automatically. For example, a student was asked how many 5 × 5 + 10, he did not really think but automatically answered 35. The second stage of thinking is basic thinking, a more general form of thinking. When someone is faced with the problem of buying 2 pairs of clothes, each costing Rp100.000,00. shirt price and Rp200.000.00. price of trousers; think will multiply 2 by Rp100.000.00, and 2 times Rp200.000.00, the result of the price of 2 shirts and 2 pants summed, resulting in Rp600.000.00 ,. Therefore one already uses his reasoning by performing multiplying and summing operations. Critical thinking is the third stage of thinking, characterized by analyzing problems, determining the adequacy of facts to solve problems, and deciding on the need for more information in a problem, and analyzing something. At this stage of thinking also includes recognizing the consistency of data, can explain the conclusions of a set of data, and can decide the validation of a conclusion. The fourth stage of thinking is creative

thinking, characterized by the ability to solve problems in unusual, unique and different ways. As Gauss was asked to add numbers 1 through 100, only a few minutes were able to complete. In the aspect of reasoning is needed in the name of reflective thinking reconstruction. This is in accordance with the results of research submitted by Suharna (2015) states that, reflective thinking reconstruction is a process that begins with the perplexity and overcome perplexity by connecting between concepts (conceptualization). The conceptualization in question is to match all related concepts, principles, and mathematical processes.

3. Research Methods

3.1. Types of research

Based on the problems and objectives of the research, the type of Action Research is descriptive or action research.

3.2. Research focus

First year This research focuses on describing the reconstruction of student's thinking structure in doing reflection of math problem solving.

3.3. Data collection technique

Data collection techniques in the study developed the mathematical reasoning structure of students thinking reflective students through reflective thinking reconstruction that is (1) think aloud or think out aloud, (2) conducting interviews, (3) acquisition of data intent is the result of interview, (4) researchers as the main instrument to observe, analyze research data, interpret research data and make conclusions based on the characteristics of students' mathematical reasoning structure reflective thinking through reflective thinking reconstruction, (5) last to collect data through discussion centered, aiming to find meaning related to the formulation of the issues raised.

3.4. Data analysis

Description of the process of analysis of research data can be seen in Diagram 2. Flow analysis of research data can be presented in Diagram 2. as follows:

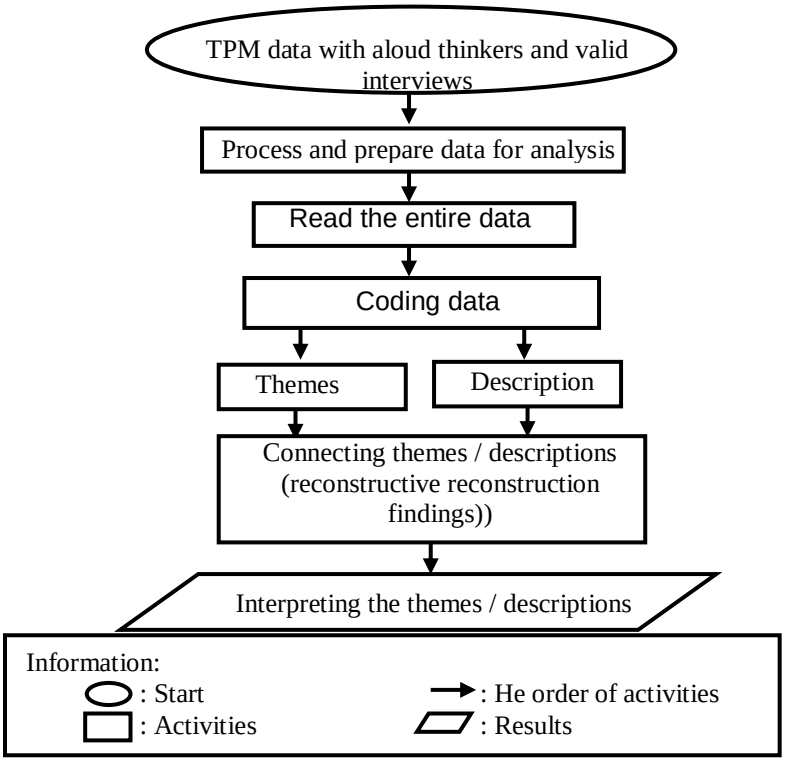

Diagram 2. *Data Analysis Process*

4. Research Results

1. Student's convective mathematical reasoning in solving math problems

This study found 3 (three) mathematical reasoning process of students in solving math problems. The three findings are: (1) convective mathematical reasoning, (2) inductive mathematical reasoning, and (3) deductive mathematical reasoning. Students who are the subject of research is S-1. S-1 thinking process in solving mathematical problems, **dominated** by the characteristics of convective reasoning. The following is exposure and analysis of S-1 data. Furthermore the work of S-1 in solving math problems as follows.

