Equitable Coloring Of Prism Graph And It's Central,Middle, Total And Line Graph

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Abstract— A proper vertex coloring of a graph is equitable if the sizes of color classes differ by atmost one. The notion of equitable coloring was introduced by Meyer in 1973. In this paper we find the equitable chromatic number for Prism graph(Y_n), the central graph of prism graph C(Y_n), the middle graph of the prism graph M(Y_n), the total graph of the prism graph T(Y_n) and the line graph of the prism graph L(Y_n).

Index Terms—central graph, equitable coloring, equitable chromatic number, line graph, middle graph, prism graph, total graph.

1 Introduction

All graphs considered in this paper are connected, finite and simple. i.e.,undirected, loopless and without multiple edges.

If the set of vertices of a graph G can be partitioned into k classes $V_1, V_2, \dots, V_k$ such that each V_i is independent set and the condition $||V_i| - |V_j|| \leq 1$ holds for every pair of (i, j), then G is said to be equitably k-colorable. The smallest integer k for which G is equitably k-colorable is known as Equitable chromatic number of G and it is denoted by $\chi_{=}(G)$ [7, 5, 10, 1]. Since Equitable coloring is a proper coloring with additional constraints, we have $\chi(G) \leq \chi_{=}(G)$ for any graph G [3]. In some discrete industrial systems one can encounter the problem of equitable partitioning of a system with binary conflicting relations into conflict-free sub systems. Such situations can be modelled by means of a equitable graph coloring.

For example, in garbage collection problem [8] the vertices of the graph represent the garbage collection routes and pair of vertices is joined by an edge if the corresponding routes should not be run on the same day. The problem of assigning one of the six days of the week to each route thus reduces to the problem of six-coloring of the graph. In practice it might be desirable to have an approximately equal number of routes run on each of the six days. So one have to color the graph in an equitable way with six colors. In contrast to the ordinary proper coloring of the graph, the equitable coloring does not possess monotonicity, namely a graph could be equitably k-colorable without being equitably $(k + 1)$ colorable. It seems to be that maximum degree plays a crucial role here [2].

In this paper we investigate the equitable chromatic number of Prism Graph $(\chi_{=}(Y_n))$, the central graph of prism graph $\chi_{=}(\mathrm{C}(Y_n))$ the middle graph of the prism graph $\chi_{=}(\mathrm{M}(Y_n))$, the total graph of the prism graph $\chi_{=}(\mathrm{T}(Y_n)$ and the line graph of the prism graph$\chi_{=}(\mathrm{L}(Y_n))$.

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2.1 Preliminaries:

Some basic definitions are stated below for the work,

Definition: 2.1

For a given graph $G = (V, E)$, we do an operation on G by subdividing each edge exactly once and joining all the non-adjacent vertices of G. The graph obtained by this process is called *central graph* [6] of G denoted by $C(G)$.

Definition: 2.2

Let G be a graph with vertex set $V(G)$ and the edge set $E(G)$. The *middle graph* [9, 10, 11] of the graph G denoted by $M(G)$ is defined as follows: The vertex set of $M(G)$ is $V(G) \cup E(G)$ in which two vertices x and y are adjacent in $M(G)$ if the following conditions holds

- $x,y \in E(G)$, x,y are adjacent in G.
- $x \in V(G)$, $y \in E(G)$ and they are incident in G.

Definition: 2.3

Let G be a graph with vertex set $V(G)$ and the edge set $E(G)$. The *total graph* [9, 10, 11] of the graph G is denoted by $T(G)$ and is defined as follows: The vertex set of $T(G)$ is $V(G) \cup E(G)$. Two vertices x and y are adjacent in $T(G)$ if the following conditions holds

- $x,y \in E(G)$, x,y are adjacent in G.
- $x,y \in V(G)$, x,y are adjacent in G.
- $x \in V(G)$, $y \in E(G)$, x,y are adjacent in G.

