Three-Phase To Two-Phase Transformation Of Asynchronous Machine By Reference Frame Theory

Avinash R. Waghmare, Dr. Vandana A. Kulkarni

Abstract: An investigative three-phase asynchronous machine mathematical model is proposed, validated and derived in this paper. This paper investigates the dynamic mathematical model of pseudo reference frame theory of three-phase to two-phase conversion for the asynchronous electric machine (three-phase induction machine) by using Clark's transformation. In this paper, also studies mathematical derivation of the hypothetical machine, primitive machine and behavior of asynchronous machine with the two-phase operation. A system of different reference frames and related transformations (three-phase to two-phase) is investigated for the presentation of asynchronous machine equations, which enchanting knowledge of all winding MMF equations. This contains transformations of three-phase to two-phase rotating reference frames and a novel transformation to two axis components. These transformations decrease to the already well-known transformations of generalized electrical machine modeling and analysis theory. This article also provides the mathematical equations of a three-phase to two-phase transformations using natural reference frame theory for an asynchronous electric machine. Reference frame theory, a mathematical model for approximating the equivalent circuit parameters of the asynchronous machine from the readily available performance characteristics is presented.

Index Terms: Clark's transformation, mathematical model, pseudo reference frame, three-phase to two-phase transformation, stator reference frame.

1 Introduction

THE Induction machines are playing a very important role in industries, transport sectors. It provides very good advantage due to its simple construction, low cost, and robustness [3]. Induction machine mathematical model is extensively used in many industries and academics including its model characterization and extraction for power electronic design, control design, fault extraction, loss reduction and many others [1]. The asynchronous electrical machine is intended for full load conditions where the copper loss is leading. Various asynchronous electrical machine model proposed in the literature such as by using Clark's transformation, two-phase reference frame, and pseudo reference frame dynamic mathematical model is proposed. The Clarks mathematical transformation model is studied only for analysis purpose with two-phase and pseudo reference frame mathematical modelling. Derivation of the mathematical equations of asynchronous electrical machine for three phases to two-phase reference frame is beneficial for the mathematical investigation, associating the calculated and real curves of currents, phase voltages and torque which are appropriate without extra transformation of mathematical equations for the deliberation of modes of asynchronous electrical machine process at asymmetrical characteristics parameters of asynchronous electrical machine [3].

- *Avinash Ramarao Waghmare is currently pursuing master's degree program in electric machine Drives from Government College of Engineering, Aurangabad 431005, India, PH- +918308407238. E-mail: waghmare1431@gmail.com*
- *Dr. Vandana Abhay Kulkarni is Associate Professor in Government College of Engineering, Aurangabad 431005, India.*

The mathematical equations are derived for three phases to two-phase transformation using pseudo reference frame is a slightly complex method. In this paper, the suggested mathematical model of the three-phase asynchronous electrical machine is depending on two-phase electrical machine mathematical model at stator reference frame coordinate system axis $\alpha\beta$, which are associated with stator phase axis ABC for the conversion of three phases to two phases asynchronous mathematical machine model [3]. A pseudo-reference frame in which maximum control algorithms are designed, and is able to express analytically and derive torque and MMF equations. As compared to other models, the analytical mathematical model is most suitable for design considerations like flux observer and machine controllers. For loss minimization control algorithms, the optimal control variables can be solved by using mathematical analysis, numerical or iteratively based on the analytical connection between the machine loss and control variables [1]. The two-phase reference frame is done with the provision of a sinusoidally scattered MMF wave in the machine air gap, it also done mathematical analysis of two windings on stator and two windings on rotor of induction machine which contains a transformation to two-axis components of stator and the two-axis space vector components transformation of rotor [2]. The existence reference model phenomenon that is normally neglected in the modelling of the induction machine is more difficult to deal with, meanwhile these characteristically require a suitable modification of the basic induction machine model. The available models of induction machine (steady state and dynamic), is account for various stray losses components under the study of conditions of inverter and grid supply [5]. Two-phase reference frame mathematical models are used for the study of various solutions for steady-state and transient problems require machine parameters that are usually considered for mathematical modelling parameters. These machine parameters are including various resistance and inductance equations in matrix form which representing stator, rotor and magnetizing branches [6]. For mathematical modelling of the asynchronous machine, dynamic machine modelling attracts much attention due to their substantial impacts on the correctness of reference frame modelling [8], due to high load the temperature distribution between windings is a crucial problem, for two-phase reference frame modelling temperature sensitivity for two phases are considered for load distribution. The estimated methods described in this paper is to provide asynchronous machine mathematical modelling which represents a parameter estimation procedure for induction machine by two-phase reference frame and pseudo reference frame, that required knowledge stator and rotor resistance, inductance equations, the behaviour of asynchronous machine parameters [6].

