# Numerical Modelling And Solution Of Microstrip Antenna By Using Caupto Time Fraction RDT Method

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**Abstract**: A mathematical model for the second-order one-dimensional time fractional differential equation for a microstrip line feed microstrip antenna is presented in this paper. Along with it, a mathematical equation is presented that is solved by caupto time fractional reduced differential transformation methodology (RDTM). The fractional derivative has shown in Caupto sense. The fractional reduced differential transform method (FRDTM) is a very modern and effective method to find an approximate solution of a partial differential and differential equation. The FRDTM method comes to be a useful mathematical tool for resolving problems arising in electrical circuits, microwave, RF field, communication systems, and science and technology fields. Two numerical examples has solved also here to check the validity and effectiveness of FRDTM method.

**Index Terms**: FRTDM, Telegraph Equation, Microstrip Antenna, Exact Solution, Caupta method, Transmission Line, Impedance Matching.

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## 1 INTRODUCTION

Amicrostrip antenna, referred to as Patch antenna, is consisting of a metallic patch on one aspect of the substrate and alternative aspect is a metallic ground plane [1],[2]. The metallic top patch is the main module of the antenna that is chargeable for radiation. According to circuit theory, electric resistance matching is needed between patch and feedline to make sure most transfer of electrical signal power from the supply source to patch. The electrical signals are fed to patch either by contacting or non-contacting strategies [3],[4]. In contacting method, the electrical signal power is provided to the patch by employing a connecting line component like microstrip line. Whereas the coupling is done to provide the electrical signal power between the microstrip line and the patch in the case of non-contacting method, Among these techniques, microstrip feeding line configuration is prescribed within the modeling of microstrip antenna owing to its straightforward analysis and simple fabrication approach. A mathematical derivation for the microstrip feed line, as a function of voltage, has been developed, in this paper. The mathematics generates a partial equation which is solved by Fractional Reduced Differential Transformation (FRDT) method [5]. This method has an effective mathematical tool in providing a solution to many problems in the area of circuit theory and RF field theory. The numerical methods are thought to see the accuracy, potency, and convergence of the method along with the two-dimensional differential wave equation. An equivalent electrical circuit of the microstrip antenna is shown in Fig. 1.

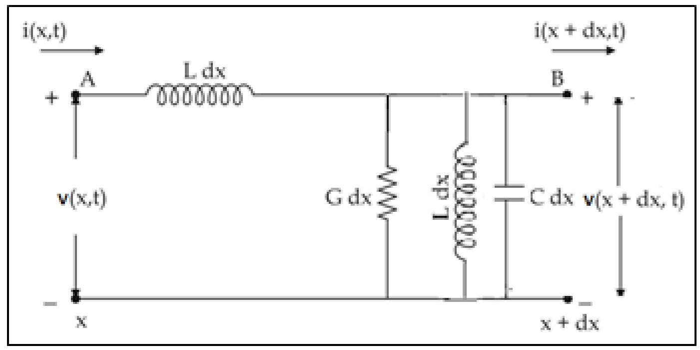


***Fig.1:*** *Equivalent electrical circuits of microstrip line fed antenna.*

Notating G, C and L as the Conductance, Capacitance and Inductance of the patch antenna, and α denote the time fractional order. Here, v(x,t)and $i(x,t)$ are the representation of voltage and current. From the basic circuit theory, the expression of voltage drop across the inductor and capacitor is given as

v = - $L\frac{di}{dt}$ (1)

v = $\frac{1}{c}\int idt$ (2)

The voltage at pt B, in figure 1, is given as

$$v(x+dx,t)-v(x,t) = -[-Ldx]\frac{di}{dt} \quad (3)$$

Differentiating this with respect to $x$ , we have

$$\partial v/\partial x = L\, di/dt \quad (4)$$

Similarly, from basic circuit rules we get

$$\frac{\partial i}{\partial x} = -GV - I_L - I_c$$

$$\Rightarrow v = -\frac{1}{G}\frac{\partial i}{\partial x} - \frac{1}{G}I_c - \frac{1}{G}I_L \quad (5)$$

Now the current across inductor and capacitor is

$v = -L\frac{dI_L}{dt}$ and Ic = c $\frac{\partial v}{\partial t}$ (6)

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Double differentiating equation (6) with respect to x, we get

$$\frac{\partial^2 u}{\partial x^2}=L\frac{\partial^2 i}{\partial x dt} \qquad (7)$$

With respect to circuit theory, this equation extends to

$$\frac{\partial V}{\partial t}=-\frac{1}{G}\frac{\partial^2 i}{\partial x dt}-\frac{\partial I_C}{\partial t}\frac{1}{G}-\frac{\partial I_L}{\partial t}\frac{1}{G}$$

$$\Rightarrow\frac{1}{GL}\frac{\partial^2 V}{\partial x^2}=-\frac{C}{G}\frac{\partial^2 V}{\partial t^2}-\frac{\partial V}{\partial t}+\frac{1}{GL}V \qquad (8)$$

This equation (8) is the one-dimensional partial differential equation for microstrip antenna.

