Black-Scholes Partial Differential Equation In The Mellin Transform Domain

Fadugba Sunday Emmanuel, Ogunrinde Roseline Bosede

Abstract: This paper presents Black-Scholes partial differential equation in the Mellin transform domain. The Mellin transform method is one of the most popular methods for solving diffusion equations in many areas of science and technology. This method is a powerful tool used in the valuation of options. We extend the Mellin transform method proposed by Panini and Srivastav [7] to derive the price of European power put options with dividend yield. We also derive the fundamental valuation formula known as the Black-Scholes model using the convolution property of the Mellin transform method. **2010 Mathematics Subject Classification**: 44A15, 60H30, 91G99

Keywords: Black-Scholes Model, Black-Scholes Partial Differential Equation, Dividend Yield, European Option, Mellin Transform Method, Option

1 INTRODUCTION

An option is an instrument whose value derives from that of another asset; hence it is called a “derivative”. In other words an option on an underlying asset is an asymmetric contract that is negotiated today with the following conditions in the future. The holder has either the right, but not the obligation to buy, as it is the case with the European call option, or the possibility to sell, as in the case of the European put option, an asset for a certain price at a prescribed date in the future. The American type of option can be exercised at any time up to and including the date of expiry. The distinctive features of American option are its early exercise privilege. The pricing of American options has been the subject of extensive research in the last decades. There is no known closed form solution and many numerical and analytic approximations have been proposed. Black and Scholes [1] published their seminal work on option pricing in which they described a mathematical frame work for finding the fair price of a European option. They used a no-arbitrage argument to describe a partial differential equation which governs the evolution of the option price with respect to the maturity time and the price of the underlying asset. The Black-Scholes model for pricing options has been applied to many different commodities and payoff structures.

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In spite of the market crash of 1987, in practice simple Black-Scholes models are widely used because they are very easy to use [4]. We present an overview of the Mellin transform method for the valuation of options in the context of Black and Scholes [1]. The Mellin transform is an integral transform named after the Finnish mathematician Hjalmar Mellin (1854-1933). The Mellin transforms in option theory were introduced by Panini and Srivastav [6]. They derived the expression for the free boundary and price of an American perpetual put as the limit of a finite-lived option. For mathematical backgrounds, the Mellin transform method in derivatives pricing and some numerical methods for options valuation see [3], [5], [8], [9], [10] just to mention a few.

2 BLACK-SCHOLES MODEL

Let us consider a market where the risk neutral asset price $S_t, 0 \le t \le T$, which is governed by the stochastic differential equation of the form

$$ds_t = (\mu - \lambda)S_t + \sigma S_t dW_t \tag{1}$$

Where λ is the dividend yield, r is the riskless interest rate, σ is called the volatility, T is the maturity date and W_t is called the Wiener process or Brownian motion.

2.1 Derivation of Black-Scholes Partial Differential Equation

Black and Scholes derived the famous Black-Scholes partial differential equation that must be satisfied by the price of any derivative dependent on a non-dividend paying stock. The Black-Scholes model can also be extended to deal with European call and put options on dividend-paying stocks. In the sequel, we derive here the Black-Scholes partial differential equation with a dividend paying stock using portfolio approach. We recall from (1) that;

$$dS_t = (\mu - \lambda)S_t dt + \sigma S_t dW_t$$

where W_t follows a Wiener process on a filtered probability space $(\Omega, B, \mu, F(B))$ in which filtration $F(B)=\{S_t : 0 \le t \le T\}$. The concept of pricing European options on dividend paying stock will now be briefly outlined below. The constant continuous dividend yield is represented by $\lambda = \lambda(S_t, t)$. In other words, it is the dividend payment per unit of time, which always represents the same fraction λ of the stock price. The holder then receives dividend payment equal to $\lambda S_t dt$ within the interval dt. As the price of the underlying asset falls by the amount of the dividend, the asset price dynamics based on the geometric Brownian motion model becomes

$$\frac{ds_t}{S_t} = (\mu - \lambda)dt + \sigma dW_t \tag{2}$$

For every asset held $\lambda S_t dt$ is received. The holder of the portfolio, who holds Δ assets, earns an amount equal to $\Delta\rho$ and dividend payment equals to $\lambda S_t \Delta dt$ in the interval dt. The change in value of the portfolio ρ is given by

