Properties Of Prime And Composite Pseudo Intrinsic Edge-Magic Graphs

M. Kaliraja, M. Sasikala

Abstract: In this paper we introduced prime pseudo intrinsic edge-magic graphs using the concept of prime strength. Also we focussed prime double pseudo intrinsic edge-magic graphs. We examined these concepts in fuzzy complete graphs, banner graphs, cycle graphs, gem graphs and butterfly graphs.

Keywords: intrinsic constant, mock constant, prime strength, prime pseudo graph, non-prime pseudo graph and composite pseudo graph.

1. Introduction:

Fuzzy set was firstly introduced by [1]. Then various researches added productive concepts to develop fuzzy sets theory like [3] and [8]. In 1987 Bhattacharya has succeeded to develop the connectivity notions between fuzzy bridge and fuzzy cut nodes [5]. A fuzzy graph contains many properties similar to crisp graph due to generalization of crisp graphs but it diverge at many places. A crisp graph G is an order pair of vertex-set V and edge set E such that $E \subseteq V \times V$. In addition $v = |V|$ is said to order and $e = |E|$ is called size of the graph G respectively. In a crisp graph, a bijective function $\rho : V \cup E \to N$ that produced a unique positive integer (To each vertex and/or edge) is called a labelling [4]. Introduced the notion of magic graph that the labels vertices and edges are natural numbers from 1 to $|V| + |E|$ such that sum of the labels of vertices and the edge between them must be constant in entire graph [6]. Extended the concept of magic graph with added a property that vertices always get smaller labels than edges and named it super edge magic labelling. Numerous other authors have explored diverse types of different magic graphs [7], [11] & [12]. The subject of edge-magic labelling of graphs had its origin in the work of Kotzig and Rosa on what they called magic valuations of graphs [2]. These labelling are currently referred to as either edge-magic labelling or edge-magic total labelling. Fuzzy graphs are generalization of graphs. In graphs two vertices are either related or not related to each other. Mathematically, the degree of relationship is either 0 or 1. In fuzzy graphs, the degree of relationship takes values from [0, 1]. A fuzzy graph has ability to solve uncertain problems in a wide range of fields. The first definition of a fuzzy graph was introduced by Kaufmann in 1973. Azriel Rosenfield in 1975 [3] developed the structure of fuzzy graphs and obtained analogs of several graph theoretical concepts. In [10], NagoorGani et. al. introduced the concepts of fuzzy labelling graphs, fuzzy magic graphs.

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In our earlier work, we have discussed the construction of fuzzy intrinsic edge-magic graphs and fuzzy perfect intrinsic edge-magic graphs in [13] and [14] , In this paper we have developed the concept of fuzzy prime pseudo intrinsic graphs using prime strength. Also we discussed the prime double pseudo intrinsic edge-magic graphs. Those graphs are developed by the nature of mock constant and intrinsic constant.

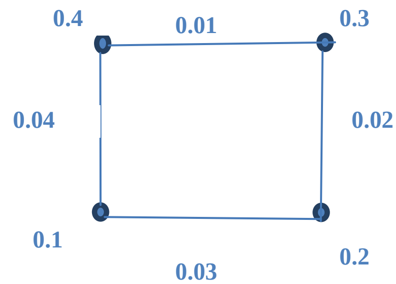
2. Preliminaries:

Definition 2.1: A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$ where for all $u, v \in V$, we have $\mu(u,v) \le \sigma(u) \wedge \sigma(v)$.

Definition 2.2: A path P is called a cycle if $v_1 = v_n$ and $n \ge 3$ and a cycle is called a fuzzy cycle if it contains more than one weakest arc.

Definition 2.3: A bijection ω is a function from the set of all nodes and edges of to [0, 1] which assign each nodes $\sigma^{\omega}(a)$, $\sigma^{\omega}(b)$ and edge $\mu^{\omega}(a,b)$ a membership value such that $\mu^{\omega}(a,b) \le \sigma^{\omega}(a) \wedge \sigma^{\omega}(b)$ for all $a, b \in V$ is called fuzzy labelling.