## 2 Fractional Calculus and Reduced Differential Transform Method (RDTM)

A various physical problems in the RF field can be successfully developed using fractional calculus theory. Before the nineteenth century, there was no analytical answer obtainable for the linear fractional differential equations until Keskin and Oturanc [6] had answered by developing the reduced differential transform method (RDTM) and showed this to be ease in using in the semi-analytical methodology. This method offers the most precise answer and applicable for both the linear and nonlinear differential equations. The primary major contribution to provide the correct definition is according to Liouville [7].The Riemann-Liouville derivatives [8],[ 9] of fractional differential equations fails to justify real-world problems with the existing model. Caputo and Mainardi [10] planned a revised fractional differentiation operator in their work. The idea of work relates viscoelasticity. The Caputo fractional derivatives permit the use of boundary conditions that have whole number order derivatives. This presents a clear physical interpretation. In their study of the work on the fractional derivative, the Caputo fractional derivative is chosen. This permits ancient boundary conditions within the design of the problem. Some literature [11],[12] quoted other properties of a fractional derivative.

Consider a function of two variables $w(x,t)$, and assume it to be a product of two single-variable functions, i.e. $w(x,t)=F(x)G(t)$. The function $w(x,t)$, by evaluating the properties of the one-dimensional differential transformation, is represented as

$$w(x,t)=\sum_{i=0}^{\infty}F(i)x^i\sum_{j=0}^{\infty}G(j)t^j=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}W(i,j)x^it^j, \qquad (9)$$

$W(i,j)=F(i)G(j)$ represents the spectrum of $w(x,t)$.

The basic definitions of RTDM along with its operations [14-18] are formulated in the literature. The idea this transform method is obtained from the power series expansion of a function.

## 3 Computational Illustrations

Here, the methodology explained in section 2 is shown. This is done by taking few examples of the wave equation of microstrip antenna. This is required to validate the potency and reliability of RTDM.

**Example3.1:** The wave equation of microstrip antenna equation is given as

$$\frac{\partial^2 E}{\partial^2 t}+\frac{\partial E}{\partial t}+\frac{\partial^2 E}{\partial x^2}-E=0 \qquad (10)$$

initial conditions are subjected to $E_0(x)=e^x$ and $E_1(x)=-e^x$

Rearrange equation (11) , we get the below equation

$$\frac{\partial^2 E}{\partial x^2}-E=-\frac{\partial^2 E}{\partial^2 t}-\frac{\partial E}{\partial t} \qquad (11)$$

Applying RDTM to the above equation following recurrence relation is obtained:

$$\frac{\partial^2 E}{\partial x^2}E_k(x)-E_k(x)=-\frac{\Gamma(k\alpha+2\alpha+1)}{\Gamma(k\alpha+1)}E_{(k+2)}(x)-\frac{\Gamma(k\alpha+\alpha+1)}{\Gamma(k\alpha+1)}E_{(k+1)}(x)$$

$$e^x-e^x=-\frac{\Gamma(k\alpha+2\alpha+1)}{\Gamma(k\alpha+1)}E_{(2)}(x)-\frac{\Gamma(k\alpha+\alpha+1)}{\Gamma(k\alpha+1)}E_{(1)}(x)$$

$$-\frac{\Gamma(k\alpha+2\alpha+1)}{\Gamma(k\alpha+1)}E_{(2)}(x)=\frac{\Gamma(k\alpha+\alpha+1)}{\Gamma(k\alpha+1)}(-e^x) \qquad (12)$$

$$E_{(2)}(x)=-\frac{\Gamma(\alpha+1)}{\Gamma(2\alpha+1)}(e^x) \qquad (13)$$

At α=1

$$E_{(2)}(x)=\frac{\Gamma(2)}{\Gamma(3)}(e^x)$$

$$E_{(2)}(x)=\frac{1}{!2}(e^x)$$

$$E_{(2)}(x)=\frac{\Gamma\left(\frac{1}{\lambda}+1\right)}{\Gamma\left(\frac{2}{\lambda}+1\right)}(e^x) \qquad (14)$$