$$d\rho = -df + \Delta dS_t + \lambda \Delta S_t dt \tag{3}$$

Where

$$\left.\begin{aligned} df &= \left(\frac{\partial f}{\partial t} + \mu S_t \frac{\partial f}{\partial S_t} + \frac{\sigma^2 s^2}{2}\frac{\partial^2 f}{\partial S_t^{\,2}}\right)dt + \sigma S_t \frac{\partial f}{\partial S_t} dW_t \\ \Delta &= \frac{\partial f}{\partial S_t} \end{aligned}\right\} \tag{4}$$

and $f(S_t, t) = f$ is the fair price of the underlying asset. Substituting (1) and (4) into (5), we have

$$\left.\begin{aligned} d\rho = &-\left(\frac{\partial f}{\partial t} + (\mu - \lambda) S_t \frac{\partial f}{\partial S_t} + \frac{\sigma^2 S_t^{\,2}}{2}\frac{\partial^2 f}{\partial S_t^{\,2}}\right)dt \\ &-\sigma S_t \frac{\partial f}{\partial S_t} dw_t + \frac{\partial f}{\partial S_t}\left((\mu - \lambda) S_t dt + \sigma S_t dw_t\right) \\ &+\lambda S_t \frac{\partial f}{\partial S_t} dt \end{aligned}\right\} \tag{5}$$

Therefore,

$$d\rho = -\left(\frac{\partial f}{\partial t} + \frac{\sigma^2 S_t^{\,2}}{2}\frac{\partial^2 f}{\partial S_t^{\,2}} - \lambda S_t \frac{\partial f}{\partial S_t}\right)dt \tag{6}$$

Notice that the last term in (3) $\lambda \Delta S_t dt$ denotes the wealth added to the portfolio due to the dividend yields by applying the no-arbitrage argument, the hedged portfolio should earn the risk-free interest rate, so that

$$d\rho = r\rho dt \tag{7}$$

where

$$\rho = -f + S_t \frac{\partial f}{\partial S_t} \tag{8}$$

Substitute (6) and (8) into (7) we have

$$\left.\begin{aligned} &-\left(\frac{\partial f}{\partial t} + \frac{\sigma^2 s^2}{2}\frac{\partial^2 f}{\partial s^2} - \lambda s_t \frac{\partial f}{\partial s}\right)dt \\ &= r\left(-f + S_t \frac{\partial f}{\partial S_t}\right)dt \end{aligned}\right\} \tag{9}$$

Equation (9) becomes

$$\left.\begin{aligned} &\frac{\partial f}{\partial t} + (r - \lambda) S_t \frac{\partial f}{\partial S_t} + \frac{\sigma^2 S_t^{\,2}}{2}\frac{\partial^2 f}{\partial S_t^{\,2}} \\ &= rf \end{aligned}\right\} \tag{10}$$

Equation (10) is called Black-Scholes partial differential equation on a dividend paying stock.

3 THE MELLIN TRANSFORM METHOD FOR THE VALUATION OF EUROPEAN PUT OPTION ON A DIVIDEND PAYING STOCK

Setting $f = E_p(S_t, t)$ in equation (10), we have European put option of the form

$$\left.\begin{aligned} &\frac{\partial E_p(S_t,t)}{\partial t} + (r - \lambda) S_t \frac{\partial E_p(S_t,t)}{\partial S_t} \\ &+\frac{1}{2}\sigma^2 S_t^{\,2}\frac{\partial^2 E_P(S_t,t)}{\partial S_t^{\,2}} \\ &-rE_p(S_t,t) = 0 \end{aligned}\right\} \tag{11}$$

with boundary conditions

$$\lim_{S_t \to \infty} E_p(S_t, T) = 0 \text{ on } [0, T] \tag{12}$$

$$\left.\begin{aligned} E_P(S_t, T) &= J(S) = (K - S_t)^+ \\ &= \max(K - S_t, 0) \end{aligned}\right\} \tag{13}$$

on $[0, \infty)$

$$E_p(0, t) = Ke^{-r(T-t)} \text{ on } [0, T) \tag{14}$$

The Mellin Transform for European put option, E_p is given by

$$\bar{E}_p(w, t) = \int_0^\infty E_p(S_t, t) S^{w-1} dS \tag{15}$$

where w is a complex variable with $0 < \mathrm{Re}(w) < \infty$. The inversion of the Mellin transform is also given by

$$E_p(S_t, t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \bar{E}_p(w, t) S^{-w} dw \tag{16}$$