Definition: 2.4

The *line graph* [9, 10] of a graph G, denoted by $L(G)$ is a graph whose vertices are the edges of G and if $u,v \in E(G)$ then $uv \in E(L(G))$ if u and v share a vertex in G.

Definition: 2.5

A *Prism graph* Y_n sometimes also called a circular ladder graph and denoted by CL_n is a graph corresponding to the skeleton of an n-prism. An n-prism has $2n$ nodes and $3n$ edges. Y_n is isomorphic to the graph cartesian product [4] $Ln = K2 \square Cn$ where K_2 is the complete graph with 2 vertices and C_n is the cycle of n nodes. The prism graph Y_n consists an inner cycle and an outer cycle both connected by joining the corresponding vertices.

Throughout this paper, let $\{v_i: 1 \le i \le n\}$ and $\{u_i: 1 \le i \le n\}$ denote the vertices of inner and outer cycle taken in cyclic order, respectively. Let $\{e_i: 1 \le i \le n\}$ and $\{e_i': 1 \le i \le n\}$ denote the edges of inner and outer cycle taken in cyclic order, respectively. Let $\{s_i: 1 \le i \le n\}$ denote the edge $\{u_i v_i: 1 \le i \le n\}$.

3 Equitable coloring of Prism graph

Theorem 3.1

The Equitable chromatic number of prism graph Y_n, where n is any positive integer is

$$\chi_=(Y_n) = \begin{cases} 2, & n \text{ is even} \\ 3, & n \text{ is odd} \end{cases}$$

Proof

Let $V(Y_n) = \{v_1, v_2, \dots, v_n\} \cup \{u_1, u_2, \dots, u_n\}$ where v_i's are the vertices of the inner cycle taken in cyclic order and u_i's are the vertices of the outer cycle taken in cyclic order such that each $v_i u_i$ is the edge connecting the two cycles.

Case 1: If n is even

Let us partition the vertex set of the prism graph $V(Y_n)$ as

$$V_1 = \{v_1, v_3, \dots, v_{n-1}\} \cup \{u_2, u_4, \dots, u_n\}$$

and

$$V_2 = \{v_2, v_4, \dots, v_n\} \cup \{u_1, u_3, \dots, u_{n-1}\}$$

Clearly V_1 and V_2 are two independent sets of $V(Y_n)$. Also $|V_1| = |V_2| = n$. This holds the inequality $||V_i| - |V_j|| \le 1$ ∀ (i,j) satisfying equitable coloring.

$$\Rightarrow \chi_=(Y_n) \le 2$$

Since $\chi(Y_n) \ge 2$, $\chi_=(Y_n) \ge \chi(Y_n) \ge 2$, we have $\chi_=(Y_n) \ge 2$.
Therefore $\chi_=(Y_n) = 2$.

Case 2: If n is odd

The equitable coloring is done in three different ways

1. If $n = 3k$, $k = 1,3,5,7,9, \cdots$ then the partition of V is done as follows:

$$V_1 = \{v_{3i-2} : 1 \le i \le k\} \cup \{u_{3i-1} : 1 \le i \le k\}.$$
$$V_2 = \{v_{3i-1} : 1 \le i \le k\} \cup \{u_{3i} : 1 \le i \le k\}.$$
$$V_3 = \{v_{3i} : 1 \le i \le k\} \cup \{u_{3i-2} : 1 \le i \le k\}.$$

Clearly V_1, V_2 and V_3 are independent sets of $V(Y_n)$. Also $|V_1| = |V_2| = |V_3| = 2k$. This holds the inequality $||V_i| - |V_j|| \le 1$ ∀ (i,j) satisfying equitable coloring.