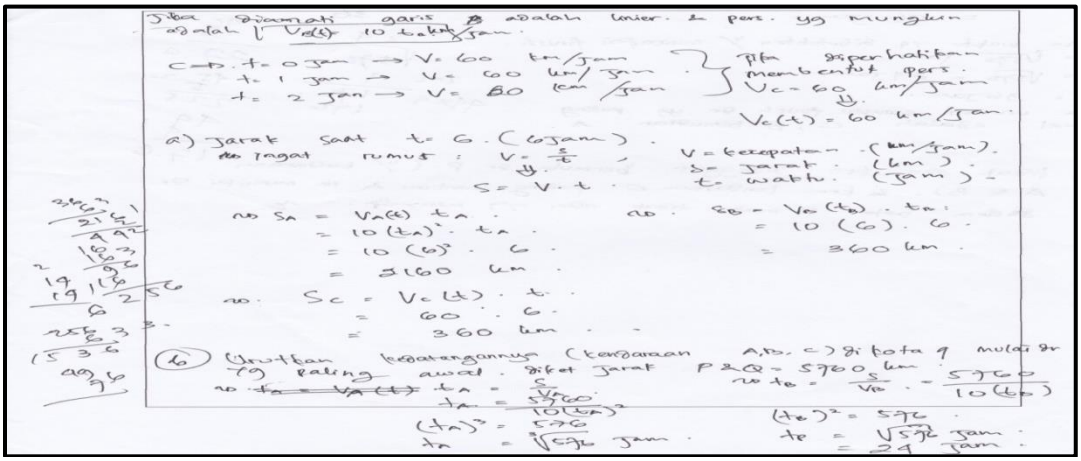

Figure 1. *The work of S1 at the planning stage of completion*

At the time of solving the mathematical problems, seen S-1 in the process of thinking, there is confusion (perplexity), then the effort made in answering the question (a) $t = 6$ (6 hour), by the formula: $v = \frac{s}{t}$ so $s = v.t$ when t v = speed (km / hour), $s =$ distance (km) and t: time. Distance of vehicle A moment $t = 6$ is $S_A = V_A(t_A)^2.t_A$ same 10 $(t_A)^2.t_A$ same 10 $(6)^2.6$ same 2160 km. Distance of vehicle B moment $t = 6$ is $S_B = V_B.(t_B)\,t_B$ same 10 $(t_B).t_B$ sama dengan 10 $(6).6$ same 360 km. Jarak kendaraan C saat $t = 6$ is $S_C = V_C.(t)\,t_C$ same 60.6 same 360 km. Next arrival (vehicles A, B, C) in Q city start from the earliest. Given distance P and Q = 5760 km. Then the arrival of vehicle A is $t_A = \frac{s}{v_A}$ so $t_A = \frac{5760}{10\,(t_A)^2}$ when $(t_A)^3 = 576$ so $t_A = \sqrt[3]{576}$ jam, the arrival of vehicle B is $t_B = \frac{s}{v_B}$ so $t_B = \frac{5760}{10\,(t_B)}$ so $(t_B)^2 = 576$ so $t_B = \sqrt{576}$ so 24 jam. The arrival of vehicle C is $t_C = \frac{s}{v_C}$ same $\frac{5760}{60}$ same $t_C = 96$ jam. Because the time it takes to reach the finish is $t_A = \sqrt[3]{576}$ jam, $t_B = \sqrt{576}$ so 24 jam, and $t_C = 96$ jam, then the sequence to finish from the earliest is vehicle C, vehicle B then vehicle A. As for the question (c) is there a vehicle that precedes the other? If so, please specify which vehicle and at what time? The answer is no, because it starts from P (for vehicle A and B) and because vehicle C is slowest and starts from before town P then no one crucified. The occurrence of mathematical reasoning S-1 when solving math problems can be seen in Diagram 3. below.