Equation (16) holds everywhere on $(0, \infty)$, where $E_p(S_t, t)$ and $\bar{E}_p(w, t)$ are called a Mellin transform pair with the following conditions

$$E_p(S_t, t) = O(1), \text{ for } S_t \to 0^+ \tag{17}$$

$$E_p(S_t, t) = O(S_t), \text{ for } S_t \to \infty \tag{18}$$

Taking the Mellin transform of (11), we have

$$M\left(\begin{aligned} &\frac{\partial E_p(S_t, t)}{\partial t} + (r - \lambda) S_t \frac{\partial E_p(S_t, t)}{\partial S_t} \\ &+ \frac{\sigma^2 S_t^{\ 2}}{2} \frac{\partial^2 E_p(S_t, t)}{\partial S_t^{\ 2}} - rE_p(S_t, t) \end{aligned} \right) = M(0)$$

$$\left.\begin{aligned} &M\left(\frac{\partial E_p(S_t, t)}{\partial t}\right) + M\left((r - \lambda) S_t \frac{\partial E_p(S_t, t)}{\partial s}\right) \\ &+ M\left(\frac{\sigma^2 S_t^{\ 2}}{2} \frac{\partial^2 E_p(S_t, t)}{\partial s^2}\right) \\ &- M\left(rE_p(S_t, t)\right) = M(0) \end{aligned}\right\} \tag{19}$$

From the properties of the Mellin transform we have that

$$\left.\begin{aligned} &M\left(\frac{\partial E_p(S_t, t)}{\partial t}\right) = \frac{\partial \bar{E}_p(w, t)}{\partial t} \\ &M\left((r - \lambda) S_t \frac{\partial E_p(S_t, t)}{\partial S_t}\right) = -w(r - \lambda)\bar{E}_p(w, t) \\ &M\left(\frac{\sigma^2 S_t^{\ 2}}{2} \frac{\partial^2 E_p(S_t, t)}{\partial S_t^{\ 2}}\right) = w(w - 1)\frac{\sigma^2}{2}\bar{E}_p(w, t) \\ &M\left(rE_p(S_t, t)\right) = r\bar{E}_p(w, t),\ M(0) = 0 \end{aligned}\right\} \tag{20}$$

Substituting (20) into (19) yields

$$\left.\begin{aligned} &\frac{\partial \bar{E}_p(w, t)}{\partial t} + \frac{1}{2}\sigma^2\left[-\frac{2w(r - \lambda)}{\sigma^2}\bar{E}_p(w, t)\right] \\ &+ \frac{1}{2}\left[w(w + 1)\bar{E}_p(w, t) - \frac{2r}{\sigma^2}\bar{E}_p(w, t)\right] = 0 \end{aligned}\right\}$$

Setting

$$\bar{E}_p(w, t) = \bar{E}_p$$

Therefore,

$$\left.\begin{aligned} &\frac{1}{2}\sigma^2\left[(w^2 + w)\bar{E}_p - \frac{2w}{\sigma^2}(r - \lambda)\bar{E}_p - \frac{2r}{\sigma^2}\bar{E}_p\right] \\ &= -\frac{\partial \bar{E}_p}{\partial t} \end{aligned}\right\}$$

$$\left.\begin{array}{l}\frac{1}{2}\sigma^2\left[\left(w^2-w-\frac{2w}{\sigma^2}(r-\lambda)-\frac{2r}{\sigma^2}\right)\bar{E}_p\right] \\ =-\frac{\partial \bar{E}_p}{\partial t}\end{array}\right\}$$

$$\left.\begin{array}{l}\frac{1}{2}\sigma^2\left[\left(w^2+w\left(1-\frac{2(r-\lambda)}{\sigma^2}\right)-\frac{2r}{\sigma^2}\right)\bar{E}_p\right] \\ =-\frac{\partial \bar{E}_p}{\partial t}\end{array}\right\} \quad (21)$$

Setting

$$z_1=\frac{2(r-\lambda)}{\sigma^2} \quad (22)$$

and

$$z_2=\frac{2r}{\sigma^2}$$

(23)

