Prime Coloring Of Some Graphs

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Abstract: In the recent era, the coloring of the graphs is arousing the attention of researchers. Among the mathematical models, coloring technique plays a vital role in a broad range of applications in real life problems and it solves several complexities of computer networks. Considering the above facts, in this paper, prime coloring and its chromatic number of some graphs are depicted and its results are validated with few theorems. **Subject Classification Code:** 05C78, 05C15

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1. INTRODUCTION:

We use the terminology as Bondy.J.A. and Murty U.S.R [1].let G= (V, E) be neither a Pseudo graph nor a disconnected graph and here order of V be | V |= n. In this paper, we discuss Prime graphs and Coloring. The Prime labeling emanate with Entringer et.al 1980 [2,3]. Be the graph G = (V, E) with vertex set V is said to have a Prime labeling if there exists a bijection f: V(G) $\rightarrow Z^+ - \{0\}$, Z^+ is the set of positive integers such that for each edge $xy \in E(G)$,gcd {f(x),f(y)}=1[5].It begin to be an interesting problem to investigate the Prime coloring as possible in a graph with n vertices. Moreover, the general graph G is proper colorable but not prime graph forever. The Coloring of a graph is explained as pair wise adjacent vertices receive conflict colors.

2. PRIME COLORING OF SOME STANDARD GRAPHS:

In this entire paper, we consider ψ is the bijective function and ψ' is the Chromatic number of G as well as η is also chromatic number of prime graph. Now, Let us define a Prime coloring of graph as follows,

Definition 2.1:

Prime Coloring is defined as G be a loop less and Without multiple edges with n distinct Vertices on Color class C={$c_1,c_2,c_3,\ldots c_n$} a bijection

ψ:V→{$c_1,c_2,c_3,\ldots c_n$} if for each edge e = c_ic_j ,i≠j , gcd{ ψ (c_i), ψ (c_j)}=1, ψ (c_i) and ψ (c_j) receive distinct Colors. The Chromatic number of Prime coloring is minimum cardinality taken by all the Prime colors. It is denoted by $\eta(G)$.

Theorem 2.2:

Every path P_n graph is Prime if it is Prime colorable graph.

Proof:

Let the path P_n with n distinct vertices, the bijection

ψ :V(P_n)→{ $c_1,c_2,c_3,\ldots c_n$ } graph if for each edge

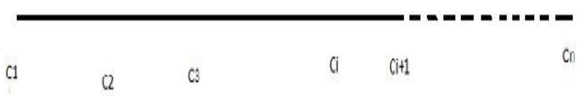
e = c_ic_j , gcd{ ψ (c_i), ψ (c_j)}=1, i≠j,

ψ (c_i) =c_i

ψ (c_j)=c_j receive distinct colors. Then the path P_n Graph is Prime colorable graph.

Corollary2.3: If path P_n graph is prime then $\eta(P_n) = 2$.

Example 2.4: P_n is Prime colorable graph



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gcd { ψ (c_i), ψ (c_j)}=1, $\eta(P_n) = 2$.

Figure2.1

Theorem 2.5: Every Cycle C_n , n≥4 if it is Prime colorable graph.

Proof: Assume that Cycle C_n on n≥4 vertices with the bijection ψ :V(C_n)→{ $c_1,c_2,c_3,\ldots c_n$ } graph if for each edge e = c_ic_j, gcd{ ψ (c_i),ψ (c_j)}=1,i≠j ,

ψ (c_i) =c_i

ψ (c_j)=c_j

receive distinct colors on cycle C_n on n≥4 .

Corollary2.6: If Cycle graph C_n , n≥4 is Prime, then

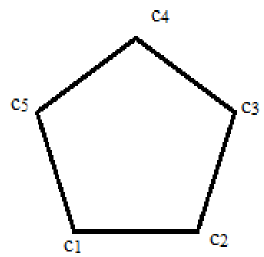
$$\eta(C_n) = \begin{cases} 2, n \text{ is even} \\ 3, n \text{ is odd} \end{cases}$$

Example2.7: Cycle C_4

gcd { ψ (c_i), ψ (c_j)}=1, $\eta(C_4) = 2$

Figure 2.2

Example2.8: Cycle C_5



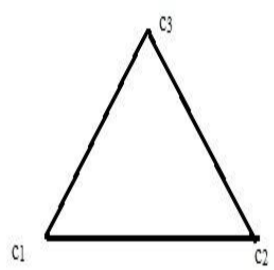
gcd {ψ (c_i), ψ (c_j)}=1, $\eta(C_5) = 3$

Figure 2.3

Theorem2.9: If a Complete graph K_n , n≤3 is Prime graph ,then K_n is Prime colorable graph with n≤3.

Proof:
Proof by induction n=1 obviously holds.
n=2 then the graph is a path by the theorem (2.2) holds.
n=3 then the graph is triangle every
gcd {ψ (c_i), ψ (c_j)} = 1, i≠j then the graph is prime and also receive distinct colors.
n=4 then the vertices assigns distinct colors but
gcd {ψ (c_i), ψ (c_j)} ≠ 1. But e = $c_i c_j$, then the graph is not a prime. Therefore n>3, K_n is not a prime colorable graph.