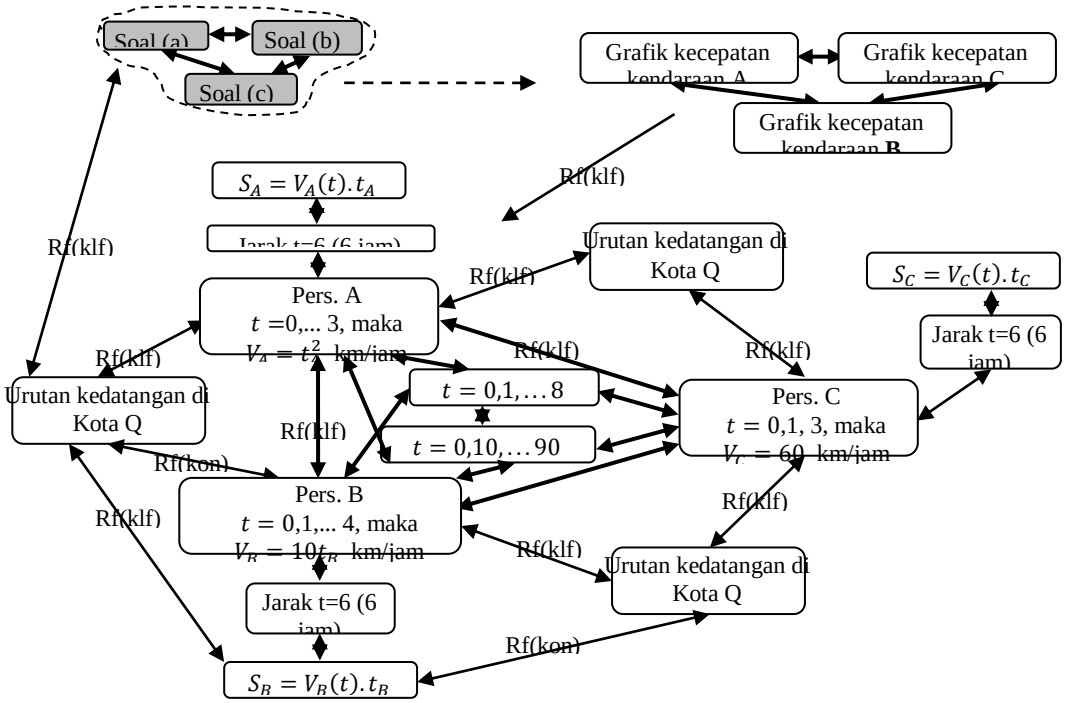

Diagram 3. *The occurrence of mathematical reasoning S-1 connectivity*

Based on Diagram 3, we can illustrate the occurrence of the S-1 constructive reasoning structure in solving mathematical problems. Constructive reasoning structure is the process of thinking in solving mathematical problems through connecting a concept with other related concepts.

The S-1 thinking process can be done by examining the flow of the accommodation process. The process of the occurrence of S-1 connectivity can be seen in Diagram 4. below.

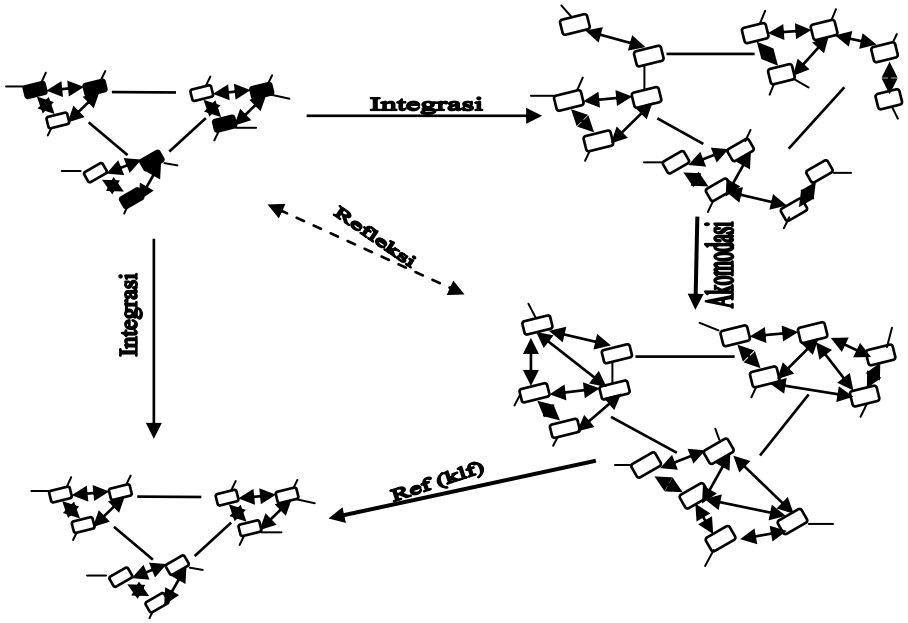

Diagram 4. *Occurrence of S-1 Conflict Reasoning Structure in Solving Mathematical Problems.*

Based on the above description, what is meant by students' mathematical reasoning that is konektif is the process of thinking in solving mathematical problems through the process of connecting a concept with other related concepts.