2. Two-Phase Reference Frame Mathematical Model Equations.

The following fig.1 shows the mathematical model for two-phase transformation and it also depends on the Clarks transformation model.

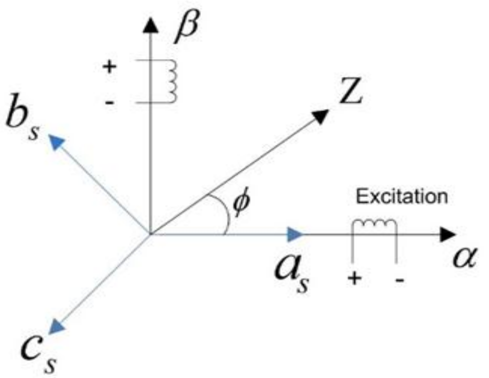

Fig. 1 *α and β components on MMF Waveform.*

Fig.1 shows that a_s, b_s, c_s is the stator three phase components with current flow from each phase is i_a, i_b, i_c. Two-phase component α and β is used for two-phase transformation, it also consists of two excitation windings having a number of turns N_s on stator phase, the resultant parameter of α and β is Z, is a resultant MMF component of stator phases. From above fig.1 MMF of Z phase at a phase angle ϕ is,

$$N_s\left[i_a\cos\phi + i_b\cos\left(\frac{2\pi}{3}-\phi\right) + i_c\cos\left(\frac{2\pi}{3}+\phi\right)\right] \quad (1)$$

By solving equation (1),

$$N_s\left[\left(i_a - \frac{i_b}{2} - \frac{i_c}{2}\right)\cos\phi + \left(\frac{\sqrt{3}}{2}i_b - \frac{\sqrt{3}}{2}i_c\right)\sin\phi\right] \quad (2)$$

For transforming into α and β, let's consider,

$$i_\alpha = i_a - \frac{i_b}{2} - \frac{i_c}{2}\,; \quad i_\beta = \frac{\sqrt{3}}{2}i_b - \frac{\sqrt{3}}{2}i_c \quad (3)$$

Rewriting equation (2),

$$N_s\left[i_\alpha\cos\phi + i_\beta\sin\phi\right] \quad (4)$$

Hence, from the above investigation the two-phase component α, β in matrix form are,

$$N_{\alpha\beta}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = N_s\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \quad (5)$$

$N_{\alpha\beta}$ is the number of turns of α, β components and current flow from α, β components are i_α and i_β, is fictitious variables are considered only for transformation into two phases, from the matrix (5) it is observed that is not a square matrix. However, it is not possible to convert two phases into three phases (i_α, i_β to a_s, b_s, c_s). For solving this complicated issue, symmetrical method going to be used. In a power system, any unbalanced three-phase system either voltage or flow of current can be resolved into a positive sequence component plus zero sequence component and then zero sequence component. A α and β component having positive sequence voltage components, v_α, v_β is equivalent to knowing v_+, v_- components. Consider, another one component that is missing as per the requirement of symmetry for the matrix is the zero-sequence component is v_0. Current i_α and i_β flow from two winding, it is not affected or makes a difference in net MMF as concerned fig. 1. The symmetrical component expression is,

$$v_0 = K[v_a + v_b + v_c] \quad (6)$$

K = any number that can be multiplied to v_a, v_b, v_c. Symmetrical matrix for two-phase transformation is,

$$N_{\alpha\beta}\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = N_s\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ K & K & K \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \quad (7)$$

For getting exact relation put K = 1/3, from this it is possible to convert two phases to three-phase transformation for that must find out the exact value of K, Let us consider, Matrix M and its inverse M^{-1} is,

$$M = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ K & K & K \end{bmatrix} \text{ and } M^{-1} = \begin{bmatrix} 1 & 0 & \frac{1}{2K} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{2K} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & \frac{1}{2K} \end{bmatrix}$$

M and M^{-1} is transpose (T) to each other.

K= $\frac{1}{2K}$ from this we can get, K = $\frac{1}{\sqrt{2}}$ and it is the exact value of K for transformation. From above conclusion, equation (8) are,

$$M^{-1} = \frac{2}{3}M^T \quad (8)$$

For solving the relation between $N_{\alpha\beta}$ and N_s two approaches going to be taken, During the transformation of three-phase windings to two-phase windings the produced MMF during transformation or after transformation must be same, but the effect of power invariance and power non-invariance is must be taken for observations of the behavior of MMF. This can be investigated by two ways that is by taking total power unchanged and per phase power unchanged.