Substituting (22) and (23) into (21), then we have

$$\left.\begin{array}{l}\frac{1}{2}\sigma^2\left[\left(w^2+w(1-z_1)-z_2\right)\bar{E}_p\right] \\ =-\frac{\partial \bar{E}_p}{\partial t}\end{array}\right\} \quad (24)$$

Setting

$$G(w)=\left(w^2+w(1-z_1)-z_2\right) \quad (25)$$

Substituting (25) into (24) yields

$$\frac{\partial \bar{E}_p}{\partial t}=-\frac{1}{2}\sigma^2 G(w)\bar{E}_p \quad (26)$$

Separating variables in (26) and upon integration, we have

$$\int_0^t\left(\frac{\partial \bar{E}_p(w,\tau)}{\bar{E}_p(w,0)}\right)=\int_0^t\left(-\frac{1}{2}\sigma^2 G(w)\right)\partial\tau$$

$$\ln\left(\frac{\bar{E}_p(w,t)}{\bar{E}_p(w,0)}\right)=\int_0^t-\frac{1}{2}\sigma^2 G(w)\partial\tau$$

Therefore,

$$\bar{E}_p(w,t)=\bar{E}_p(w,0)e^{-\frac{1}{2}\sigma^2 G(w)t} \quad (27)$$

Where $\bar{E}_p(w,0)$ is a constant that depends on the boundary conditions. Now, let us consider the terminal condition given by (13) as

$$E_P(S_t,t)=J(S_t)=(K-S_t)^+=\max(K-S_t,0)$$

So,

$$\bar{E}_p(w,o)=\bar{J}(w,t)=\bar{J}(w,t)e^{\frac{1}{2}\sigma^2 G(w)T} \quad (28)$$

Where

$$\bar{J}(w,t)=\bar{J}(w)=\left(\frac{K^{w+1}}{w(w+1)}\right) \quad (29)$$

Equation (29) is called the Mellin transform of (13), then (27) becomes

$$\bar{E}_p(w,t)=\bar{J}(w,t)e^{\frac{1}{2}\sigma^2 G(w)(T-t)} \quad (30)$$

Using the inverse of the Mellin transform, we can write for the price of a European put option as

$$E_P(S_t,t)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\bar{E}_p(w,t)S^{-w}dw \quad (31)$$

Substitute equation (30) into (31) we have

$$E_P(S_t,t)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\bar{J}(w,t)e^{\frac{1}{2}\sigma^2 G(w)(T-t)}s^{-w}dw \quad (32)$$

where $c \in (0,\infty)$ a constant, $\{w \in \mathbb{C} \mid 0 < \mathrm{Re}(w) < \infty\}$ and $(S_t, t) \in (0,\infty) \times (0,T)$

3.1 Solution of Black-Scholes Partial Differential Equation with a Dividend Paying Stock in the Mellin Transform Domain

We derive here the solution of Black-Scholes partial differential equation using Mellin transform by means of algebraic transformations

Let

$$\alpha_1 = \frac{1}{2}\sigma^2 (T-t) \tag{33}$$

We have that

$$\left.\begin{aligned} &\frac{1}{2}\sigma^2(T-t)G(w) = \frac{1}{2}\sigma^2(T-t)\left(w^2 + w(1-z_1) - z_2\right) \\ &= \frac{1}{2}\sigma^2(T-t)\left(\left(w + \frac{1-z_1}{2}\right)^2 - \left(\frac{1-z_1}{2}\right)^2 - z_2\right) \\ &= \frac{1}{2}\sigma^2(T-t)\left((w+\alpha_2)^2 - \alpha_2^2 - z_2\right) \\ &= \alpha_1\left((w+\alpha_2)^2 - \alpha_2^2 - z_2\right) \end{aligned}\right\}$$

Therefore,

$$\frac{1}{2}\sigma^2(T-t)G(w) = \alpha_1(w+\alpha_2)^2 - \alpha_1(\alpha_2^2 - z_2) \tag{34}$$

Thus, we can write for the put price by substituting (34) into (32), we have

$$E_p(S_t, t) = e^{-\alpha_1(\alpha_2^2 + z_2)} \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \bar{J}(w,t) e^{\alpha_1(w+\alpha_2)^2} s^{-w} dw \tag{35}$$

Now, $\beta^*(w)$ is the Mellin transform of $e^{\alpha_1(w+\alpha_2)^2}$

$$e^{\alpha_1(w+\alpha_2)^2} = \int_0^\infty \beta(S_t) S_t^{\,w-1} dS_t \tag{36}$$