2. If $n = 3k - 1$, $k = 2,4,6,8,10, \cdots$ then the partition of V is done as follows:

$$V_1 = \{v_{3i-2} : 1 \le i \le k\} \cup \{u_{3i-1} : 1 \le i \le k\}.$$
$$V_2 = \{v_{3i-1} : 1 \le i \le k\} \cup \{u_{3i} : 1 \le i \le k-1\}.$$
$$V_3 = \{v_{3i} : 1 \le i \le k-1\} \cup \{u_{3i-2} : 1 \le i \le k\}.$$

Clearly V_1, V_2 and V_3 are independent sets of $V(Y_n)$. Also $|V_1| = 2k - 1, |V_2| = |V_3| = 2k$. This holds the inequality $||V_i| - |V_j|| \le 1$ ∀ (i,j) satisfying equitable coloring.

3. If $n = 3k - 2$, $k = 3,5,7,9,11,\dots$, then the partition of V is done as follows:

$$V_1 = \{v_{3i-2} : 1 \le i \le k-1\} \cup \{u_{3i-1} : 1 \le i \le k\}.$$
$$V_2 = \{v_{3i-1} : 1 \le i \le k\} \cup \{u_{3i} : 1 \le i \le k-1\}.$$
$$V_3 = \{v_{3i} : 1 \le i \le k-1\} \cup \{u_{3i-2} : 1 \le i \le k-1\}.$$

Clearly V_1, V_2 and V_3 are independent sets of $V(Y_n)$. Also $|V_1| = |V_2| = 2k-1, |V_3| = 2k-2$. This holds the inequality $||V_i| - |V_j|| \le 1 \forall (i,j)$ satisfying equitable coloring.
From case 2, $\chi_=(Y_n) \le 3$.
Since $\chi(Y_n) \ge 3$ and $\chi_=(Y_n) \ge \chi(Y_n) \ge 3$, we have $\chi_=(Y_n) \ge 3$.
Therefore $\chi_=(Y_n) = 3$.

4 Equitable coloring of Central graph Of Prism graph

Theorem 4.1

The Equitable chromatic number of central graph of prism graph Y_n, where n is any positive integer is

$$\chi_=(C(Y_n)) = n,\ n \ge 3.$$

Proof

Let $V(Y_n) = \{v_1, v_2, \cdots, v_n\} \cup \{u_1, u_2, \cdots, u_n\}$ where v_i's are the vertices of the inner cycle taken in cyclic order and u_i's are the vertices of the outer cycle taken in cyclic order and $E(Y_n) = \{e_i : 1 \le i \le n-1\} \cup \{e_n\} \cup \{e_i' : 1 \le i \le n-1\} \cup \{e_n'\} \cup \{s_i' : 1 \le i \le n\}$ where e_i is the edge $v_i v_{i+1}$ $(1 \le i \le n-1)$, e_n is the edge $v_n u_1$, e_i' is the edge $u_i u_{i+1}$ $(1 \le i \le n-1)$, e_n' is the edge $u_n u_1$ and s_i is the edge $v_i u_i$ $(1 \le i \le n)$.

By the definition of central graph $V(C(Y_n)) = V(Y_n) \cup E(Y_n) = \{v_i : 1 \le i \le n\} \cup \{u_i : 1 \le i \le n\} \cup \{v_i' : 1 \le i \le n\} \cup \{u_i' : 1 \le i \le n\} \cup \{s_i' : 1 \le i \le n\}$, where v_i', u_i' and s_i' represents the edge e_i, e_i' and s_i respectively and joining all the non-adjacent vertices of Y_n in $C(Y_n)$.

Case 1: n is even

Assign the colors $\{c_1, c_1, c_2, c_2, \dots, c_{n/2}, c_{n/2}\}$ to the consecutive vertices $\{v_1, v_2, v_3, v_4 \dots, v_{n-1}, v_n\}$ and $\{u_1', u_2', u_3', u_4', \dots, u_{n-1}', u_n'\}$ the colors $\{c_{(n/2)+1}, c_{(n/2)+1}, c_{(n/2)+2}, c_{(n/2)+2}, \dots, c_n, c_n\}$ to the consecutive vertices $\{u_1, u_2, u_3, u_4 \dots, u_{n-1}, u_n\}$ and $\{v_1', v_2', v_3', v_4', \dots, v_{n-1}', v_n'\}$ and the colors $\{c_2, c_3, c_4, \dots, c_{n-2}, c_{n-1}, c_n, c_1\}$ to the consecutive vertices $\{s_1', s_2', s_3', s_4', \dots, s_{n-1}', s_n'\}$.
Now we partition $V(C(Y_n)$ as follows