Example2.10: The complete graph K_3 is Prime Coloring graph.



gcd { ψ (c_i), ψ (c_j)}=1, $\eta(K_3) = 3$

Figure2.4

Illustrative Example2.11: Complete graph K_4

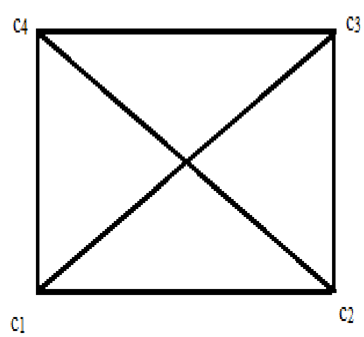


Figure 2.5

Here gcd {ψ (c_1), ψ (c_2)}=1, gcd {ψ (c_1), ψ (c_4)}=1,
gcd {ψ (c_1), ψ (c_3)}=1,But gcd{ψ (c_2), ψ (c_4)}≠1.
Consider the general **K_4** is proper colorable with

Chromatic number $\psi'(K_4) = 4$,
But $\eta(K_4)$ not defined.

Illustrative Example2.12:
Complete graph K_5

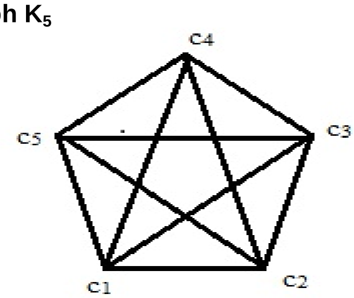


Figure 2.6

Here,
gcd {ψ (c_1), ψ (c_2)}=1,
gcd {ψ (c_1), ψ (c_4)}=1,
gcd {ψ (c_1), ψ (c_3)}=1,
gcd {ψ (c_1), ψ (c_5)}=1,
gcd {ψ (c_3), ψ (c_5)}=1.

But gcd {ψ (c_2), ψ (c_4)}≠1.consider the General K_5 is proper colorable with chromatic number $\psi'(K_5) = 5$. But $\eta(K_5)$ not defined.

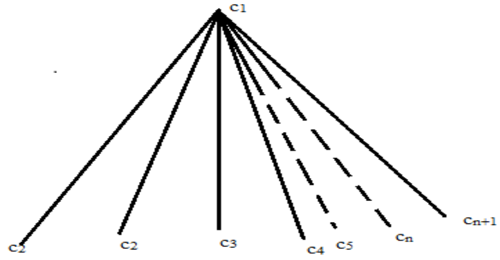
Remark 2.13: Every Complete graph **K_n** is Proper Colorable but not Prime colorable.
Consider K_n is complete graph pair wise adjacent vertices are connected by the definition of graph coloring it has to assign distinct colors, but the labeled graph K_n with {1,2,….n} ,gcd (1,n) not equal to 1 for n ≥3,K_n is not Prime graph. Thus K_n is not a Prime colorable.

Remark2.14: If G is the prime graph and Δ is the maximum degree of the graph then
$\eta(G) \leq \Delta + 1$

Theorem2.15: If Star graphs $K_{1,n}$ is Prime graph then $\eta(K_{1,n}) = 2$.

Proof:
Let V={c_1,c_2,c_3…….c_{n+1}} vertices of Star graph $K_{1,n}$.the graph labeled as well as colored ,by
the bijection ψ :V($K_{1,n}$)→{ c_1,c_2,c_3,…..c_{n+1} } graph if for each edge e= $c_i c_j$,gcd{ ψ (c_i), ψ (c_j)}=1, i≠j
ψ (c_i) and ψ (c_j) receive distinct colors .Therefore the coloring pairs of are the distinct. The coloring pairs are at most 2 and also every gcd (i,j)=1 is a prime graph.
Thus,
ψ (c_i)=c_i
ψ (c_j)=c_j ,for 1≤i,j≤n+1 then every i≠j is Prime colorable. Therefore minimum coloring needed for the Star graph is $\eta(K_{1,n}) = 2$.

Example 2.16: Star graph $K_{1,n}$



gcd { ψ (c), ψ (c_j)}=1, $\eta(K_{1,n}) = 2.$

Figure 2.7

Corollary2.17: Every spanning sub graph of prime graph is prime colorable graph.

Seoud and youssef [6] proved every spanning sub graph is prime graph. Now coloring proper by the graph become an prime colorable gcd{ ψ (c_i), ψ (c_j)}=1, i≠j , ψ (c_i) and ψ (c_j) receive distinct colors. We observe that every spanning sub graph is also prime colorable.

Corollary2.18: Every tree is prime colorable graph.

Every tree is prime is proved by Entringer n≤50 vertices (4). The tree have been properly colored as the color class C={ $c_1,c_2,c_3,\ldots.c_n$ }. Tree is a spanning sub graph of the general graph G by the above corollary **(2.17)** holds, there for every tree is prime colorable.

Theorem2.19: If Wheel graph W_n, n≥4 is Prime graph ,

then $\eta(W_n) = 4$,When n is even.