2.1 Power Invariant Transformation

It must know from basic power equation a, b, c to α, β, 0. $I^T_{abc}.V^T_{abc} = I^T_{\alpha\beta0}.V^T_{\alpha\beta0}$; I is a vector consisting of i_a, i_b, i_c current components, and V is a vector consisting of v_a, v_b, v_c voltage components. From this conclusion, we can write the active power equation,

$P_{active} = v_a i_a + v_b i_b + v_c i_c$; for transformation

$$I_{\alpha\beta0} = \frac{N_s}{N_{\alpha\beta}} M\, I_{abc}$$

$$I_{abc}^{T}V_{abc} = \left(\frac{N_s}{N_{\alpha\beta}} I_{abc}M\right)^{T} V_{\alpha\beta 0} = \frac{N_s}{N_{\alpha\beta}} I_{abc}^{T}M^{T}\, V_{\alpha\beta 0} \qquad (9)$$

From equation (9) we can write voltage and current equation for three phases to two phase transformation.

$$\left.\begin{aligned} V_{abc} &= \frac{N_s}{N_{\alpha\beta}}\ M^{T}V_{\alpha\beta 0} \\ I_{abc} &= \frac{N_s}{N_{\alpha\beta}}\ M^{-1}\, I_{\alpha\beta 0} \end{aligned}\right\} \qquad (10)$$

For achieving invariant power between three phases to two-phase the relation between V & I must be identical.

$$\frac{N_s}{N_{\alpha\beta}}\ M^{T} = \frac{N_{\alpha\beta}}{N_s}\ M^{-1} = \frac{N_{\alpha\beta}}{N_s}\ \frac{2}{3}\ M^{T} \qquad (11)$$

From equation (11) it can be written as,

$$\frac{N_s}{N_{\alpha\beta}} = \sqrt{\frac{2}{3}} \qquad (12)$$

and the equation (12) value shows that net power by transforming three phases into two phases is invariant.

2.2 Power non-Invariant Transformation

Per phase power unchanged is observed by the following equation,

$$\frac{I_{abc}^{T}V_{abc}}{3} = \frac{I_{\alpha\beta 0}^{T}V_{\alpha\beta 0}}{2} \qquad (13)$$

For two-phase transformation components $\alpha\,\beta$, the current is divided by 2 because of the system consisting of balanced positive and negative sequence and absence of zero sequence component. Above equation is,

$$\frac{I_{abc}^{T}V_{abc}}{3} = \frac{N_s}{N_{\alpha\beta}} I_{abc}^{T}M^{T}\, V_{\alpha\beta 0}\ \frac{1}{2} \qquad (14)$$

Equation (13) must be identical if,

$$\left.\begin{aligned} V_{abc} &= \frac{3}{2}\frac{N_s}{N_{\alpha\beta}}\ M^{T}V_{\alpha\beta 0} \\ I_{abc} &= \frac{N_{\alpha\beta}}{N_s}\ M^{-1}\, I_{\alpha\beta 0} \end{aligned}\right\} \qquad (15)$$

$\frac{3}{2}\frac{N_s}{N_{\alpha\beta}}\ M^{T} = \frac{N_{\alpha\beta}}{N_s}\ M^{-1}$ Transpose of this equation is,

$$\frac{3}{2}\frac{N_s}{N_{\alpha\beta}}\ M^{T} = \frac{N_{\alpha\beta}}{N_s}\ \frac{2}{3}\ M^{T} \qquad (16)$$

By solving equation (16),

$$\frac{N_s}{N_{\alpha\beta}} = \frac{2}{3} \qquad (17)$$

Equation (16) this value shows that net per phase power by transforming three phases into two phases is non-invariant. The relation between V and I is,

$$\left.\begin{aligned} V_{abc} &= M^{T}V_{\alpha\beta 0} \\ I_{abc} &= M^{T}\ I_{\alpha\beta 0} \end{aligned}\right\} \qquad (18)$$

By taking the inverse of equation (17),

$$\left.\begin{aligned} V_{\alpha\beta 0} &= \frac{2}{3} M V_{abc} \\ I_{\alpha\beta 0} &= \frac{2}{3} M I_{abc} \end{aligned}\right\} \qquad (19)$$

Equation (17) and (18) shows the relation between V & I during three phases to two-phase transformation and two phases to three-phase transformation this all transformation is based on Clark's transformation theory.