$$V_1 = \{v_1, v_2, u_1', u_2', s_n'\}$$

$$V_2 = \{v_3, v_4, u_3', u_4', s_1'\}$$

$$\vdots$$

$$V_{n-1} = \{u_{n-3}, u_{n-2}, v_{n-3}', v_{n-2}', s_{n-1}'\}$$

$$V_n = \{u_{n-1}, u_n, v_{n-1}', v_n', s_{n-2}'\}$$

Clearly $V_1, V_2, \cdots, V_{n-1}, V_n$ are independent sets of $V(C(Y_n))$. Also $|V_1| = |V_2| = |V_3| = \cdots = |V_n| = 5$. This holds the inequality $||V_i| - |V_j|| \le 1 \ \forall \ (i,j)$ satisfying equitable coloring.
$\Rightarrow \chi_{=}(C(Y_n)) \le n$.
For each i, v_i is non-adjacent with v_{i-1} and v_{i+1}.
Hence $\chi(C(Y_n)) \ge n$.
$\Rightarrow \chi_{=}C(Y_n) \ge \chi(C(Y_n)) \ge n$.
Hence $\chi_{=}C(Y_n) \ge n$.
Therefore $\chi_{=}C(Y_n) = n$

Case 2: n is odd

Assign the colors $\{c_1, c_1, c_2, c_2 \ldots c_{\lfloor (n/2) \rfloor}, c_{\lfloor (n/2) \rfloor}\}$ to the consecutive vertices $\{v_1, v_2, v_3, v_4, \cdots, v_{n-2}, v_{n-1}\}$ and $\{u_1', u_2', u_3', u_4', \cdots, u_{n-2}', u_{n-1}'\}$,
the colors $\{c_{\lfloor (n/2)+2 \rfloor}, c_{\lfloor (n/2)+2 \rfloor}, c_{\lfloor (n/2)+3 \rfloor}, c_{\lfloor (n/2)+3 \rfloor}, \ldots c_n, c_n\}$ to the consecutive vertices $\{u_1, u_2, u_3, u_4, \cdots, u_{n-2}, u_{n-1}\}$, the colors $\{c_{\lfloor (n/2)+1 \rfloor}, c_{\lfloor (n/2)+1 \rfloor}, c_{\lfloor (n/2)+2 \rfloor}, c_{\lfloor (n/2)+2 \rfloor}, \ldots c_{n-1}\}$to $\{v_1', v_2', v_3', v_4', \cdots, v_{n-2}', v_{n-1}'\}$, the colors $\{c_2, c_3, c_4, \ldots, c_{n-2}, c_{n-1}, c_n, c_1\}$ to the vertices $\{s_1', s_2', s_3', \cdots, s_{n-2}', s_{n-1}', s_n'\}$. The vertices u_n and v_n are assigned the color $c_{\lfloor (n/2)+1 \rfloor}$ and the vertices e_n and e_n' are assigned the color c_n.