Where $\alpha=\frac{1}{\lambda}, \lambda>0$

At k=1

$$\frac{\partial^2 E}{\partial x^2} E_k(x) - E_k(x) = -\frac{\Gamma(k\alpha+2\alpha+1)}{\Gamma(k\alpha+1)} E_{(k+2)}(x) - \frac{\Gamma(k\alpha+\alpha+1)}{\Gamma(k\alpha+1)} E_{(k+1)}(x)$$

At k=1

$$-e^x + e^x = -\frac{\Gamma(3\alpha+1)}{\Gamma(\alpha+1)} E_3(x) - \frac{\Gamma(2\alpha+1)}{\Gamma(\alpha+1)} E_2(x)$$

$$\frac{\Gamma(k\alpha+2\alpha+1)}{\Gamma(k\alpha+1)} E_3(x) = -\frac{\Gamma(k\alpha+\alpha+1)}{\Gamma(k\alpha+1)} E2(x) \qquad (15)$$

$$E_3(x) = -\frac{\Gamma(2\alpha+1)}{\Gamma(3\alpha+1)} E_2(x)$$

$$E_3(x) = -\frac{\Gamma(2\alpha+1)}{\Gamma(3\alpha+1)} \frac{\Gamma(\alpha+1)}{\Gamma(2\alpha+1)} (e^x)$$

$$E_3(x) = -\frac{\Gamma(\alpha+1)}{\Gamma(3\alpha+1)} (e^x) \qquad (16)$$

At α=1

$$E_{(3)}(x) = -\frac{1}{!3}(e^x)$$

Similarly

$$E_{(4)}(x) = \frac{1}{!4}(e^x) \qquad (17)$$

Using the differential inverse reduced transform of $E_K$, we get

$$E(x,t) = \sum_{k=0}^{\infty} E_k(x) t^{k\alpha} = \sum_{k=0}^{\infty} E_K(x) t^{k/\lambda} \qquad (18)$$

$$E(x,t) = E(0)(x) + E(1)(x)t^{k/\lambda} + E(2)(x)t^{k/\lambda} + E(3)(x)t^{k/\lambda} + \cdots \ldots \ldots ..$$

Put the value of

$$E_{(0)}(x), E_{(1)}(x), E_{(2)}(x) E_{(3)}(x) \ldots \ldots \ldots \ldots \ldots,$$

$$= (e^x) + (-1)t(e^x) + t^2\frac{(-1)^2}{2!}(e^x) + t^3\frac{(-1)^3}{3!}(e^x) + t^3\frac{(-1)^4}{4!}(e^x) \ldots$$

$$= (e^x)\left[1 + (-1)t + t^2\frac{(-1)^2}{2!} + t^3\frac{(-1)^3}{3!} + t^3\frac{(-1)^4}{4!} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots\right]$$

$$=(e^x)(e^{-t}) \qquad (19)$$

$$\Rightarrow E(x,t) = (e^{x-t}) \qquad (20)$$

**Example 3.2** Another wave equation of microstrip antenna equation is given as

$$\frac{\partial^2 E}{\partial^2 t} + 2\frac{\partial E}{\partial t} + \frac{\partial^2 E}{\partial x^2} - E = 0 \qquad (21)$$

initial conditions are subject to
$E_0(x) = e^x$ and $E_1(x) = -2\,e^x$

Again rearranging equation (21), we get the below equation

$$\frac{\partial^2 E}{\partial x^2} - E = -\frac{\partial^2 E}{\partial^2 t} - 2\frac{\partial E}{\partial t} \qquad (22)$$