3. Natural Reference Frame Mathematical Model

Fig. 2 shows the stator and rotor three phase displacement by an angle θ_r,

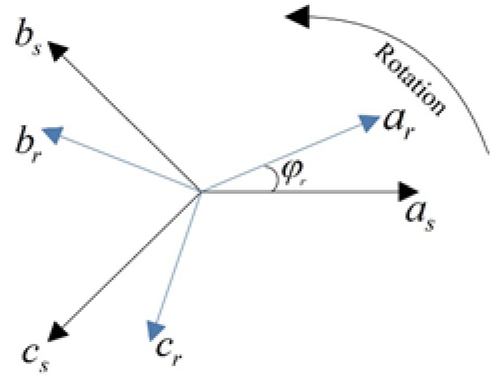

Fig.2 *Stator and rotor phase displacement*

Apply above investigation results to Induction machine, $V^{abc}, V^{\alpha\beta}$ = voltage of stator phases a, b, c and α , β phase. $V_{abc}, V_{\alpha\beta}$ = voltage of rotor phases a, b, c and α , β phase. Similar for current also, Induction machine equation is, $\lambda\,\psi$ is the vector of flux linkages between stator and rotor windings.

$$V_{abc}^{abc} = R\, I_{abc}^{abc} + \lambda\, \psi_{abc}^{abc}$$

$$V_{abc}^{abc} = R\, I_{abc}^{abc} + \lambda \left[L\, I_{abc}^{abc}\right]$$

For transformation of $\alpha\beta 0$ to abc voltage equation is,

$$V_{\alpha\beta 0}^{\alpha\beta 0} = A\, V_{abc}^{abc}\ ;\ A = \begin{bmatrix} A_{11} & 0 \\ 0 & A_{11} \end{bmatrix};$$

$$A = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

A is the transformation matrix for three-phase stator and rotor transformation.

$$\begin{bmatrix} V_{\alpha\beta 0} \\ V^{\alpha\beta 0} \end{bmatrix} = \begin{bmatrix} A_{11} & 0 \\ 0 & A_{11} \end{bmatrix} \begin{bmatrix} V_{abc} \\ V^{abc} \end{bmatrix} \qquad (20)$$

Similarly, abc to $\alpha\beta 0$ transformation,

$$V_{abc}^{abc} = A^{-1}\, V_{\alpha\beta 0}^{\alpha\beta 0}\ ,$$

$$A^{-1} = A^{T} \qquad (21)$$

We can derive similar equations for current using equation (20) & (21). From equation (19) induction machine modified equation will be written as,

$$A^{-1}\, V_{\alpha\beta 0}^{\alpha\beta 0} = R\, A^{-1}\, I_{\alpha\beta 0}^{\alpha\beta 0} + \lambda \left[L\, A^{-1}\, I_{\alpha\beta 0}^{\alpha\beta 0}\right] \qquad (22)$$

Multiplying equation (22) by matrix A,

$$V_{\alpha\beta 0}^{\alpha\beta 0} = A\, R\, A^{-1}\, I_{\alpha\beta 0}^{\alpha\beta 0} + A\, L\, \lambda\, A^{-1}\, I_{\alpha\beta 0}^{\alpha\beta 0} + A\, \lambda\, [L\, A^{-1}]\, I_{\alpha\beta 0} \qquad (23)$$

Where $\lambda = \frac{d}{dt}$ operator.

From the above equation, it is concluded that matrix A does not depend on time t and matrix A is a fixed matrix for transformation. Inductance L of induction machine windings is depending on time t because it changes its behavior according to rotor phase angle. For simplifying equation (23), it can be considered that after solving $A\, R\, A^{-1} = R$ and $A\, L\, A^{-1} = L$. From this consideration expanding the value of L & C for the natural reference frame, Inductance matrix,

$$L = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \text{ where,}$$

L_{11}= stator self-inductance, L_{12}= Stator to Rotor mutual inductance,

L_{22}= Rotor self-inductance, L_{21}= Rotor to stator mutual inductance. Stator self-inductance,

$$L_{11} = \begin{bmatrix} L_{ls}+L_{ms} & m_s & m_s \\ m_s & L_{ls}+L_{ms} & m_s \\ m_s & m_s & L_{ls}+L_{ms} \end{bmatrix} \quad (24)$$