Now we partition $V(C(Y_n)$ as follows

$$V_1 = \{v_1, v_2, u_1', u_2', s_6'\}$$

$$V_2 = \{v_3, v_4, u_3', u_4', s_1'\}$$

$$\vdots$$

$$V_{n-1} = \{u_{n-4}, u_{n-3}, v_{n-2}', v_{n-1}', s_{n-2}'\}$$

$$V_n = \{u_{n-2}, u_{n-1}, v_n', u_n', s_n'\}$$

Clearly $V_1, V_2, \cdots, V_{n-1}, V_n$ are independent sets of $V(C(Y_n))$. Also $|V_1| = |V_2| = \cdots = |V_{n-1}| = |V_n| = 5$. This holds the inequality $||V_i| - |V_j|| \le 1 \ \forall \ (i,j)$ satisfying equitable coloring.
$\Rightarrow \chi_{=}(C(Y_n) \le n$.
For each i, v_i is non-adjacent with v_{i-1} and v_{i+1}. Hence $\chi(C(Y_n) \ge$ n. $\Rightarrow \chi_{=}C(Y_n) \ge \chi(C(Y_n)) \ge n$, Hence $\chi_{=}C(Y_n) \ge$ n.
Therefore $\chi_{=}C(Y_n) = n$.

5. Equitable coloring of middle graph of a Prism graph

Theorem 5.1 *The Equitable chromatic number of middle graph of prism graph Y_n, where n is any positive integer is*

$$\chi_{=}(M(Y_n)) = 5, \ n > 4.$$

Proof
Let $V(Y_n) = \{v_1, v_2, \cdots, v_n\} \cup \{u_1, u_2, \cdots, u_n\}$ where v_i's are the vertices of the inner cycle taken in cyclic order and u_i's are the vertices of the outer cycle taken in cyclic order and $E(Y_n) = \{e_i : 1 \le i \le n-1\} \cup \{e_n\} \cup \{e_i' : 1 \le i \le n-1\} \cup \{e_n'\} \cup \{s_i' : 1 \le i \le n\}$ where e_i is the edge $v_i v_{i+1}$ $(1 \le i \le n-1)$, e_n is the edge $v_n u_1$, e_i' is the edge $u_i u_{i+1}$ $(1 \le i \le n-1)$, e_n' is the edge $u_n u_1$ and s_i is the edge $v_i u_i$ $(1 \le i \le n)$.

By the definition of middle graph $V(M(Y_n)) = V(Y_n) \cup E(Y_n) = \{v_i : 1 \le i \le n\} \cup \{u_i : 1 \le i \le n\} \cup \{v_i' : 1 \le i \le \text{n}\} \cup \{u_i' : 1 \le i \le n\} \cup \{s_i' : 1 \le i \le n\}$, where $v_i', u_i'$and s_i'represents the edge $e_i, e_i'$and s_i,respectively.

Case 1: n is even.
Now, partition the vertex set $V(M(Y_n))$ as follows

$$V_1 = \{v_{2i-1} : \{1 \le i \le n/2\}\} \cup \{u_{2i-1}' : \{1 \le i \le n/2\}\}$$
$$V_2 = \{v_{2i} : \{1 \le i \le n/2\}\} \cup \{u_{2i-1} : \{1 \le i \le n/2\}\}$$
$$V_3 = \{u_{2i} : \{1 \le i \le n/2\}\} \cup \{v_{2i-1}' : \{1 \le i \le n/2\}\}$$
$$V_4 = \{v_{2i}' : \{1 \le i \le n/2\}\} \cup \{u_{2i}' : \{1 \le i \le n/2\}\}$$
$$V_5 = \{s_i' : \{1 \le i \le n\}\}$$

Clearly V_1, V_2, V_3, V_4 and V_5 are independent sets of $M(Y_n)$. Also $|V_1| = |V_2| = |V_3| = |V_4| = |V_5| = n$. This holds the inequality $||V_i| - |V_j|| \le 1 \ \forall \ (i,j)$ satisfying equitable coloring.
$\Rightarrow \chi_{=}M(Y_n) \le 5$.
Since $M(Y_n)$ contains a clique of order 5, $\chi(M(Y_n) \ge 5$.
$\Rightarrow \chi_{=}(M(Y_n)) \ge \chi(M(Y_n)) \ge 5$.
Hence $\chi_{=}(M(Y_n)) \ge 5$.
Therefore, $\chi_{=}(M(Y_n)) = 5$.