2. Inductive mathematical reasoning in solving mathematical problems

Students who are the subject of research are S-2. The process of S-2 thinking in solving mathematical problems, dominated by inductive reasoning traits. Following exposure and analysis of S-2 data. Furthermore, the work of S-2 in solving math problems as follows

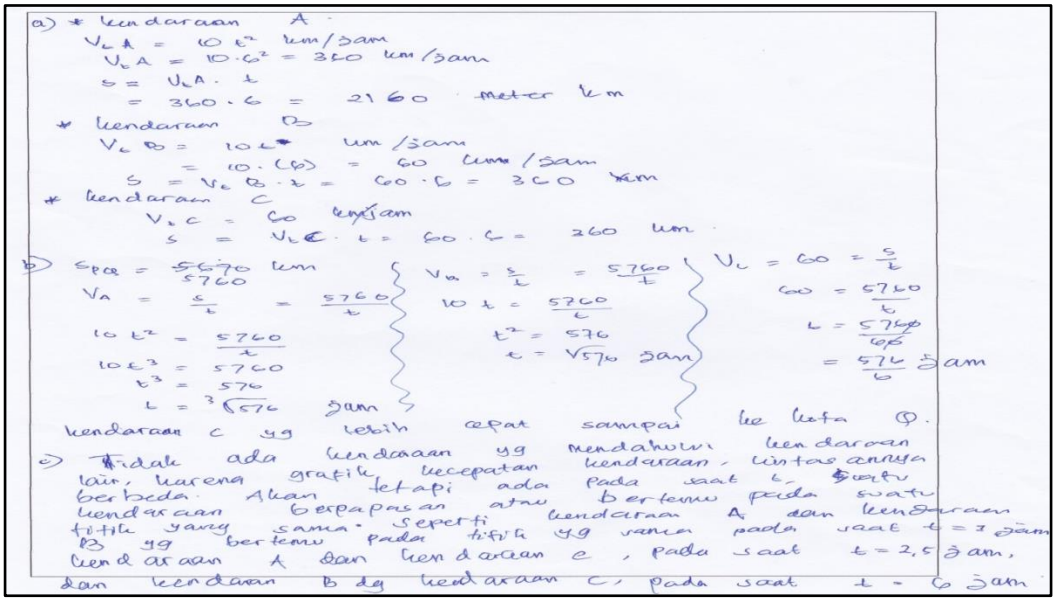

Figure 2. *The work of S-1 at the planning stage of completion*

The work of S-2 in Figure 2 in solving the mathematical problem, S-2 looks confusion (perplexity), then the effort made in answering the question (a) for vehicle A, $V_tA = 10t^2$ km/hour $\Leftrightarrow V_tA = 10.6^2 \Leftrightarrow V_tA = 360$ km/hour, so $S = V_tA.t$ obtained $S = 2160$ km. Vehicle B, $V_tB = 10t$ km/hour $\Leftrightarrow V_tB = 10.(6) \Leftrightarrow V_tB = 60$ km/hour, so $S = V_tB.t$ obtained $S = 60.6 = 360$ km. Vehicle C, $V_tC = 60$ km/hour, so $V_tC = 60$ km/hour, so $S = V_tC.t$ obtained$S = 60.6 = 360$ km. The reflection process that occurs in the S-2 is clarification, it can be seen from S-2 directly substituting the value at each t (time). Furthermore, in answering the question (b) the order of vehicle arrival in Q city starting from the earliest. The process of thinking S2 in answering the problem (b) solve that distance $S_{PQ} = 5760$ km, so $V_A = \frac{s}{t} \Leftrightarrow V_A = \frac{5760}{t}$ and $V_A = 10t^2$ so $10t^2 = \frac{5760}{t} \Leftrightarrow 10t^3 = 5760 \Leftrightarrow t^3 = 576$ obtained $t = \sqrt[3]{576}$ hour. For Vehicle B is $V_B = \frac{s}{t} \Leftrightarrow V_B = \frac{5760}{t}$ so $V_B = 10t$ so $10t = \frac{5760}{t} \Leftrightarrow 10t^2 = 5760 \Leftrightarrow t^2 = 576$ obtained $t = \sqrt{576}$ hour. Vehicle C is $V_C = 60 = \frac{s}{t}$ so $60 = \frac{5760}{t} \Leftrightarrow 60t = 5760 \Leftrightarrow t = \frac{576}{6}$ diperoleh $t = 96$ hour. The process of reflection that occurs in S-2 in answer question (b) is clarification, it can be seen from S-2 directly substituting value at each t (time). S-2 reasoning process for problem (c) no vehicle precedes another vehicle, because the speed graph of the vehicle, the path is different but there is at t, a vehicle passes or meets at a similar point. Such as vehicle A and vehicle B meet at the same point at time t = 1 hour vehicle A and vehicle C, at t = 2.5 hours, and vehicle B with vehicle C, at t = 6 hours. The occurrence of S-2 inductive reasoning in the problem solving stage can be seen in Diagram 5. below.

