Transmutation Of The Two Parameters-Rayleigh Distribution On The Effect Of Physical Training On Age-Related Reduction Of GH Secretion During Exercise In Normally Cycling Women

M. Kaliraja, K. Perarasan, Vimala Subramanian

Abstract: Several life-time data are investigated using statistical analysis based on a respective statistical distribution. However, many of the life time data are still needed an attention in favour of statistical view. In this paper, we have employed a Transmuted Two Parameters Rayleigh Distribution to offer the mathematical elucidation for the effect of physical training on age-related reduction of GH secretion during exercise in normally cycling women.

Index Term: Rayleigh distribution, Mathematical tools, Growth hormone, Physical training.

1. INTRODUCTION

Rayleigh distribution (RD) is a continuous probability distribution for positive-valued random variables. It is a special case of the Weibull distribution and it is broadly used for survival analysis [1]. Lord Rayleigh (1880) introduced the single scale parameter Rayleigh distribution [2]. Rayleigh distribution application emerged in engineering sciences. This was first proposed by Longuet-Higgins [1], who demonstrated that in the case of linear waves with a narrow-band frequency spectrum, wave heights are Rayleigh distributed. Further, the applications of RD are extended to social sciences, medical sciences and marketing etc., [3-5]. In literature, it has been shown that the two parameters Rayleigh distribution with location and scale parameters. They have discussed about the some more properties of two parameters Rayleigh distribution. Here, we used the Rayleigh distribution to analyse the life data for the effect of physical training on age-related reduction of GH secretion during exercise in normally cycling women.Growth hormone (GH) is a 191 amino-acid single chain polypeptide, which is secreted by the somatotrophs in the anterior pituitary. With the recognition of its multiple and complex effects in the early 1960s, the physiology and regulation of GH has become a major area of research interest in the field of endocrinology [6-10].

- *M. Kaliraja, Assistant Professor, PG and Research Department of Mathematics, H. H. The Rajah's College, Pudukottai- 622 001. Email: mkr.maths009@gmail.com*
- *K. Perarasan, Research Scholar, PG and Research Department of Mathematics, H. H. The Rajah's College, Pudukottai- 622 001. Email: rinomathz502@yahoo.com*
- *Vimala Subramanian, Department of Epidemiology and Public Health, Central University of Tamil Nadu, Thiruvaur- 610 005. Email: vimala@cutn.ac.in*

It has been reported that the GH levels various with the physical activity. Physical exercise is a persuasive physiological stimulus for growth hormone (GH) secretion, and both aerobic and resistance exercise resulted in significant, acute increases in GH secretion [7-8]. Numerous lifetime data used in statistical analysis depends on a particular statistical distribution. Knowledge of suitable distribution of real data will extremely ameliorates the efficiency and the power of the statistical tests involved with it. Therefore, several distributions are suggested for modelling lifetime data. However, there are still many life time data that does not follow any distribution and hence there is a need to extend some new distributions. Here we suggested a new distribution for fitting lifetime data using one of the well known distribution function generation methods. Moreover, in the present study, we indented to analyse the V. Coiro et al., 2010 [11] stated data using transmutation two parameter Rayleigh Distribution.

2. METHODOLOGY

2.1. MATHEMATICAL MODEL

2.1.1 TRANSMUTED TWO PARAMETERS RAYLEIGH DISTRIBUTION

A random variable X has a transmuted distribution if its satisfy the relationship that is given by Shaw and Buckley [12, 13] which is known as quadratic rank transmutation map $F(x) = G(x)[(1+\lambda) - \lambda G(x)], |\lambda| \le 1$

Which on differentiation yields

$$f(x) = g(x)[1 + \lambda - 2\lambda G(x)]$$

Where λ is an additional parameter which is known as transmuted parameter and $g(x)$ and $G(x)$ are the pdf and cdf of base distribution.

