An Application Of Generalised Johansen-Ledoit-Sornette (GJLS) Model To Detect Size Of Rational Speculative Bubble In DJIA Stock Market During Year 1997

Devendran Indiran, Nurfadhlina Abdul Halim, Wan Muhamad Amir W. Ahmad

Abstract: Rational speculative bubble can be defined as transient upward movements of stock prices above fundamental value due to speculative investors. The Generalised Johansen-Ledoit-Sornette (GJLS) model have been developed as a flexible tool to identify the size of rational speculative bubble. This model is combines the economic theory of rational expectation bubbles with finite-time singular crash hazard rates, behavioural finance on imitation and herding of investors and traders as well as mathematical statistical physics of bifurcations and phase transitions. It has been employed successfully to a large variety of stock bubbles in many different markets. The purpose of this study is to prediction bubble size of DJIA during economic crisis 1997.

Keywords: Bubbles, Intrinsic Value, GJLS, Economic Crises, DJIA, Prediction

I. INTRODUCTION

Rational speculative bubbles can be defined as positive acceleration of prices above fundamental value [1]-[3], [6]. A sudden rise in the price of a continuous process also can be named as rational speculative bubble [4]. Rational speculative bubbles are one of the serious issue that give negative implications to the development of country’s economy. This is because of economic bubble formation and dramatic bursts in financial markets [5]. Many recent theories portrays that economic bubbles can be created because of positive feedback trading by noise traders, heterogeneous beliefs of investors together with a limitation on arbitrage and synchronization failures among rational traders. Researches done by [7]-[13] proved that the combined effects of heterogeneous beliefs and short-sales constrained may lead large movements in asset. In this kind of models which assume heterogeneous beliefs and short-sales, the asset prices are determined at equilibrium to the extent that they reflect the heterogeneous beliefs about payoffs, but short sales boundaries force the pessimistic investors disappear from the market, leaving only optimistic investors and thus magnified asset price levels. However, when short sales limitations no longer tie investors, then prices fall back downwards.

In another class of models, the role of “noise traders” in fostering positive feedback trading has been emphasized. The term “noise trader” was proposed first by [14] and [15] to portray irrational investors. These noise positive feedback traders purchase securities when prices rises and sell when prices drop. Due to this positive feedback mechanism, the divergence between the market price and the intrinsic value has been bloated [16]-[19]. The empirical evidences on this theory are mainly from the studies on momentum trading strategies. Stocks which performed poorly in the past will perform better in a long-term perspective (over the next three to five years) than stocks which performed well in the past [20]. In contrast, at intermediate horizon (three to twelve months), the stocks which performed well previously will still perform better [21]. Apart from that, the factors that cause inflation have relationship to economic bubbles. The following figure 1 portrays the relation between inflation and economic bubbles.

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Figure 1: The relation between inflation and economic bubble

- The increase in demand for goods and services and at the same time lack in the supply of goods and services
- Increase in the price of goods and services drastically
- Economic Bubbles
- The value of currency slumps and the price of consumer goods and services are increasing.
- Inflation
However, identifying the existence of economic bubbles remains an unsolved problem in standard econometric and financial economic methods [24], [25]. This is due to the fact that the intrinsic value is in general poorly constrained and it is impossible to differentiate between exponentially growing bubble prices. Diagnosing the bubble ex-ante could help to take several actions to stop from bubble bursting. But none of the theories mentioned above can diagnose bubble ex-ante. This may be due to the fact that all these theories cannot distinguish between intrinsic and bubble price and cannot give a price dynamics which leads to a crash. The Standard Johansen-Ledoit-Sornette (SJLS) model or Johansen-Ledoit-Sornette Model was developed by Sornette and his colleagues has potential to describe the price dynamics during a bubble regime by analysing the cumulative human behaviour in new perspectives. It also has the ability to predict the most probable crash time after a bubble ex-ante. Generalized Johansen-Ledoit-Sornette (GJLS) Models have been developed as flexible tools to detect bubble size by predicting fundamental value by [26]. This study focused on estimating bubble size that formed in DJIA stock market during year 1997.