Where, L_{ls} = stator winding leakage inductance; L_{ms}= Magnetizing stator inductance; m_s= Stator phase Mutual Inductance. Stator to rotor mutual inductance matrix with each phase of the stator to the rotor is,

$$L_{12} = \begin{bmatrix} m_{sr}\cos\theta & m_{sr}\cos\left(\theta+\frac{2\pi}{3}\right) & m_{sr}\cos\left(\theta+\frac{4\pi}{3}\right) \\ m_{sr}\cos\left(\theta+\frac{4\pi}{3}\right) & m_{sr}\cos\theta & m_{sr}\cos\left(\theta+\frac{2\pi}{3}\right) \\ m_{sr}\cos\left(\theta+\frac{2\pi}{3}\right) & m_{sr}\cos\left(\theta+\frac{4\pi}{3}\right) & m_{sr}\cos\theta \end{bmatrix} \quad (25)$$

m_{sr} is the maximum value of stator to rotor mutual inductance. Rotor
self-inductance,

$$L_{22} = \begin{bmatrix} L_{sr}+L_{mr} & m_r & m_r \\ m_r & L_{sr}+L_{mr} & m_r \\ m_s & m_r & L_{sr}+L_{mr} \end{bmatrix} \quad (26)$$

where, $m_r = -\frac{L_{mr}}{2}$, L_{sr}= Rotor leakage inductance, L_{ms}= Magnetizing rotor inductance, m_r= rotor mutual inductance. From above investigation concludes that combine inductance matrix for two phase transformation is,

$$L = \begin{bmatrix} L_{ls}+\frac{3}{2}L_{ms} & 0 & 0 & \frac{3}{2}m_{sr}\cos\theta & -\frac{3}{2}m_{sr}\sin\theta & 0 \\ 0 & L_{ls}+\frac{3}{2}L_{ms} & 0 & \frac{3}{2}m_{sr}\sin\theta & \frac{3}{2}m_{sr}\cos\theta & 0 \\ 0 & 0 & L_{ls} & 0 & 0 & 0 \\ \frac{3}{2}m_{sr}\cos\theta & \frac{3}{2}m_{sr}\sin\theta & 0 & L_{sr}+\frac{3}{2}L_{mr} & 0 & 0 \\ -\frac{3}{2}m_{sr}\sin\theta & \frac{3}{2}m_{sr}\cos\theta & 0 & 0 & L_{sr}+\frac{3}{2}L_{mr} & 0 \\ 0 & 0 & 0 & 0 & 0 & L_{sr} \end{bmatrix} \quad (27)$$

From induction machine speed considered then two-phase reference frame equation will be,

$$V_{\alpha\beta0}^{\alpha\beta0} = R\, I_{\alpha\beta0}^{\alpha\beta0} + L_{\alpha\beta0}^{\alpha\beta0}\,\lambda\, I_{\alpha\beta0}^{\alpha\beta0} + G_{\alpha\beta0}^{\alpha\beta0}\,\frac{d\theta}{dt}\, I_{\alpha\beta0}^{\alpha\beta0} \quad (28)$$

G is the reference frame speed matrix, $\frac{d\theta}{dt}$ is the rotor speed.

$$G = \begin{bmatrix} 0 & 0 & 0 & -\frac{3}{2}m_{sr}\sin\theta & -\frac{3}{2}m_{sr}\cos\theta & 0 \\ 0 & 0 & 0 & \frac{3}{2}m_{sr}\cos\theta & -\frac{3}{2}m_{sr}\sin\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{3}{2}m_{sr}\sin\theta & \frac{3}{2}m_{sr}\cos\theta & 0 & 0 & 0 & 0 \\ -\frac{3}{2}m_{sr}\cos\theta & -\frac{3}{2}m_{sr}\sin\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (29)$$

Above inductance and speed matrix investigated by considering two windings on the rotor and two windings on stator, both are located at axis 90^0 displace to each other.

4. Pseudo Reference Frame Mathematical Model For Induction Machine

Fig. 3 shows the two-phase stator and two-phase rotor pseudo reference frame phase displacement by an angle θ_r and MMF angle displaced by ϕ.

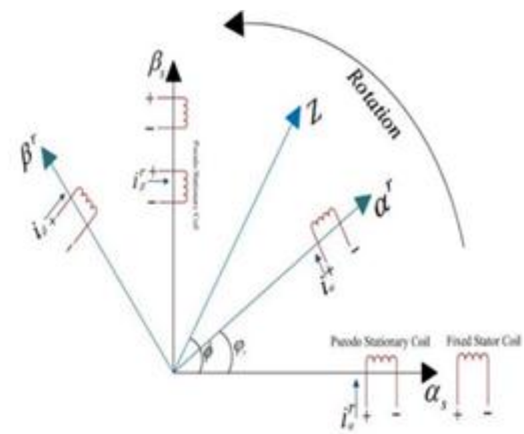

Fig.3 *Pseudo Reference Frame*

Voltage equation for the asynchronous machine is,

$$V = [R + L\,\lambda + G\,\omega]\, I \quad (30)$$

By recasting equation (30),

$$L\lambda I = V - R\,I - G\,\omega I,$$

The equation for the electrical system is,

$$\lambda I - L^{-1}[V - R\,I - G\,\omega I]$$

From fig. 3 MMF due to the α^r coil and the β^r coil is going to be calculated as expression,

$$F(\phi) = N^{\alpha} i^{\alpha}\cos(\phi - \theta_r) + N^{\beta} i^{\beta}\sin(\phi - \theta_r) \quad (31)$$