The pdf of the two parameters Rayleigh distribution with location and scale parameters is $g(x;\alpha,\beta) = 2\beta(x-\alpha)e^{-\beta(x-\alpha)^2}$ $x > \alpha, \beta > 0$ Where β and α are scale and location parameters respectively and the corresponding cdf is defined as $G(x;\alpha,\beta) = 1 - e^{-\beta(x-\alpha)^2}$, $x > \alpha$ By using above equation, we obtained the cdf of the transmuted two parameters Rayleigh distribution $F_{TR}(x;\alpha,\beta,\lambda) = \left[1 - e^{-\beta(x-\alpha)^2}\right]\left[1 + \lambda e^{-\beta(x-\alpha)^2}\right]$ This gives the corresponding pdf of transmuted two parameters Rayleigh distribution ie, defined a $f_{TR}(x;\alpha,\beta,\lambda)=2\beta(x-\alpha)e^{-\beta(x-\alpha)^2}\left[1-\lambda+2\lambda e^{-\beta(x-\alpha)^2}\right]$

Theorem
Let X be is a random variable that has $T_R(x;\alpha,\beta,\lambda)$ with $|\lambda|\le 1$ then the rth moment $E(x^r)$ of transmuted two parameters Rayleigh distribution is

$$\mu_r'=\sum_{k=0}^{r}\binom{r}{k}\alpha^{r-k}\left[(1-\lambda)\left\{\frac{\Gamma\left(\frac{k}{2}+1\right)}{\beta^{\frac{k}{2}}}\right\}+\lambda\left\{\frac{\Gamma\left(\frac{k}{2}+1\right)}{2\beta^{\frac{k}{2}}}\right\}\right]$$

Proof:
The rth moment is given by,
$\mu_r'=E(X^r)=\int_\alpha^\infty x^r 2\beta(x-\alpha)e^{-\beta(x-\alpha)^2}\left[1-\lambda+2\lambda e^{-\beta(x-\alpha)^2}\right]dx$
Substitute $x=\alpha+z$, then

$$\mu_r'=2\beta\int_0^\infty(\alpha+z)^r z e^{-\beta z^2}\left[(1-\lambda)+2\lambda e^{-\beta z^2}\right]dz$$

$$\mu_r'=2\beta\int_0^\infty\sum_{k=0}^{r}\binom{r}{k}z^k\alpha^{r-k}\,ze^{-\beta z^2}\left[(1-\lambda)+2\lambda e^{-\beta z^2}\right]dz$$

$$\mu_r'=2\beta\sum_{k=0}^{r}\binom{r}{k}\alpha^{r-k}\int_0^\infty z^{k+1}e^{-\beta z^2}\left[(1-\lambda)+2\lambda e^{-\beta z^2}\right]dz$$

Again for convenience we substitute $y=z^2$, then

$$\mu_r'=2\beta\sum_{k=0}^{r}\binom{r}{k}\alpha^{r-k}\left[(1-\lambda)\int_0^\infty\frac{y^{\left(\frac{k+1}{2}\right)}e^{-\beta y}}{2\sqrt{y}}dy+2\lambda\int_0^\infty\frac{y^{\left(\frac{k+1}{2}\right)}e^{-2\beta y}}{2\sqrt{y}}dy\right]$$

$$\mu_r'=2\beta\sum_{k=0}^{r}\binom{r}{k}\alpha^{r-k}\left[\frac{1-\lambda}{2}\right]\int_0^\infty y^{\left(\frac{k}{2}\right)}e^{-\beta y}\,dy+\lambda\int_0^\infty y^{\left(\frac{k}{2}\right)}e^{-2\beta y}dy$$

$$\mu_r'=\sum_{k=0}^{r}\binom{r}{k}\alpha^{r-k}\left[(1-\lambda)\left\{\frac{\Gamma\left(\frac{k}{2}+1\right)}{\beta^{\frac{k}{2}}}\right\}+\lambda\left\{\frac{\Gamma\left(\frac{k}{2}+1\right)}{2\beta^{\frac{k}{2}}}\right\}\right]$$