II. Generalised Johansen LeoditSornette Model

The JLS model of financial bubbles and crashes is an extension of the rational expectation bubble model proposed by [17]. A financial bubble is modelled as a regime of accelerating or super-exponential power law growth punctuated by short-lived corrections organized according the symmetry of discrete scale invariance [18]. The super-exponential power law is argued to result from positive feedback resulting from noise trader decisions that tend to enhance deviations from fundamental valuation in an accelerating spiral. We firstly consider the purely speculative asset that pays no dividends, so that we do not take into account the interest rate, information asymmetry, risk aversion, and the market clearing condition. The rational expectations are simply corresponding to the familiar martingale hypothesis in (1).

\[ E_t[p(t')] = p(t) \quad \forall t' > t \quad (1) \]

where \( p(t) \) denotes the price of the asset at time \( t \) and \( E_t[\cdot] \) indicates the expectation conditional on information revealed up to time \( t \). Then lets the cumulative distribution function (cdf) of the time of crash is called \( Q(t) \), the probability density function (pdf) is \( q(t) = \frac{dQ}{dt} \) and the hazard rate is \( h(t) = \frac{q(t)}{1 - Q(t)} \). The hazard rate is the probability per unit of time that the crash will happen in the next instant if it has not happened yet. In the JLS model, the stock market dynamics is described as (2).

\[ \frac{dp}{p} = \mu(t)dt - \kappa dj \quad (2) \]

where \( p \) is the stock market price and the term \( dj \) indicates a discontinuous jump such that \( dj = 0 \) before the crash and \( dj = 1 \) after the crash happens. The parameter \( \kappa \) determined the loss amplitude associated with the occurrence of a crash. The time-dependent drift \( \mu(t) \) is chosen so that the price process satisfies the martingale condition given as (3) and (4), respectively.

\[ E_t[dp] = \mu(t)p(t)dt - \kappa p(t)\delta(t)dt = 0 \quad (3) \]

\[ \mu(t) = \kappa h(t) \quad (4) \]

And (5) is corresponding to the price.

\[ \log \left[ \frac{p(t)}{p(t_0)} \right] = \kappa \int_{t_0}^{t} h(t)dt \quad (5) \]

This gives the logarithm of the price as the relevant observable. The higher the probability of a crash, the faster the price grow (conditional on having no crash) in order to obey the martingale condition. Intuitively, investors must be remunerated by a higher return in order to be induced to hold an asset that might crash. The sensitivity of the market reaction to news or external influences accelerate on the approach to this transition in a specific way characterized by a power law divergence at the critical time \( t_c \) of the form

\[ F(t) = (t_c - t)^{-z}, \quad z \text{ is called a critical exponent. This form amounts to the following property of (6).} \]

\[ \frac{d \ln f}{d \ln (t_c - t)} = -z \quad (6) \]

(6) is a constant, namely that the behaviours of the observable \( F \) become self-similar close to \( t_c \). The symmetry of self-similarity in the present context refers to the fact that the relative variations \( d \ln F = \frac{dF}{F} \) of the observable with respect to relative variations

\[ d \ln (t_c - t) = \frac{d(t_c - t)}{(t_c - t)} \]

of the time-to-crash is independent of time \( t \), as expressed by the constancy of the exponent \( z \). The crash hazard rate follow the same dependence as (7).

\[ h(t) = B'(t_c - t)^{m-1} \quad (7) \]

where \( B \) is a positive constant and \( t_c \) is the critical point or theoretical date of the bubble end. The term \( m \) must in the range of \( 0 < m < 1 \) for an important economic reason’s otherwise; the price would go infinity when approaching \( t_c \) (if the bubble has not crashed yet). The first order expansion for (7) (the hazard rate) is given by (8).
\[ h(t) \approx B (t_c - t)^{m-1} + c'(t_c - t)^{m-1} \cos(\omega \ln(t_c - t) + \phi) \]  