By simplifying equation (31),

$$N\left[i_{\alpha}^{r}\cos\phi + i_{\beta}^{r}\sin\phi\right]$$

here we can say that the MMF produced by i_{α}^{r} and i_{β}^{r} is same as MMF produced by i_{α} and i_{β}.

$$\left.\begin{aligned} i_{\alpha}^{r} &= i^{\alpha}\cos\theta_r - i^{\beta}\sin\theta_r \\ i_{\beta}^{r} &= i^{\alpha}\sin\theta_r + i^{\beta}\cos\theta_r \end{aligned}\right\} \quad (32)$$

Fig. (32) fictious current for two phase transformation,

$$\begin{bmatrix} i_{\alpha}^{r} \\ i_{\beta}^{r} \end{bmatrix} = \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix} \begin{bmatrix} i^{\alpha} \\ i^{\beta} \end{bmatrix}$$

$$i_{\alpha\beta}^{r} = D_{22}\, i^{\alpha\beta}\ ,\ D_{22} = \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix}$$

Equation of power invariance is,

$$v_{\alpha\beta}^{\alpha\beta} = R\, i_{\alpha\beta}^{\alpha\beta} + \lambda\left[L_{\alpha\beta}^{\alpha\beta}\, i_{\alpha\beta}^{\alpha\beta}\right] \quad (33)$$

Inductance matrix for two-phase pseudo reference transformation is,

$$L_{\alpha\beta}^{\alpha\beta} = \begin{bmatrix} L_s & 0 & m_{sr}\cos\theta_r & -m_{sr}\sin\theta_r \\ 0 & L_s & m_{sr}\sin\theta_r & m_{sr}\cos\theta_r \\ m_{sr}\cos\theta_r & m_{sr}\sin\theta_r & L_r & 0 \\ -m_{sr}\sin\theta_r & m_{sr}\cos\theta_r & 0 & L_r \end{bmatrix} \quad (34)$$

Two fictitious coils on stator having α_s axis and β_s axis, rotor current is

$$I^{\alpha\beta} = {D_{22}}^{T} I_{\alpha\beta}^{r}$$

from this equation power, invariance equation for power in $\alpha\beta$ and power in the fictitious coil is equal,

$$I^{\alpha\beta^{T}} V^{\alpha\beta} = I_{\alpha\beta}^{r^{T}} \quad (35)$$

$$\left[{D_{22}}^{T} I_{\alpha\beta}^{r}\right]^{T} V^{\alpha\beta} = I_{\alpha\beta}^{r^{T}} V_{\alpha\beta}^{r} \quad (36)$$

From equation (34) voltage equation is,

$$V_{\alpha\beta}^{r} = D_{22} V^{\alpha\beta} \quad (37)$$

For both stator and rotor voltage matrix with the fictitious coil is,

$$\begin{bmatrix} V_\alpha \\ V_\beta \\ V_\alpha^r \\ V_\beta^r \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\theta_r & -\sin\theta_r \\ 0 & 0 & \sin\theta_r & \cos\theta_r \end{bmatrix} \begin{bmatrix} V_\alpha \\ V_\beta \\ V^\alpha \\ V^\beta \end{bmatrix}$$

$$V^r_{\alpha\beta_s\alpha\beta_r} = DV^{\alpha\beta}_{\alpha\beta} \tag{38}$$

Substituting $I^r_{\alpha\beta\alpha\beta}$ & $V^r_{\alpha\beta\alpha\beta}$ in equation (32) by machine description is,

$$D^{-1}V^r_{\alpha\beta\alpha\beta} = RD^{-1}I^r_{\alpha\beta\alpha\beta} + \lambda\left[L^{\alpha\beta}_{\alpha\beta}D^{-1}I^r_{\alpha\beta\alpha\beta}\right] \tag{39}$$

Multiply equation (32) by matrix D and $D = \begin{bmatrix} X & 0 \\ 0 & D_{22} \end{bmatrix}$ and $D^{-1} = D^T$, $X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is an identity matrix.