The mean of the transmuted two parameters Rayleigh distribution using above equation, can be defined as follows, $Mean=E(X)=\alpha+\frac{\sqrt{\pi}}{2\beta}\left[(1-\lambda)+\frac{\lambda}{2}\right]$

r^{th} *Moment expression of the transmuted two parameters Rayleigh distribution*

$$\mu_r'=\alpha+\frac{\sqrt{\pi}}{2\beta}\left[(1-\lambda)+\frac{\lambda}{2}\right]$$

$$\mu_1'=\alpha+\frac{\sqrt{\pi}}{2\sqrt{\beta}}\left[(1-\lambda)+\frac{\lambda}{2}\right]$$

$$\mu_2'=\alpha^2+\frac{\alpha\sqrt{\pi}}{\sqrt{\beta}}\left[(1-\lambda)+\frac{\lambda}{2}\right]+\frac{1}{\beta}\left[(1-\lambda)+\frac{\lambda}{2}\right]$$

And hence, can find $\mu_3',\mu_4',\ldots..$

Theorem
Let the random variable X follows transmuted two parameters Rayleigh distribution, and then its variance has the following form $\sigma^2=Var(X)=\frac{1}{\beta}\left[(1-\lambda)+\frac{\lambda}{2}\right]-\frac{\pi}{4\beta}\left[(1-\lambda+\frac{\lambda}{2})\right]^2$

Proof:
the variance of transmuted parameters Rayleigh distribution is given as

$$Var(X)=E(X^2)-[E(X)]^2$$

Then

$$Var\ (X)=a^2+\frac{a\sqrt{\pi}}{\sqrt{\beta}}\left[(1-\lambda)+\frac{\lambda}{\sqrt{2}}\right]+\frac{1}{\beta}\left[(1-\lambda)+\frac{\lambda}{2}\right]-\left[a+\frac{\sqrt{\pi}}{2\sqrt{\beta}}\left[(1-\lambda)+\frac{\lambda}{\sqrt{2}}\right]\right]^2$$

$$Var\ (X)=a^2+\frac{a\sqrt{\pi}}{\sqrt{\beta}}\left[(1-\lambda)+\frac{\lambda}{\sqrt{2}}\right]+\frac{1}{\beta}\left[(1-\lambda)+\frac{\lambda}{2}\right]-a^2-\frac{\pi}{4\beta}[(1-\lambda)+\frac{\lambda}{2}]^2-\frac{a\sqrt{\pi}}{\sqrt{\beta}}\left[(1-\lambda)+\frac{\lambda}{\sqrt{2}}\right]$$

$$Var\ (X)=\frac{1}{\beta}\left[(1-\lambda)+\frac{\lambda}{2}\right]-\frac{\pi}{4\beta}\left[(1-\lambda)+\frac{\lambda}{2}\right]^2$$

Theorem:
Let X has a transmuted two parameters Rayleigh distribution, and then the moment generating function of transmuted two parameters Rayleigh distribution is

$$M_X(t)=\sum_{r=a}^{\infty}\sum_{k-0}^{r}\frac{t^r}{r!}\binom{r}{k}a^{r-k}\left[(1-\lambda)\left\{\frac{\Gamma\left(\frac{k}{2}+1\right)}{\beta^{\left(\frac{k}{2}\right)}}\right\}+\lambda\left\{\frac{\Gamma\left(\frac{k}{2}+1\right)}{(2\beta)^{\left(\frac{k}{2}\right)}}\right\}\right]$$

Proof: the mgf for X is given as

$$M_X(t)=E(e^{tX})=\int_a^\infty e^{tx}f_{TR}(x,a,\beta,\lambda)dy$$

We know that

$$e^{tx}=1+tx+\frac{t^2x^2}{2!}+\frac{t^3x^3}{3!}+\cdots\frac{t^rx^r}{r!}+\cdots$$