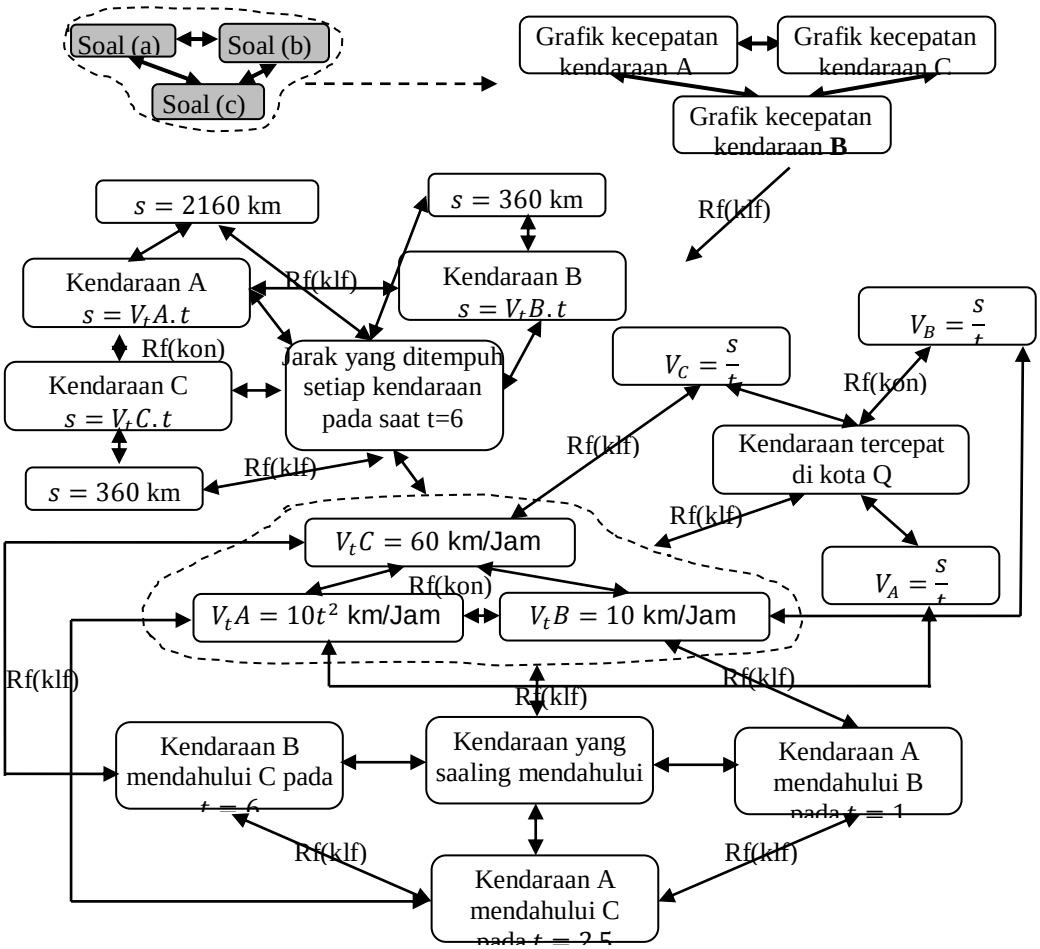

Diagram 5. *The occurrence of inductive reasoning S-2*

Based on Diagram 5, we can illustrate the occurrence of S-1 constructive reasoning structure in solving mathematical problems. The process of inductive mathematical reasoning is the process of thinking in solving mathematical problems through pattern recognition, guess and generalization formation. The occurrence of mathematical reasoning S2 in solving math problems can be seen in Diagram 6 below.