Case 2: n is odd
Let us partition the vertex set $V(M(Y_n))$ as follows

$$V_1 = \{v_{2i-1} : \{1 \le i \le n-1/2\}\} \cup \{u_{2i-1}' : \{1 \le i \le n-1/2\}\} \cup s_n'$$
$$V_2 = \{v_{2i} : \{1 \le i \le n-1/2\}\} \cup \{u_{2i}' : \{1 \le i \le n-1/2\}\} \cup \{v_n'\}$$
$$V_3 = \{u_{2i-1} : \{1 \le i \le n-1/2\}\} \cup \{v_{2i-1}' : \{1 \le i \le -1\,n/2\}\} \cup \{v_n\}$$
$$V_4 = \{u_{2i} : \{1 \le i \le n-1/2\}\} \cup \{v_{2i}' : \{1 \le i \le n-1/2\}\} \cup \{u_n'\}$$
$$V_5 = \{s_i' : \{1 \le i \le n-1\}\} \cup \{u_n\}$$

Clearly V_1, V_2, V_3, V_4 and V_5 are independent sets of $M(Y_n)$. Also $|V_1| = |V_2| = |V_3| = |V_4| = |V_5| = n$.

This holds the inequality $||V_i| - |V_j|| \leq 1 \ \forall \ (i,j)$ satisfying equitable coloring.
$\Rightarrow \chi_= (M(Y_n)) \leq 5$.
Since $M(Y_n)$ contains a clique of order 5, $\chi(M(Y_n)) \geq 5$.
$\Rightarrow \chi_= (M(Y_n)) \geq \chi(M(Y_n)) \geq 5$.
Hence we have $\chi_= (M(Y_n)) \geq 5$.
Therefore $\chi_= (M(Y_n)) = 5$.

6. Equitable coloring of total graph of a Prism graph

Theorem 6.1 *The Equitable chromatic number of total graph of prism graph Y_n, where n is any positive integer is*

$$\chi_= (T(Y_n)) = 5, \ n > 4.$$

Proof

Let $V(Y_n) = \{v_1, v_2, \cdots, v_n\} \cup \{u_1, u_2, \cdots, u_n\}$ where v_i's are the vertices of the inner cycle taken in cyclic order and u_i's are the vertices of the outer cycle taken in cyclic order and $E(Y_n) = \{e_i : 1 \leq i \leq n-1\} \cup \{e_n\} \cup \{e_i' : 1 \leq i \leq n-1\} \cup \{e_n'\} \cup \{s_i' : 1 \leq i \leq n\}$ where e_i is the edge $v_i v_{i+1}$ $(1 \leq i \leq n-1)$, e_n is the edge $v_n u_1$, e_i' is the edge $u_i u_{i+1}$ $(1 \leq i \leq n-1)$, e_n' is the edge $u_n u_1$ and s_i is the edge $v_i u_i$ $(1 \leq i \leq n)$.

By the definition of total graph $V(T(Y_n)) = V(Y_n) \cup E(Y_n) = \{v_i : 1 \leq i \leq n\} \cup \{u_i : 1 \leq i \leq n\} \cup \{v_i' : 1 \leq i \leq n\} \cup \{u_i' : 1 \leq i \leq n\} \cup \{s_i' : 1 \leq i \leq n\}$ where v_i', u_i' and s_i' represents the edge e_i, e_i' and s_i, respectively.

The proof of the theorem follows as in Theorem 5.1.

Note: For $n = 4$, $\chi_= (M(Y_n)) = \chi_= (T(Y_n)) = 4$.