Then

$$M_X(t)=\int_a^\infty\left(1+tx+\frac{t^2x^2}{2!}+\frac{t^3x^3}{3!}+\cdots\frac{t^nx^n}{n!}+\cdots\right)f_{TR}(x,a,\beta,\lambda)dy$$

$$M_X(t)=\sum_{r=a}^{\infty}\frac{t^rE(X)^r}{r!}$$

$$M_X(t)=\sum_{r=a}^{\infty}\frac{t^r}{r!}\left(\sum_{k=0}^{r}\binom{r}{k}a^{r-k}\left[(1-\lambda)\left\{\frac{\Gamma\left(\frac{k}{2}+1\right)}{\beta^{\left(\frac{k}{2}\right)}}\right\}+\lambda\left\{\frac{\Gamma\left(\frac{k}{2}+1\right)}{(2\beta)^{\left(\frac{k}{2}\right)}}\right\}\right]\right)$$

$$M_X(t)=\sum_{r=a}^{\infty}\sum_{k-0}^{r}\frac{t^r}{r!}\binom{r}{k}a^{r-k}\left[(1-\lambda)\left\{\frac{\Gamma\left(\frac{k}{2}+1\right)}{\beta^{\left(\frac{k}{2}\right)}}\right\}+\lambda\left\{\frac{\Gamma\left(\frac{k}{2}+1\right)}{(2\beta)^{\left(\frac{k}{2}\right)}}\right\}\right]$$

Reliability analysis

The reliability function gives the probability of surviving of an item at reach on t time. The reliability function of transmuted two parameters Rayleigh distribution written of the form

$$R_{TR}(t) = P(T > t) = \int_t^\infty f(t)dt = 1 - F_{TR}(t)$$

$$R_{TR}(t) = e^{-\beta(x-\alpha)^2}\left(1 - \lambda\left(1 - e^{-\beta(x-\alpha)^2}\right)\right)$$

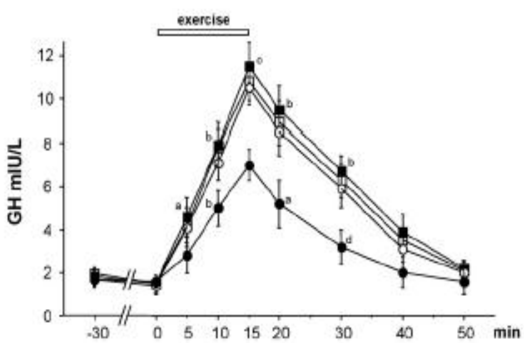
.

3. RESULTS

3.1. APPLICATION

3.1.1. Background

To establish a transmutation two parameters Rayleigh distribution, the life time medical data was adopted from the published work of [11]. Where, authors have performed the study with twenty younger (26–30 years) and twenty older (42–46 years) normally cycling healthy women were participated. Please refer V. Coiro et al., 2010 [11] for further explanation on experimental and study design. In the present study, we have acquired the following data (shown in Figure 1)