A graph is said to be fuzzy labelling graph if it has a fuzzy labelling and it is denoted by G^{ω}



Fuzzy labelling graph

Definition 2.4:[14] A fuzzy labelling graph G is said to be fuzzy perfect intrinsic labelling if $f:\sigma \rightarrow [0,1]$ and $f:\mu \rightarrow [0,1]$ is bijective such that the membership values of edges are $\{z, 2z\, 3z, ..., \in z\}$ and vertices are $\{(\varepsilon+1)z, (\varepsilon+2)z, ...(\varepsilon+\upsilon)z\}$ where $\varepsilon+\upsilon = N$ is the total number of vertices and edges and let z = 0.1 for N≤6 & z = 0.01 for N>6 .

Definition 2.5:[14] An edge-magic constant in a fuzzy perfect intrinsic edge-magic graph is said to be mock constant 'λ_m' if it is equal to $\sigma(v_i)+\mu(v_i v_j)+\sigma(v_j)$ for some $v_i, v_j \in V$ with $\lambda_s \neq \lambda_w$.

Definition 2.6: [13] A fuzzy graph is said to be a pseudo-intrinsic edge-magic graph if it contains a mock constant 'λ_m' which is denoted by 'G_p' .

3. Prime pseudo intrinsic edge-magic graphs

Definition 3.1:
Let G be a fuzzy pseudo intrinsic edge-magic graph. The prime strength of G is denoted by δ and is defined as $\delta = \{ \lambda_c + \lambda_m \quad if \quad \lambda_m \; is \; prime \}$

Definition 3.2:

Let G be a FPSIEM graph. If δ is prime then the graph G is called the prime pseudo intrinsic edge-magic graph and it is denoted by G_α .

Definition 3.3:

Let G be a FPSIEM graph. If δ is not prime then the graph G is called the non-prime pseudo intrinsic edge-magic graph and it is denoted by G_β .

Definition 3.4:

Let G be a FPSIEM graph. If the mock constant 'λ_m' which is not prime, then the graph is called the composite pseudo intrinsic edge-magic.

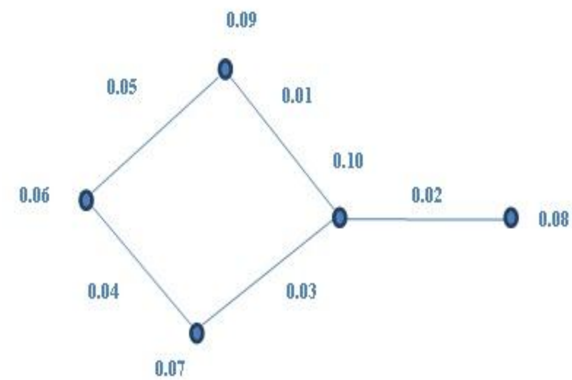
Definition 3.5:
A fuzzy intrinsic edge-magic graph with two mock constants, the graph is said to be double pseudo intrinsic edge-magic.

Definition 3.6:
Let G be a fuzzy double pseudo intrinsic edge-magic graph. The prime strength of G is denoted by δ_d and is defined as $\delta_d = \{\lambda_c + \lambda_{m_i}\}$ where 'λ_{m_i}' is a mock constant.

Definition 3.7:

Let G be a FDPSIEM graph. If δ_d is prime then the graph is said to be prime double pseudo intrinsic edge-magic.

Theorem 3.8:
A 4-pan graph (Banner graph) is a fuzzy prime pseudo intrinsic edge-magic. Let G be a 4-Pan graph. Consider the fuzzy intrinsic edge-magic labelling,



4-pan graph (Banner graph)

$$\sigma(v_1)+\mu(v_1 v_2)+\sigma(v_2) = 0.09 + 0.06 + 0.05 = 0.20 = \lambda_c$$
$$\sigma(v_2)+\mu(v_2 v_3)+\sigma(v_3) = 0.06 + 0.07 + 0.04 = 0.17 = \lambda_m$$
$$\sigma(v_3)+\mu(v_3 v_4)+\sigma(v_4) = 0.07 + 0.10 + 0.03 = 0.20 = \lambda_c$$
$$\sigma(v_4)+\mu(v_4 v_5)+\sigma(v_5) = 0.10 + 0.08 + 0.02 = 0.20 = \lambda_c$$
$$\sigma(v_1)+\mu(v_1 v_4)+\sigma(v_4) = 0.09 + 0.10 + 0.01 = 0.20 = \lambda_c$$