$$V^r_{\alpha\beta\alpha\beta} = DRD^T I^r_{\alpha\beta\alpha\beta} + D\lambda\left[L^{\alpha\beta}_{\alpha\beta}D^T I^r_{\alpha\beta\alpha\beta}\right]$$

$$V^r_{\alpha\beta\alpha\beta} = RI^r_{\alpha\beta\alpha\beta} + DL^{\alpha\beta}_{\alpha\beta}D^T\lambda I^r_{\alpha\beta\alpha\beta} + D\lambda[LD^T]I^r_{\alpha\beta\alpha\beta} \tag{40}$$

For simplifying equation (38),

$$L^{\alpha\beta}_{\alpha\beta}\,D^T = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} X & 0 \\ 0 & {D_{22}}^T \end{bmatrix}$$

$$L_{12}{D_{22}}^T = \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix} \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ -\sin\theta_r & \cos\theta_r \end{bmatrix} m_{sr}$$

$$L_{12}{D_{22}}^T = m_{sr}X$$

By solving,

$$DL^{\alpha\beta}_{\alpha\beta}D^T = \begin{bmatrix} L_{11} & m_{sr}X \\ m_{sr}X & L_{22} \end{bmatrix} \tag{41}$$

By taking derivative of LD^T

$$\lambda\left[L^{\alpha\beta}_{\alpha\beta}D^T\right] = \begin{bmatrix} 0 & 0 \\ \lambda L^T_{12} & \lambda L_r{D_{22}}^T \end{bmatrix} \tag{42}$$

$$DL^{\alpha\beta}_{\alpha\beta}D^T\lambda = \begin{bmatrix} 0 & 0 \\ D_{22}\lambda L^T_{12} & D_{22}\lambda L_r{D_{22}}^T \end{bmatrix}$$

Expanding $D_{22}\lambda L^T_{12}$,

$$m_{sr}\begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix} \begin{bmatrix} -\sin\theta_r & \cos\theta_r \\ -\cos\theta_r & -\sin\theta_r \end{bmatrix} = m_{sr}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\frac{d\theta_r}{dt}$$

Similarly,

$$D_{22}\lambda L_r{D_{22}}^T = L_r\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\frac{d\theta_r}{dt}$$

$$D\lambda\left[L^{\alpha\beta}_{\alpha\beta}D^T\right] = \begin{bmatrix} 0 & 0 \\ m_{sr}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} & L_r\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \end{bmatrix}$$

$$V^r_{\alpha\beta_s\alpha\beta_r} = RI^r_{\alpha\beta_s\alpha\beta_r} + \begin{bmatrix} L_{11} & m_{sr}X \\ m_{sr}X & L_{22} \end{bmatrix}\lambda I^r_{\alpha\beta_s\alpha\beta_r} + \begin{bmatrix} 0 & 0 \\ m_{sr}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} & L_r\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \end{bmatrix}\frac{d\theta_r}{dt}\, I^r_{\alpha\beta_s\alpha\beta_r} \tag{43}$$

Equation (42) doesn't have rotor angle, by compressing and rewrite equation (42) in matrix form, Operational Impedance Matrix for two-phase pseudo reference frame of the asynchronous electrical machine is,

$$\begin{bmatrix} V_\alpha \\ V_\beta \\ V_\alpha^r \\ V_\beta^r \end{bmatrix} = \begin{bmatrix} r_r + L_s\lambda & 0 & m_{sr}\lambda & 0 \\ 0 & r_r + L_s\lambda & 0 & m_{sr}\lambda \\ m_{sr}\lambda & m_{sr}\omega_r & r_r + L_r\lambda & L_r\omega_r \\ -m_{sr}\omega_r & m_{sr}\lambda & -L_r\omega_r & r_r + L_r\lambda \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \\ i_\alpha^r \\ i_\beta^r \end{bmatrix} \tag{44}$$

5. Hypothetical Two Phase Machine Transformation

Following fig.4 shows the hypothetical three windings on stator $\alpha\beta$ axis, and mathematical expression study of one winding.

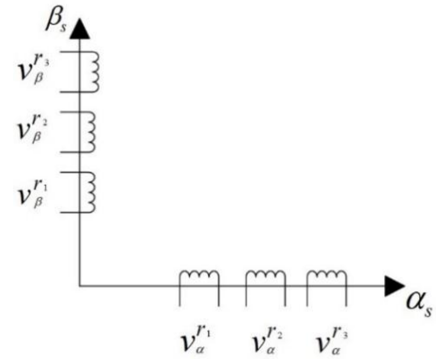

Fig. 4 *Hypothetical machine winding transformation*

Stator having more than one coil as shown in fig. 4

$$V_\alpha^{r_1} = m_{sr_1}\lambda I_\alpha + m_{sr_1}\omega_r I_\beta - m_{sr_2}\lambda i_\alpha^r + m_{sr_2}\omega_r i_\beta^r + \left(R_{r_1} + L_{r_1}\lambda\right)i_\alpha^{r_1} + L_{r_1}\omega_r i_\beta^{r_1} \tag{45}$$

Similarly, we can write for other voltage equations, $V_\beta^{r_1}, V_\alpha^{r_2}, V_\beta^{r_2}, V_\alpha^{r_3}, V_\beta^{r_3}$.