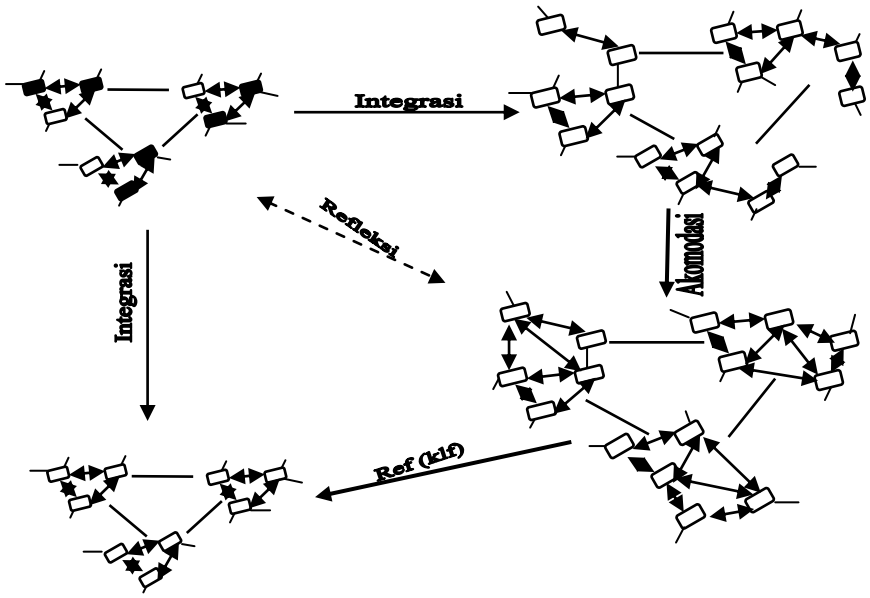

Diagram 6. *The occurrence of Inductive S-2 Reasoning Structure in Mathematical Problems Solving*

Based on the above description, what is meant by students' inductive mathematical reasoning is the process of thinking in solving mathematical problems through pattern recognition, conjecture and generalization formation.

3. Deductive reasoning in solving mathematical problems

Students who are the subject of research are S-3. The process of S-3 thinking in solving mathematical problems, dominated by the characteristics of deductive reasoning. The following is exposure and analysis of S-3 data. Further work of S-3 in solving the math problem as follows.