7. Equitable coloring of Line graph of a Prism graph

Theorem 7.1 *The Equitable chromatic number of Line graph of prism graph Y_n, where n is any positive integer is*

$$\chi_= (L(Y_n)) = 3, \ n \geq 3.$$

Proof

Let $V(Y_n) = \{v_1, v_2, \cdots, v_n\} \cup \{u_1, u_2, \cdots, u_n\}$ where v_i's are the vertices of the inner cycle taken in cyclic order and u_i's are the vertices of the outer cycle taken in cyclic order and $E(Y_n) = \{e_i : 1 \leq i \leq n-1\} \cup \{e_n\} \cup \{e_i' : 1 \leq i \leq n-1\} \cup \{e_n'\} \cup \{s_i' : 1 \leq i \leq n\}$ where e_i is the edge $v_i v_{i+1}$ $(1 \leq i \leq n-1)$, e_n is the edge $v_n u_1$, e_i' is the edge $u_i u_{i+1}$ $(1 \leq i \leq n-1)$, e_n' is the edge $u_n u_1$ and s_i is the edge $v_i u_i$ $(1 \leq i \leq n)$.

By the definition of line graph $V(L(Y_n)) = E(Y_n) = \{v_i' : 1 \leq i \leq n\} \cup \{u_i' : 1 \leq i \leq n\} \cup \{s_i' : 1 \leq i \leq n\}$ where v_i', u_i' and s_i' represents the edge e_i, e_i' and s_i respectively.

Case 1: n is even

Let $n = 2k$, $k = 2,3,4, \cdots$. Let us partition the vertex set of the prism graph $V(L(Y_n))$ as follows

$$V_1 = \{v_{2i-1}' : 1 \leq i \leq k\} \cup \{u_{2i-1}' : \{1 \leq i \leq k\}$$

$$V_2 = \{v_{2i}' : 1 \leq i \leq k\} \cup \{u_{2i}' : \{1 \leq i \leq k\}$$

$$V_3 = \{s_i' : 1 \leq i \leq 2k\}$$

Clearly V_1, V_2 and V_3 are independent sets of $V(L(Y_n))$. Also $|V_1| = |V_2| = |V_3| = n$. This holds the inequality $||V_i| - |V_j|| \leq 1 \ \forall \ (i,j)$ satisfying equitable coloring.
$\Rightarrow \chi_= (L(Y_n)) \leq 3$.
Since $L(Y_n)$ contains a clique of order 3, $\chi(L(Y_n)) \geq 3$.
$\Rightarrow \chi_= (L(Y_n)) \geq \chi(L(Y_n)) \geq 3$.
Hence, $\chi_= (L(Y_n)) \geq 3$.
Therefore, $\chi_=(L(Y_n)) = 3$.

Case 2: n is odd

Let $n = 2k - 1$, $k = 2,3,4, \cdots$. Let us partition the vertex set $V(L(Y_n))$ as follows

$$V_1 = \{v_{2i-1}' : 1 \leq i \leq k-1\} \cup \{s_{2k-1}'\} \cup \{u_{2i-1}' : \{1 \leq i \leq k-1\}$$

$$V_2 = \{v_{2i}' : 1 \leq i \leq k-1\} \cup \{s_1'\} \cup \{u_{2i}' : \{1 \leq i \leq k-1\}$$

$$V_3 = \{v_{2k-1}'\} \cup \{s_i' : 2 \leq i \leq 2k-1\} \cup \{u_{2k-1}'\}$$

Clearly V_1, V_2 and V_3 are independent sets of $V(L(Y_n))$. Also $|V_1| = |V_2| = |V_3| = n$. This holds the inequality $||V_i| - |V_j|| \leq 1 \ \forall \ (i,j)$ satisfying equitable coloring.
$\Rightarrow \chi_= (L(Y_n)) \leq 3$.
Since $L(Y_n)$ contains a clique of order 3, $\chi(L(Y_n)) \geq 3$.
$\Rightarrow \chi_= (L(Y_n)) \geq \chi(L(Y_n)) \geq 3$.
Hence, $\chi_= (L(Y_n)) \geq 3$.
Therefore, $\chi_= (L(Y_n)) = 3$.

8. Conclusion

In this paper, we have discussed the equitable chromatic number for Prism graph families. This work can be extended to find the equitable chromatic number for other families of graphs and the middle, total, central and line graphs of any graph G.

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