The crash hazard rate now displays log-periodic oscillations. This can easily be seen by taking the exponent \( z \) to be complex with a non-zero imaginary part, since the real part of \((t_c - t)^{-z+i\omega} \) is \((t_c - t)^{-m-1} \cos(\omega \ln(t_c - t)) \). The evolution of the price before the crash and critical date is then given by (9).

\[ \ln E[p(t)] \approx A + B(t_c - t)^m + C(t_c - t)^m \cos(\omega \ln(t_c - t) + \phi) \] 

The generalised Johansen LeoditSornette Model is formed by inferring fundamental value of stock in eq.(9). Extension of (9) is said to be GJLS Model that proposed by [26].

The price dynamics of an asset as

\[ dp = \mu(t)pdtd + \sigma(t)pdt - \kappa(p - p_1)^\gamma dj \]  

where the \( \mu(t)pdt + \sigma(t)pdt \) describes the statistical geometrical Brownian motion and the third term is the jump.

When the crash occurs at some time \( t^* \) (indicate \( I^*+ \)) , the price drops abruptly by amplitude

\[ \kappa\left[p\left(t^*-\right) - p_1\right]^\gamma \]  

where \( \kappa = \gamma = 1 \), the price drops from \( p(t^-) \) to \( p(t^+) = p_1 \). The price changes from its value just before crash to a fixed well-defined valuation \( p_1 \). Inferring no-arbitrage condition \( E_t[dp] = 0 \) to (10) leads to

\[ \mu(t)p = k(p - p_1)^\gamma h(t) \]  

Conditional on the absence of a crash, the dynamics of the expected price obeys the equation

\[ dp = \mu(t)pdt = k(p - p_1)^\gamma h(t)dt \] 

and the fundamental price must obey the condition

\( p_1 < \min p(t) \). For \( \gamma = 1 \), the solution is

\[ \ln[p(t) - p_1] = F_{LPPL}(t) \]  

where \( F_{LPPL}(t) \) is given by the (9); For \( \gamma \in (1,0) \), the solution is

\[ (p - p_1)^{1-\gamma} = F_{LPPL}(t) \] 

do not consider the case \( \gamma > 1 \) which would give an economically non-sensible behaviour, namely the price diverges in finite time before the crash hazard rate itself diverges. In summary, [26] considered a model as shown below.

\[ p_1 + \exp(F_{LPPL}(t)), \gamma = 1 \]  

The final model (16) was applied to the DJIA stock index to identify the size of bubble that appeared during the year 1997. This study is to obtain fundamental value of stock price and followed by identification of bubble size as well. Time interval that selected to analyse are as shown in the figure 2 below.
III. RESULT

Figure 2: Time interval of DJIA Stock Market

Figure 3: Fitted curve selected DJIA index data, 1997
TABLE 1: Bubble Size and Fundamental Value of DJIA Stock During 1997

<table>
<thead>
<tr>
<th>Stock Market</th>
<th>Time Interval</th>
<th>Market Value</th>
<th>Fundamental Value</th>
<th>Bubble Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>DJIA</td>
<td>30/08/1995-28/03/1997</td>
<td>8060.40</td>
<td>5723.17</td>
<td>2337.23%, 29%</td>
</tr>
</tbody>
</table>

The above table shows the values obtained for DJIA stock market during the interval 30/08/1995-28/03/1997. The DJIA market stopping time from the historical data is 8060.40. This meant that the rational speculative bubble start to flat from the value 8060.40. The fundamental value obtained is 5723.17; this value explains that the market value deviated from its intrinsic value about 2337 or 29%. This phenomena is called as bubble phase which caused crisis economy during year 1997. The deviation between fundamental value and market value is a size of the appeared bubble.

IV. CONCLUSION
This paper examines the possible size of existence rational speculative bubble in the DJIA stock market on the 1997. The GJLS model was successfully employed to the data to achieve our goal of study. It is essential needs for researcher to study on economic bubble. It is because the economic bubbles are one of the serious issue that give negative implications to the development of economy which is the factor leads to an economy crisis.

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VI. REFERENCES