Using the transformation in [2], we have

$$e^{Jw^2} = \int_0^\infty \frac{1}{2\sqrt{\pi J}} e^{-\left(\frac{(\ln S_t)^2}{4J}\right)} S_t^{\,w-1} dS_t, \ \mathrm{Re}(J) \geq 0 \tag{37}$$

We get

$$\beta(S_t) = \beta(S_t, t) = \frac{S_t^{\,\alpha_2}}{\sigma\sqrt{2\pi(T-t)}} e^{-\frac{1}{2}\left(\frac{\ln S_t}{\sigma\sqrt{T-t}}\right)^2} \tag{38}$$

From the convolution property of the Mellin transform, we can write that

$$E_p(S_t, t) = \int_0^\infty J(u)\beta\left(\frac{S_t}{u}\right)\frac{1}{u} du \tag{39}$$

where

$$J(u) = (K-u) \tag{40}$$

$$\beta\left(\frac{S_t}{u}\right) = \frac{\left(\frac{S_t}{u}\right)^{\alpha_2}}{\sigma\sqrt{2\pi(T-t)}} e^{-\frac{1}{2}\left(\frac{\ln\left(\frac{S_t}{u}\right)}{\sigma\sqrt{T-t}}\right)^2} \tag{41}$$

Substituting (40) and (41) into (39), then the European power put price can therefore be expressed as

$$E_p(S_t, t) = \frac{e^{-\alpha_1(\alpha_2^2 + z_2)}}{\sigma\sqrt{2\pi(T-t)}} \int_0^k (K-u)\left(\frac{S_t}{u}\right)^{\alpha_2} e^{-\frac{1}{2}\left(\frac{\ln\left(\frac{S_t}{u}\right)}{\sigma\sqrt{T-t}}\right)^2} \frac{1}{u} du$$

(42)

Equation (42) becomes

$$\left.\begin{array}{l} E_p\left(S_t,t\right)=\dfrac{e^{-\alpha_1\left(\alpha_2^2+z_2\right)}}{\sigma\sqrt{2\pi\left(T-t\right)}}\displaystyle\int_0^K K\left(\frac{S_t}{u}\right)^{\alpha_2} e^{-\frac{1}{2}\left(\frac{\ln\left(\frac{S_t}{u}\right)}{\sigma\sqrt{T-t}}\right)^2}\frac{1}{u}du \\ -\dfrac{e^{-\alpha_1\left(\alpha_2^2+z_2\right)}}{\sigma\sqrt{2\pi\left(T-t\right)}}\displaystyle\int_0^K u\left(\frac{S_t}{u}\right)^{\alpha_2} e^{-\frac{1}{2}\left(\frac{\ln\left(\frac{S_t}{u}\right)}{\sigma\sqrt{T-t}}\right)^2}\frac{1}{u}du \end{array}\right\} \quad (43)$$

where $\alpha_1=\frac{1}{2}\sigma^2\left(T-t\right),\ \alpha_2=\frac{1-z_1}{2},\quad z_1=\frac{2r}{\sigma^2}$

Equation (43) can be expressed as

$$E_p\left(S_t,t\right)=\frac{e^{-\alpha_1\left(\alpha_2^2+z_2\right)}}{\sigma\sqrt{2\pi\left(T-t\right)}}\left(KS_t^{\alpha_2}\int_0^K \frac{1}{U^{\alpha_2+1}}e^{-\frac{1}{2}\left(\frac{\ln\left(\frac{s}{u}\right)}{\sigma\sqrt{T-t}}\right)^2}du\right) \quad (44)$$

$$-\frac{e^{-\alpha_1\left(\alpha_2^2+z_2\right)}}{\sigma\sqrt{2\pi\left(T-t\right)}}\left(S_t^{\alpha_2}\int_0^K \frac{1}{u^2}e^{-\frac{1}{2}\left(\frac{\ln\left(\frac{S_t}{u}\right)}{\sigma\sqrt{T-t}}\right)^2}du\right)$$

$$E_p\left(S_t,t\right)=\frac{e^{-\alpha_1\left(\alpha_2^2+Z_2\right)}}{\sigma\sqrt{2\pi\left(T-t\right)}}\left(KS_t^{\alpha_2}\Omega_1-S_t^{\alpha_2}\Omega_2\right) \quad (45)$$