Applying the RDTM to equation (22), the following recurrence relation is obtained

$$\frac{\partial^2 E}{\partial x^2} E_k(x) - E_k(x) = -\frac{\Gamma(k\alpha+2\alpha+1)}{\Gamma(k\alpha+1)} E_{(k+2)}(x) - 2\frac{\Gamma(k\alpha+\alpha+1)}{\Gamma(k\alpha+1)} E_{(k+1)}(x)$$

$$e^x - e^x = -\frac{\Gamma(k\alpha+2\alpha+1)}{\Gamma(k\alpha+1)} E_{(2)}(x) - \frac{\Gamma(k\alpha+\alpha+1)}{\Gamma(k\alpha+1)} E_{(1)}(x)$$

$$-\frac{\Gamma(k\alpha+2\alpha+1)}{\Gamma(k\alpha+1)} E_{(2)}(x) = 2\frac{\Gamma(k\alpha+\alpha+1)}{\Gamma(k\alpha+1)}(-e^x) \qquad (23)$$

$$E_{(2)}(x) = 4\frac{\Gamma(k\alpha+\alpha+1)}{\Gamma(k\alpha+2\alpha+1)}(e^x)$$

At α=1,

$$E_{(2)}(x) = 4\frac{\Gamma(2)}{\Gamma(3)}(e^x)$$

$$E_{(2)}(x) = \frac{(-2)^2}{!2}(e^x) \qquad (24)$$

At k=1

$$\frac{\partial^2 E}{\partial x^2} E_k(x) - E_k(x) = -\frac{\Gamma(k\alpha+2\alpha+1)}{\Gamma(k\alpha+1)} E_{(k+2)}(x) - \frac{\Gamma(k\alpha+\alpha+1)}{\Gamma(k\alpha+1)} E_{(k+1)}(x)$$

$$-2e^x + 2e^x = -\frac{\Gamma(3\alpha+1)}{\Gamma(\alpha+1)} E_3(x) - \frac{\Gamma(2\alpha+1)E}{\Gamma(\alpha+1)} 2(x)$$

$$\frac{\Gamma(k\alpha+2\alpha+1)}{\Gamma(k\alpha+1)} E_3(x) = -\frac{\Gamma(k\alpha+\alpha+1)}{\Gamma(k\alpha+1)} E_2(x) \qquad (25)$$

$$E_3(x) = -\frac{\Gamma(2\alpha+1)E_2}{\Gamma(3\alpha+1)}(x)$$

$$E_3(x) = -\frac{\Gamma(2\alpha+1)}{\Gamma(3\alpha+1)} \frac{\Gamma(\alpha+1)}{\Gamma(2\alpha+1)} (e^x)$$

$$E_3(x) = -\frac{\Gamma(\alpha+1)}{\Gamma(3\alpha+1)} (e^x) \qquad (26)$$

At α=1

$E_{(3)}(x) = -\frac{1}{!3}(e^x)$

Similarly

$$E_{(4)}(x) = \frac{1}{!4}(e^x) \quad (27)$$

Using the differential inverse reduced transform, we get

$E(x,t) = \sum_{k=0}^{\infty} E_K(x) t^{k\alpha} = \sum_{k=0}^{\infty} E_K(x) t^{k/\lambda}$ ……(28)

$$E(x,t) = E_{(0)}(x) + E_{(1)}(x)\, t\text{^}(k/\lambda) + E_{(2)}(x)\, t\text{^}(k/\lambda) + E_{(3)}(x)\, t\text{^}(k/\lambda) + \cdots \ldots\ldots$$

Put the value of

$$E_{(0)}(x), E_{(1)}(x), E_{(2)}(x) E_{(3)}(x) \ldots\ldots\ldots\ldots,$$

$$= (e^x) + (-2)t(e^x) + t^2\frac{(-2)^2}{2!}(e^x) + t^3\frac{(-2)^3}{3!}(e^x) + t^3\frac{(-2)^4}{4!}(e^x)\ldots$$

$$= (e^x)[1 + (-2)t + t^2\frac{(-2)^2}{2!} + t^3\frac{(-2)^3}{3!} + t^3\frac{(-2)^4}{4!}\ldots\ldots\ldots\ldots\ldots\ldots\ldots]$$

$$=(e^x)(e^{-2t}) \quad (29)$$

$$\Rightarrow E(x,t) = (e^{x-2t}) \quad (30)$$

Both the solutions has been found by fractional FRDTM. In the case of any electric circuits, power transferred is dependent on the distance of the load $x$ and time *t*, with respect to the supply source. This view has also seen in the above two examples presented in the preceding section of this paper demonstrated in this document.

## 4 Conclusion

The paper presented the reduced differential transform method for one-dimensional and second order hyperbolic linear and nonlinear Telegraph equations. This method is directly applicable without any usage of mathematical assumptions such as linearization, transformation, restrictive or discretization. The results show that the introduced technique is extremely correct, and straightforward to use in numerous RF field-engineering cases.

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