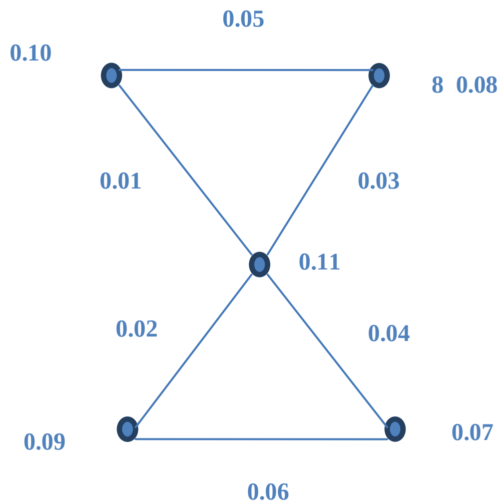
Here $\lambda_m = 0.17$ which is prime, by definition of prime strength,
$\delta = \{ \lambda_c + \lambda_m \} = 0.20 + 0.17 = 0.37 \; (which \;\; is \;\; prime)$

From the above, the prime strength of G is prime. So we can conclude that a Banner graph is a fuzzy prime pseudo intrinsic edge-magic.

Theorem 3.9:
A complete graph with four vertices K_4 is a fuzzy prime pseudo intrinsic edge-magic.

Friendship Graph (Fn):
The friendship graph F can be constructed by joining n-copies of the cycle graph C_3 with a common vertex. The graph F_2 is isomorphic to the butterfly graph.

Theorem- 3.10: The butterfly graph F_2 is a fuzzy non-prime pseudo intrinsic edge-magic graph. Let G be a butterfly graph.i.e, A butterfly graph can be construct by joining 2-copies of the cycle graph C_3 with a common vertex.



Butterfly graph

In the above graph, we applied the fuzzy intrinsic labelling for all the vertices & edges.

$$\sigma(v_1)+\mu(v_1 v_2)+\sigma(v_2)=0.10+0.01+0.11=0.22=\lambda_c$$
$$\sigma(v_2)+\mu(v_2 v_3)+\sigma(v_3)=0.11+0.09+0.02=0.22=\lambda_c$$
$$\sigma(v_3)+\mu(v_3 v_4)+\sigma(v_4)=0.09+0.07+0.06=0.22=\lambda_c$$
$$\sigma(v_4)+\mu(v_4 v_5)+\sigma(v_5)=0.07+0.11+0.04=0.22=\lambda_c$$
$$\sigma(v_5)+\mu(v_1 v_5)+\sigma(v_1)=0.08+0.10+0.05=0.23=\lambda_m$$

Here $\lambda_m = 0.23$ which is prime, by definition of prime strength,

$$\delta = \{\lambda_c + \lambda_m\} = 0.22 + 0.23 = 0.45 \ (which \ \ is \ not \ \ prime)$$

From the above, the prime strength of G is not prime.

So, we conclude that the butterfly graph F_2 is a fuzzy non-prime pseudo intrinsic edge-magic graph.

Theorem 3.11:
Every fuzzy pseudo intrinsic edge-magic is need not be a prime pseudo intrinsic edge-magic.

Proof: Let G be a fuzzy pseudo intrinsic edge-magic.

By our assumption, the mock constant λ_m should appear but it is not necessary prime.
The following cases are arise:

Case (i): λ_m is prime and λ_c is not prime.

Here λ_m is prime, the prime strength may or may not be prime.

In this case, G may be prime or non-prime pseudo intrinsic edge-magic.

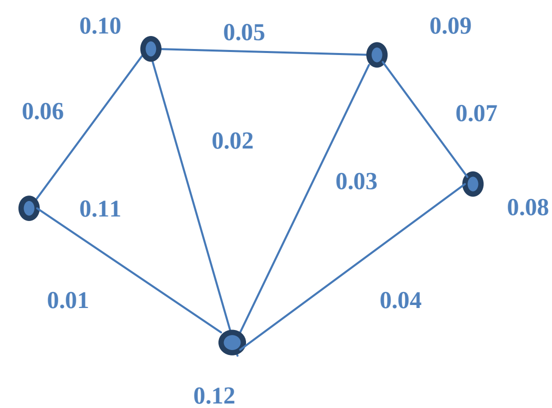
Case (ii): λ_m is prime and λ_c is prime.