where Ω_1 and Ω_2 denote first and second integrals of (44) respectively. To evaluate Ω_1 and Ω_2, we use the following variables below

$$\rho_1=\frac{1}{\sigma\sqrt{T-t}}\left(\ln\left(\frac{S_t}{u}\right)-\alpha_2\sigma^2\left(T-t\right)\right) \quad (46)$$

$$\rho_2=\frac{1}{\sigma\sqrt{T-t}}\left(\ln\left(\frac{S_t}{u}\right)-(\alpha_2-1)\sigma^2\left(T-t\right)\right) \quad (47)$$

and

$$\Omega_1^{\bullet}=\frac{e^{-\alpha_1\left(\alpha_2^2+z_2\right)}}{\sigma\sqrt{2\pi\left(T-t\right)}}\int_0^K K\left(\frac{S_t}{u}\right)^{\alpha_2} e^{-\frac{1}{2}\left(\frac{\ln\left(\frac{S_t}{u}\right)}{\sigma\sqrt{T-t}}\right)^2}\frac{1}{u}du \quad (48)$$

$$\Omega_2^{*}=-\frac{e^{-\alpha_1\left(\alpha_2^2+z_2\right)}}{\sigma\sqrt{2\pi\left(T-t\right)}}\int_0^K u\left(\frac{S_t}{u}\right)^{\alpha_2} e^{-\frac{1}{2}\left(\frac{\ln\left(\frac{S_t}{u}\right)}{\sigma\sqrt{T-t}}\right)^2}\frac{1}{u}du \quad (49)$$

$$\Rightarrow E_p\left(S_t,t\right)=\Omega_1^*+\Omega_2^* \quad (50)$$

Evaluating Ω^*_1 and Ω^*_2, we have respectively

$$\Omega_1^*=Ke^{-r(T-t)}N\left(d_2\right) \quad (51)$$

and

$$\Omega_2^*=-e^{-\lambda(T-t)}S_tN\left(d_1\right) \quad (52)$$

Substituting equations (51) and (52) into (50), we have

$$E_p\left(S_t,t\right)=Ke^{-r(T-t)}N\left(d_2\right)-e^{-\lambda(T-t)}S_tN\left(d_1\right) \quad (53)$$

where

$$\left.\begin{array}{l} d_1=\dfrac{\ln\left(\frac{S_t}{k}\right)+\left(r-\lambda+\frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}} \\ d_2=\dfrac{\ln\left(\frac{S_t}{k}\right)+\left(r-\lambda-\frac{1}{2}\sigma^2\right)T-t}{\sigma\sqrt{T-t}}=d_1-\sigma\sqrt{T-t} \end{array}\right\} \quad (54)$$

Equation (53) is called Black-Scholes model for the solution of European put option using the Mellin transform method. The above result can be summarized in the theorem below as follows:

Theorem 1: Let S_t be the price of the underlying asset, K be the strike price, r be the risk interest rate and T be the time to maturity. Using the convolution property of the Mellin transform given by

$$E_p\left(S_t,t\right)=\int_0^\infty J\left(u\right)\beta\left(\frac{S_t}{u}\right)\frac{1}{u}du$$

We have the Black-Scholes formula for the valuation of European power put option as

$$E_p\left(S_t,t\right)=Ke^{-r(T-t)}N\left(d_2\right)-e^{-\lambda(T-t)}S_tN\left(d_1\right)$$

where the parameters d_1 and d_2 are given by

$$\left.\begin{array}{l} d_1 = \dfrac{\ln\left(\dfrac{S_t}{k}\right) + \left(r - \lambda + \dfrac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}} \\ d_2 = \dfrac{\ln\left(\dfrac{S_t}{k}\right) + \left(r - \lambda - \dfrac{1}{2}\sigma^2\right)T-t}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t} \end{array}\right\}$$

The normal distribution is given by

$$N(x) = \frac{1}{2\pi}\int_{-\infty}^{x} e^{-\frac{u^2}{2}}du$$

4 CONCLUSION

In this paper, we have considered the Black-Scholes partial differential equation in the Mellin transform domain. We have established a formula for the valuation of European power put option using the Mellin transform method on a dividend yield consisting of single integral. We also show how to derive the Black-Scholes valuation formula for European power put option by means of the convolution property of the Mellin transform method.

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