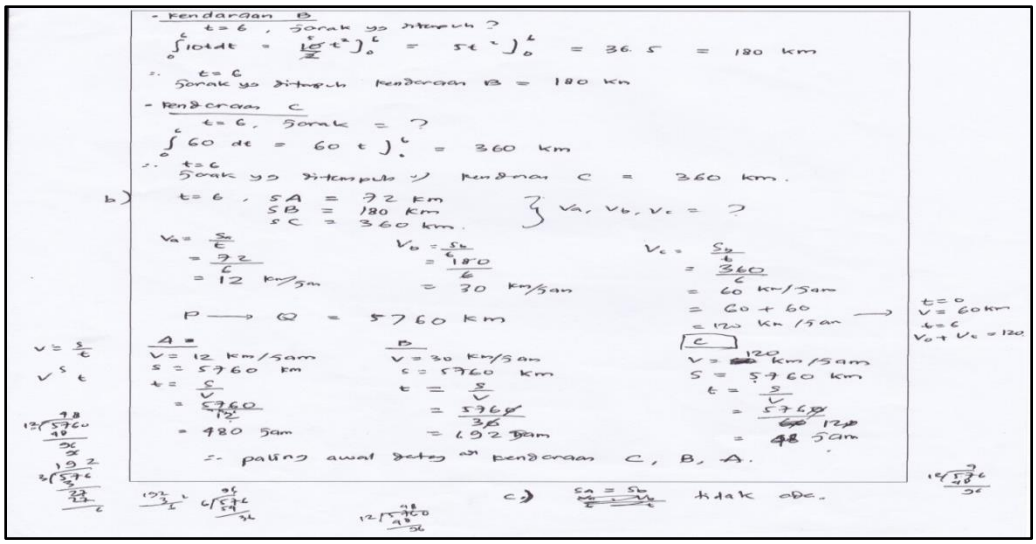

Figure 2. *The work of S-3 in solving math problems*

In the problem-solving stage, S-3 is seen in the process of thinking, there is confusion (perplexity), then efforts are made to overcome it for problem (a) when finding the distance to be used by using integral. S-3 equation graph of vehicle C so obtained 72 km, so vehicle A covering distance 72 km. The vehicle B continues to be 180 km, the distance traveled by vehicle B = 180 km. as well as the vehicle C distance traveled by vehicle C is 360 km. In the process of thinking S-3 in answer question (b) for t = 6 obtained $S_A = 72$ km, $S_B = 180$ km and $S_C = 360$ km. Likewise S-3 defines that V_a equals S_A per t equal to 72/6 obtained 12 km / h. Furthermore, V_b equals S_B per t obtained at 30 km / h, and vehicle speed C is V_c equal to $S_C$per t obtained 60 km / h, but since $V_0 = 60$ 0 means t = 0, V = 60 hold t = 0 fed $V_0 + V_t = 120$ so $V_c = 120$ km / h. Furthermore, because the distance of city P to city Q equal to 5760 km, then the time in vehicle A is if v = 12 km / h and s = 5760 km, then the required time is t = s / v equal to 5760/12 same with 480 hours. time of vehicle B ie if v = 30 km / h and s = 5760 km, then t = s / v equal to 5760/30 equals 192 hours; and the time of vehicle C is if v = 120 km / h and s = 5760 km, then t = s / v equals 5760/120 equals

48 hours. So the vehicle of the earliest order came is the vehicle C, B then vehicle A.Question (c) during the journey is there a vehicle that precedes each other's vehicle? If there is any mention of which vehicle and at what time?. In answer to this problem S3 argues that $S_a = S_b$, $S_a = S_c$, and $S_a = S_b$, $S_a = S_c$, concludes that there is no vehicle that precedes each other. The occurrence of S-3 mathematical reasoning in the problem solving phase can be seen in Diagram 7. below.