Here λ_m is prime, the prime strength is not a prime.
In this case, G should be a non-prime pseudo intrinsic edge-magic. From the above observation, it depends upon the mock constant and intrinsic constant.

Theorem 3.12:
A fuzzy cycle C_n always a composite pseudo intrinsic edge-magic for n>3.

Theorem 3.13:
A fuzzy gem graph is a prime double pseudo intrinsic edge-magic.

Proof:
Let G be a fuzzy gem graph. Apply fuzzy intrinsic edge-magic labelling,



Fuzzy gem graph

1Apply fuzzy intrinsic edge-magic labelling,

$$\sigma(v_1)+\mu(v_1 v_2)+\sigma(v_2)=0.10+0.05+0.09=0.24=\lambda_c$$
$$\sigma(v_2)+\mu(v_2 v_3)+\sigma(v_3)=0.09+0.07+0.08=0.24=\lambda_c$$
$$\sigma(v_3)+\mu(v_3 v_4)+\sigma(v_4)=0.08+0.04+0.12=0.24=\lambda_c$$
$$\sigma(v_4)+\mu(v_4 v_5)+\sigma(v_5)=0.12+0.01+0.11=0.24=\lambda_c$$
$$\sigma(v_5)+\mu(v_1 v_5)+\sigma(v_1)=0.11+0.06+0.10=0.27=\lambda_m$$
$$\sigma(v_1)+\mu(v_1 v_4)+\sigma(v_4)=0.10+0.02+0.12=0.24=\lambda_c$$
$$\sigma(v_2)+\mu(v_2 v_4)+\sigma(v_4)=0.09+0.03+0.12=0.24=\lambda_c$$

We get one mock constant is not prime. Hence pseudo intrinsic fuzzy gem graph is a composite edge- magic graphs.

Conclusion:

In this paper, we discussed the various types of fuzzy pseudo intrinsic edge-magic graphs applied in gem graph , complete graph, butterfly graph and friendship graph.

References:

[1]. Zadeh L.A. (1965), Fuzzy sets, Information and control 8: 338-358.
[2]. Kotzig, A., and A. Rosa. (1970), "Magic valuations of finite graphs." Canadian Mathematical Bulletin 13: 451-461.
[3]. Rosenfeld, A. (1975), "Fuzzy graphs: In Fuzzy Sets and Their Applications." Academic Press, USA.
[4]. Trenkler M. (1983), "Some results on magic graphs," in Graphs and Other Combinatorial Topics, M. Fieldler, Ed., vol. 59 of Textezur Mathematik Band, pp.328–332,Teubner, Leipzig,Germany.
[5]. Bhattacharya, P. (1987), Some remarks on fuzzy graphs, Pattern recognition Lett. 6: 297-302.
[6]. Enomoto, H., A. S. Llado, T. Nakamigawa and G. Ringel. (1998), "Super edge-magic graphs." SUT Journal of Mathematics 34(2):105-109.
[7]. Avadayappan S, Jeyanthi P, and Vasuki R. (2001), "Super magic Strength of a graph,"Indian Journal of Pure and Applied Mathematics,vol.32, no.11, pp.1621–1630.
[8]. Bhuttani, K. R., A. Battou. (2003), "On M-strong fuzzy graphs." Information Sciences, 155(1-2): 103-109.
[9]. Ngurah, A. A., A. N. Salman, and L. Susilowati. (2010), "H-super magic labelings of graphs." Discrete Mathematics 310(8): 1293-1300.
[10]. NagoorGani A., Akram M.(2014), Novel properties of fuzzy labeling graphs, Journal of Mathematics.
[11]. Jamil R.N., Javaid M., Rehman M.A., Kirmani K.N. (2016), On the construction of fuzzy magic graphs, Science International 28(3).
[12]. Sobha K R, Chandra Kumar and Sheeba R S. (2018), Fuzzy Magic Graphs – A Brief Study, International Journal of Pure and Applied Mathematics, Volume 119 No. 15,1161-1170.
[13]. Kaliraja M and Sasikala M, "On the construction of fuzzy intrinsic edge-magic graphs"(Communicated)
[14]. Kaliraja M and Sasikala M, "Properties of fuzzy perfect intrinsic edge-magic graphs"(Communicated).