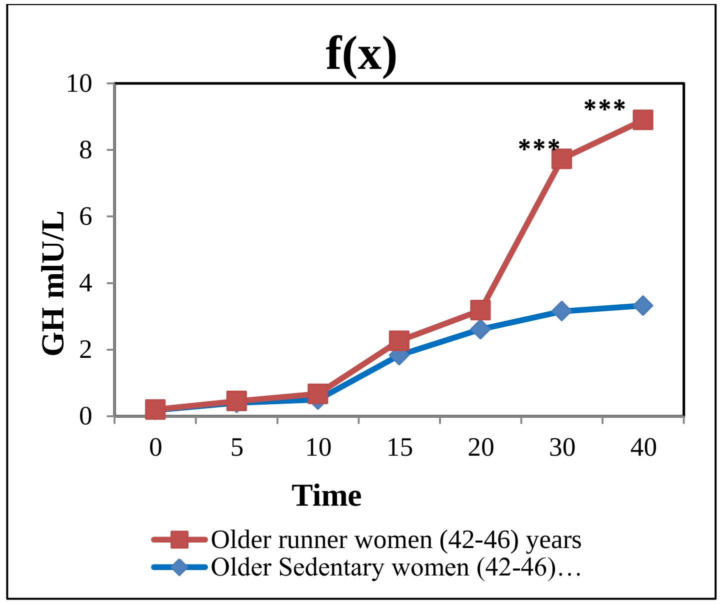
Medical figure.1

Fig. 1. GH response to exercise in younger sedentary (- - n.10, 27–29 years), younger runner (- - n.10, 26–30 years); older sedentary (- - n.10, 42–46 years), older runner women (- - n.10, 42–46 years). Each point represents the mean ± SE of 10 observations.

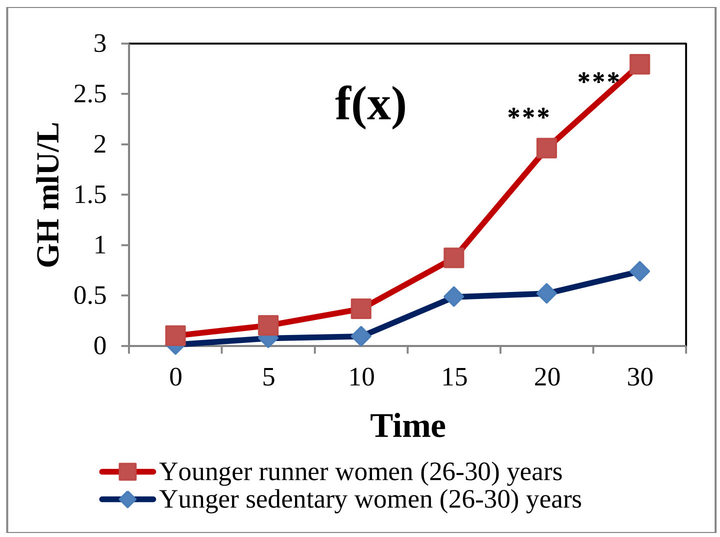
4. MATHEMATICAL RESULTS

4.1. The probability density function of the transmutation two parameters Rayleigh distribution.

Probability density functions $f(x)$ of transmutation of the two parameters Rayleigh distribution analysis on for the Effect of physical training on age-related reduction of GH secretion during exercise in normally cycling women is shown in mathematical figure 1(a-b). In parallel to V. Cario et al., [4] the two parameter RD f(x) plot shows the increased level of GH in younger runner women as compared to younger sedentary women with the age of 27-29 years (figure 1b). Similar results were observed with older runner and sedentary women's GH levels too (figure 1a).