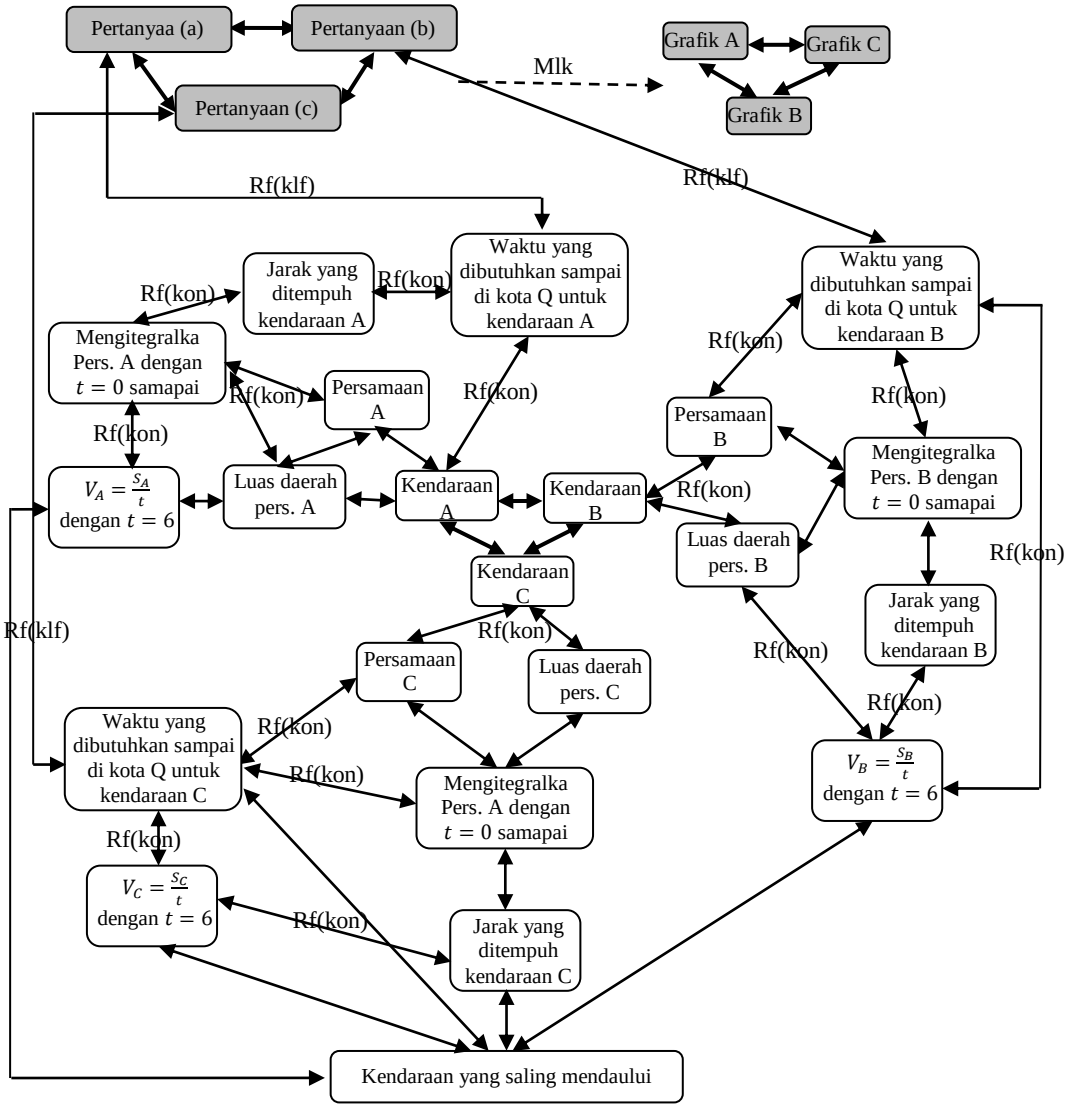

Diagram 7. *The reasoning process of reflective thinking S-3 in solving mathematical problems*

Based on diagram 7, we can illustrate the occurrence of S-3 inductive reasoning structure in solving mathematical problems. The process of thinking in solving math problems through the process of drawing conclusions from the general things to the special things. Deductive reasoning in making conclusions is based on generalized generalizations and special statements and does not accept generalizations from observations. The occurrence of mathematical reasoning S-3 in solving math problems can be seen in Diagram 8. below.

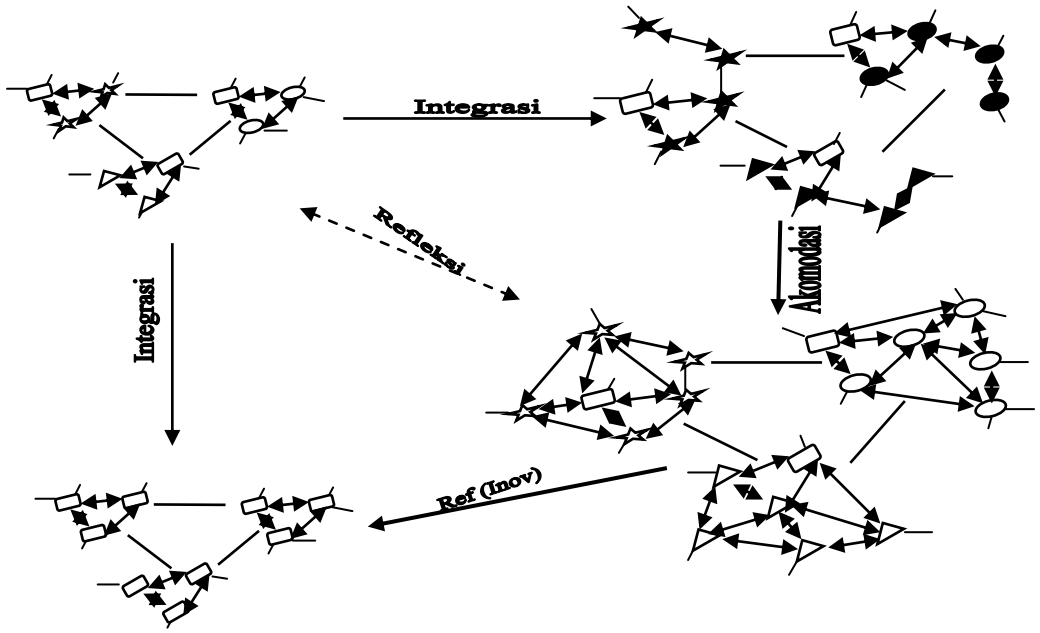

Diagram 8. *The process of occurring Structural deductive reasoning S-3 in Solving Mathematical Problems*

Deductive reasoning is the process of thinking in solving math problems through the process of drawing conclusions from the general things to the special things. Deductive reasoning in making conclusions is based on generally accepted generalizations and special statements and does not accept generalizations from observations

5. Conclusions

1. Student's mathematical reasoning that is konektif is the process of thinking in solving math problems through connecting a concept with other related concepts.
2. Inductive mathematical reasoning is the process of thinking in solving mathematical problems through pattern recognition, conjecture and generalization formation, and
3. Deductive reasoning is the process of thinking in solving math problems through the process of drawing conclusions from the general things to the special things. Deductive reasoning in making conclusions is based on generalized generalizations and special statements and does not accept generalizations from observations.

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