Proof:

Let V= {$c_1,c_2,c_3\ldots\ldots.c_{k+1}$} vertices of wheel graph W_n. The graph labeled as well as colored.

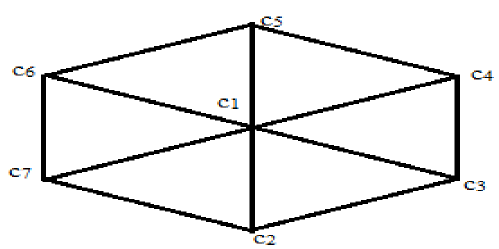
By the bijection ψ :V(W_n)→{ $c_1,c_2,c_3,\ldots.c_{k+1}$ } graph if for each edge e = c_ic_j , gcd{ ψ (c_i),(c_j)}=1, i≠j ,ψ (c_i) and ψ (c_j) receive distinct colors .Therefore the coloring pairs of are the distinct. Therefore the coloring pairs are at most four and also gcd(i,j)=1 is a prime graph.

Thus,ψ (c_i)=c_i

ψ (c_j)=c_j for 1≤i,j≤k+1

Then every i≠j is prime colorable. Therefore minimum coloring needed for the Wheel graph is $\eta(W_n) = 4.$

Example 2.20: Wheel graph W_5



gcd {ψ (c_i), ψ (c_j)}=1 , $\eta(W_n) = 4.$

Figure 2.8

Lemma 2.21: G is Prime graph ⇔ G is Prime colorable graph.

Proof:

Let us assume G is prime label with the bijection ψ :V(G)→{ $c_1,c_2,c_3,\ldots.c_n$ }graph if for each edge w = c_ic_j, gcd {ψ (c_i), ψ (c_j)}=1 , i≠j ,ψ (c_i) & ψ (c_j) receives distinct colors. Now coloring the vertices is proper coloring. Then the resulting graph with different colors.

Conversely G is Prime coloring by the definition of graph coloring holds it is a prime graph.

Remark2.22: If G is Prime Graph then $\psi'(G) = \eta(G)$

Lemma2.23:

If $\psi'(P_n) = \eta(P_n) = 2$, then the following are equivalent

i. P_n is Path.

ii. P_n is Prime graph.

iii. P_n is Prime colorable graph.

Proof:

(i) ⇒ (ii) let the path P_n with n distinct vertices then the chromatic number of the graph $\psi'(P_n) = 2$ we have to prove P_n is a prime graph with gcd{ ψ (c_i), ψ (c_j)}=1, i≠j ,then P_n is a prime graph.

(ii) ⇒ (iii)

By the theorem (2.2) refers to every prime graph P_n which assigned the distinct colors and by the definition of a prime color then $\eta(P_n) = 2$.

(iii) ⇒ (i) from the proof given prime colorable graph $\psi'(P_n) = \eta(P_n) = 2$

Then n distinct vertices are connected as a path.

Lemma2.24:

If $\psi'(C_n) = \eta(C_n) = \begin{cases} 2, \text{ n is even} \\ 3, \text{ n is odd} \end{cases}$, then the following are equivalent

i. C_n is Cycle

ii. C_n is Prime graph.

iii. C_n is Prime colorable graph.

Proof:

Case (i) n is odd:

(i) ⇒ (ii)

Consider C_n be the cycle with distinct n vertices. If Cycle C_n , n>3 is an odd cycle with 2n-1 vertices, by the reference [6] Fusion, switching, duplication of the vertices than the graph is the prime graph.

(ii) ⇒ (iii)

Consider C_n is prime graph when n is odd, be the fusion, switching, duplication is allowed then the graph is prime by the definition (2.1) C_n is prime colorable graph.

(iii) ⇒ (i)

By the definition of (2.1), all the vertices are distinct when n is odd i.e $c_1 = c_{2n-1}$, n>3 and holds $\psi'(C_n) = \eta(C_n) = 3$.

Case (ii) n is even
(i) ⇒ (ii) Clearly the even number of vertices on the cycle is prime (6) and gcd { ψ (c_i), ψ (c_j)}=1, i ≠ j
(ii) ⇒ (iii)
By the theorem (2.5) the graph gcd { ψ (c_i), ψ (c_j)}=1, i≠j, then $\psi'(C_n) = \eta(C_n)$ =2 pair wise adjacent vertices receive distinct colors.
(iii) ⇒ (i)
$\psi'(C_n) = \eta(C_n)$ =2 with the Cycle C_n , n>3 we get $c_1 = c_{2n-2}$, i≠j vertices are distinct and even vertices of C_n is cycle.

Remark 2.25: Every prime graph is proper colorable graph.

Remark 2.26: Every graph G is proper Colorable but not prime color.

CONCLUSION:
This article discussed & absorbed the upshot of graph coloring technique which associated with the prime labeled graph. We elucidated the coloring techniques of some standard graphs and also evinced the some interesting results obtained by Prime Coloring graphs.

CONFLICT OF INTEREST:
The authors confirm that there is no conflict of interest to declare for this publication.

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