Mathematical Figure: 1a

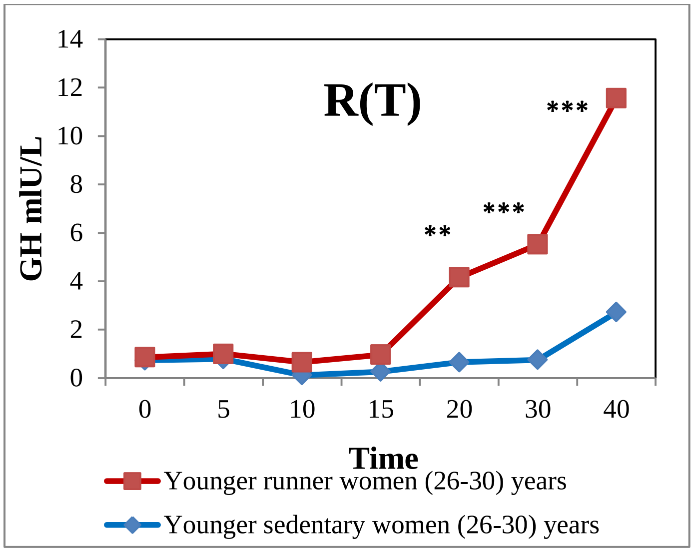


Mathematical Figure: 1b

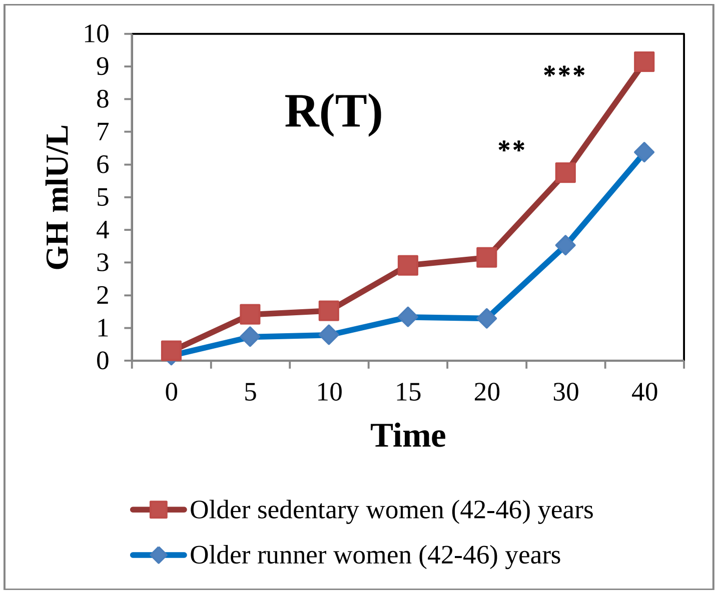
Transmutation of the two parameters Rayleigh distribution of its probability density function of graphical levels of effect of physical training on age-related changes of GH secretion during exercise in normally cycling women. Where, *** p<0.01, younger runner women vs younger sedentary women; *** p<0.01, Older runner women vs Older sedentary women.

4.2. The Reliability function of the transmutation of the two parameters Rayleigh distribution.

Mathematical Figure 2 represents the reliability function for the effect of physical training on age-related reduction of GH secretion during exercise in normally cycling women after 5, 10, 15, 20, 30, 40 and 50 min. In parallel to the probability function, the reliability function also showed the increased level of GH in younger runner women as compared to younger sedentary women with the age of 27-29 years as well as in older runner and sedentary women's GH levels too (figure 2).



Mathematical Figure: 2a



Mathematical Figure: 2b

Transmutation of the two parameters Rayleigh distribution of its Reliability function's graphical levels for the effect of physical training on age-related changes of GH secretion during exercise in normally cycling women. Where, *** $p<0.01$, younger runner women vs younger sedentary women; *** $p<0.01$, Older runner women vs Older sedentary women; ** $p<0.05$, Older runner women vs Older sedentary women.

5. DISCUSSION:

Statistical distributions are incredibly useful in recognizing and predicting the real time global phenomena. One among them is the extreme value distribution. It has been expansively used to model lifetime data and modelling natural phenomena. In this paper we have evaluated the effect of physical training on age-related changes of GH secretion during exercise in normally cycling women using mathematical model of transmutation of the two parameters Rayleigh distribution. The above results are calculated at the different times as follows in minutes 5, 10, 15, 20, 30, 40 and 50 min. The probability function (figure 1a-b) and the reliability function (figure 2) elicits that the level of GH increased in the older age runner women as compared with sedentary women. Consequently, the younger age sedentary women showed a decreased level of GH in comparison with the younger runner women. It should be noted that the obtained results are well supported by our previously published data [14-16]. There, we have analysed the life-time data, especially on GH level using different statistical tools and the results are concurrent.