The supply frequency of stator of two-phase machine is ω_s. Two phases on the stator having voltage supply and two phases on the rotor are shortened for operation of asynchronous electrical machine, the current in the rotor is at slip frequency, $\omega_s - \omega_r$, for same MMF the current i^α and i^β is flow in the rotor then,

$$\begin{bmatrix} i_\alpha^r \\ i_\beta^r \end{bmatrix} = \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix} \begin{bmatrix} I_m\cos\{(\omega_s - \omega_r)t\} + \phi \\ I_m\sin\{(\omega_s - \omega_r)t\} + \phi \end{bmatrix} \tag{46}$$

By simplifying equation (45),

$$i_\alpha^r = I_m[\cos\{\theta + (\omega_s - \omega_r)t + \phi\}] \tag{47}$$

As $\theta = \omega_r t$ put in (46),

$i_\alpha^r = I_m(\omega_s t + \phi)$; similar equation for i_β^r.

Equation (47) shows the flow of current from one winding of hypothetical machine, such a way we can find all current equations flow from different number of windings wound on stator shown in fig. 4.

Conclusion

This paper deeply investigated analytical three-phase to two-phase transformation asynchronous electrical machine mathematical model which is suitable for three-phase to two-phase $\alpha\beta$ pseudo reference frame mathematical model. The asynchronous machine model parameters can be suitably extracted from the standard induction machine mathematical characteristics. Such mathematical study of asynchronous electrical machine is expected to provide theoretical studies concerning advanced control design and mathematical modelling for machine design prospective. It has been shown that transformation matrices simplify the analysis asynchronous electrical machine taking justification of all winding MMF, Inductance and machine equations. This paper studies three-phase vector components transformation and transformation to two-axis components are included. These transformations then applied to a set of asynchronous machine equations to validate several characteristics of the transformed equations. These equations then particularly analyse and studies phase winding influences and supply voltage conditions of stator and rotor, must thus still be extracted from the rest. By separation three phase to two phase transformation many problems associated with torque and frequencies if stator and rotor will be overcome.

References

[1]. Yiqi Liu, Ali M. Bazi, "A General Analytical Three Phase Induction Machine Core Loss Model in the Arbitrary Reference Frame", IEEE Transactions on Industry Applications, 2016.

[2]. F. S. van der Merwe, Stellenbosch, "Reference Frames and Transformations for Rotating Machines with Smooth Air-Gap and MMF Harmonics", Archiv Für Elektrotechnik, Springer, 1978.

[3]. M. Yu. Pustovetov, "A Mathematical Model of The Three-Phase Induction Motor in Three-Phase Stator Reference Frame Describing Electromagnetic and Electromechanical Processes", IEEE Trans. Dynamics and systems, January 2017.

[4]. Yiqi Liu, Ali M. Bazi, "A Detailed Induction Machine Core Loss Model in the Arbitrary Reference Frame", IEEE Applied Power Electronics Conference and Exposition (APEC), May 2015

[5]. Lamine, E. Levi, "Dynamic Induction Machine Modelling Considering the Stary Load Losses", IEEE 39th international universities Power Engineering Conference (UPEC), Sept. 2004.

[6]. Somchai Ansuj, Farrokh Shokooh, Roland Schinzinger, "Parameter Estimation for Induction Machines Based on Sensitivity Analysis", IEEE Transactions on Industry Applications, Vol. 25, Nov./Dec. 1989.

[7]. Jean-Francois Stumper, Alexander Dotlinger, Ralph Kennel, "Loss Minimization of Induction Machines in Dynamic Operation", IEEE Transactions on Energy Conversion, 2013.

[8]. Xiaodong Liang, C. Y. Chung, Senior, "Bus-Split Algorithm for Aggregation of Induction Motor and Synchronous Motors in Dynamic Load Modelling", IEEE Transactions On Industry Applications, Vol. 50, No. 3, May/June 2014.

[9]. Electrical – Modelling and Analysis of Electrical Machines, NPTEL, http://nptel.ac.in

[10]. Gonzalez, C. Hernandez, M. A. Arjona, "A Novel High Efficiency Parallel Winding Connection for a Three Phase Induction Motor Fed by a Single Phase Power Supply", IEEE Transactions On Energy Conversion, Vol. 29, No. 2, June 2014.