6. CONCLUSION:

In conclusion, the results of the current study add ups and provides an extra arm to support that the physical exercise increases the growth hormone level which suggests the regular exercise benefits to lead a healthiest life. Further studies are needed to substantiate this hypothesis.

7. REFERENCES

[1] G. Maymon, Chapter 2 - Some Important Statistical Distributions, Stochastic Crack Propagation Essential Practical Aspects 2018, Pages 9-18

[2] D. Kundu and M.Z. Raqab, Generalized Rayleigh distribution: different methods of estimations. *Computational statistics & data analysis*, *49*(1), pp.187-200, 2005.

[3] H. M. Khan, S. B. Provost & A. Singh, Predictive infer-ence from a two-parameter Rayleigh life model given a doubly censored sample. Communications in Statistics—Theory and Methods,39(7),1237-1246,2010, http://dx.doi.org/10.1080/03610920902871453.

[4] C. D. Lai, Constructions and applications of lifetime distributions. Applied Stochastic Models in Business and Industry,29(2),127-140,2013, http://dx.doi.org/10.1002/asmb.948.

[5] Merovci, F. (2013). Transmuted Rayleigh distribution. Austrian Journal of Statistics, 42(1), 21-31.

[6] E. E. Kuruoglu and J. Zerubia, Modeling SAR images with a generalization of the Rayleigh distribution. IEEE Transactions on Image Processing, 13(4), pp.527-533, 2004.

[7] V. Coiro, R. Volpi, D. Gramellini, et al. Altered neuroendocrine control of GH secretion in normal women of advanced reproductive age. J Gerontol, 52:M254–8, 1997.

[8] MR. Ambrosio, A. Valentini, G. Transforini, et al. Function of the GH/IGF1 axis in healthy middled-aged male runners. Neuroendocrinology, 63:498–500, 1986.

[9] FF. Horber, SA. Kohler, K. Lippuner, P. Jaeger, Effect of regular physiol training on age associated alteration of body composition in men. Eur J Clin Invest, 26:279–85, 1996.

[10] V. Coiro, R. Volpi, L. Capretti, G. Caffarri, C. Davoli, P. Chiodera, Age-dependent decrease in the growth hormone response to growth hormone-releasing hormone in normally cycling women. Fertil Steril, 66:230–4, 1996.

[11] V. Coiro, Riccardo Volpia, Dandolo Gramellini, M. Maria Ludovica Maffei, Elio Volta, Andrea Melani, Paolo Chiodera, Effect of physical training on age-related reduction of GH secretion during exercise in normally cycling women, Maturitas 65, 392–395, 2010.

[12] W. T. Shaw & I R. Buckley, The alchemy of probability distributions: beyond Gram-Charlier expansions and a skew-kurtotic-normal distribution from a rank transmutation map. arXiv preprint arXiv:0901.0434, 2009.

[13] Ehsan Ullah , Mirza Naveed Shahzad , Transmutation of the two parameters Rayleigh distribution, International Journal of Advanced Statistics and Probability, , 4 (2) , 95-10, 2016.

[14] Kaliraja and K. Perarasan, "An Impact of Growth Hormone Releasing Hormone (GHRH) on Growth Hormone (GH) Response in Women of Reproductive age associated with Obesity– A Stochastic model of exponentiated Gumbel type", International Journal of Mathematical Sciences, Vol. 18, Nos. 1-2, January-June, pp. 17-25, 2019.

[15] M. Kaliraja and K. Perarasan, "A Mathematical Weibull Model to Unravel the Involvement of Acute Exercise on Serum Growth Hormone Response in Elite Male Water Polo Players, Vol. 9 No. 12, Page No 2047 – 2054, 2018.

[16] M. Kaliraja and K. Perarasan, "A Lindley Distribution model for Intravenous administration of Ghrelin Stimulates GH Secretion in Vagotomized Patients and normal subjects, Int. J.Sci.Res.in Mathematical and Statistical Sciences, Vol. 6, Issue 2, pp.16-20,April 2019.