Intersection Matrices Associated With Non Trivial Suborbit Corresponding To The Action Of Rank 3 Groups On The Set Of Unordered Pairs

BettyChepkorir, John K. Rotich, Benard C. Tonui, ReubenC. Langat

Abstract: In this paper we find intersection numbers and intersection matrices associated with each non trivial sub orbit corresponding to the action of rank 3 groups; The symmetric group S_5,alternating group A_5 and The dihedral group D_5 on the set of unordered pairs. We showed that the column sum of the intersection matrices associated with Δ_i is equal the length of the suborbit Δ_i. They are also square matrices and of order 3x3.

Index Terms: Intersection Matrices,Non Trivial Suborbit, Action of Rank 3 Groups,Set of Unordered Pairs

1 INTRODUCTION

In 1964, Higman introduced the rank of a group when he worked on finite permutation groups of rank 3. He showed that if *G* is a group acting transitively on a set *X*, where $|X| = n$ and if *G* is a rank 3 group of degree $n=k^2+1$, where *k* is the length of a G_x–orbit, $x \in X$ then n = 5,10, 50 or 3250 .In 1970 , Higman calculated the rank and subdegrees of the symmetric group Sn acting on a 2 element subsets from the set X={1,2,…,n} .He showed that the rank is 3 and the subdegrees are $1, 2(n-2), \binom{n-2}{2}$. In 1972, Cameron worked on suborbits of multiply permutation groups and later in 1974, he studied suborbits of primitive groups. In 1978, he dealt with the orbits of permutation groups of unordered pairs. In 1977, Neuman extended the work of Higman and Sims to finite permutation groups, edge coloured graphs and also matrices.

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2 INTERSECTION MATRICES ASSOCIATED WITH THE ACTION OF $G = S_5$. ON $X^{(2)}$.

2.1 Intersection matrix corresponding to $\Delta_1\{1,2\}$.

Taking a ={1,2} in $X^{(2)}$ and $G_{\{1,2\}}$ - orbits arranged as follows

$$\Delta_0\{1,2\} = \{1,2\}$$

$$\Delta_1\{1,2\} = \{\{1,3\},\{1,4\},\{1,5\},\{2,3\},\{2,4\},\{2,5\}\}$$

$$\Delta_2\{1,2\} = \{\{3,4\},\{3,5\},\{4,5\}\}$$

We therefore arrange G_b - orbits as follows

$$\Delta_0\{1,3\} = \{1,3\}$$

$$\Delta_1\{1,3\} = \{\{1,2\},\{1,4\},\{1,5\},\{3,2\},\{3,4\},\{3,5\}\}$$

$$\Delta_2\{1,3\} = \{\{2,4\},\{2,5\},\{4,5\}\}$$

$$\Delta_0\{3,4\} = \{3,4\}$$

$$\Delta_1\{3,4\} = \{\{3,1\},\{3,2\},\{3,5\},\{4,1\},\{4,2\},\{4,5\}\}$$

$$\Delta_2\{3,4\} = \{\{2,1\},\{2,5\},\{1,5\}\}$$

From definition 6, the intersection numbers relative to the suborbit $\Delta_1\{1,2\}$ are defined by

$$\mu_{ij}^{(l)} = |\Delta_l(b) \text{I } \Delta_i\{1,2\}|, \qquad b \in \Delta_j\{1,2\},$$

Hence we find the intersection numbers relative to $\Delta_1\{1,2\}$ as follows

$$\mu_{00}^{(1)} = \left|\Delta_1\left(\{1,2\}\right) \cap \Delta_0\left(\{1,2\}\right)\right| = 0$$

$$\mu_{10}^{(1)} = \left|\Delta_1\left(\{1,2\}\right) \cap \Delta_1\left(\{1,2\}\right)\right| = 6$$

$$\mu_{20}^{(1)} = \left|\Delta_1\left(\{1,2\}\right) \cap \Delta_2\left(\{1,2\}\right)\right| = 0$$

$$\mu_{01}^{(1)} = \left|\Delta_1\left(\{1,3\}\right) \cap \Delta_0\left(\{1,2\}\right)\right| = 1$$

$$\mu_{11}^{(1)} = \left|\Delta_1\left(\{1,3\}\right) \cap \Delta_1\left(\{1,2\}\right)\right| = 3$$

$$\mu_{21}^{(1)} = \left|\Delta_1\left(\{1,3\}\right) \cap \Delta_2\left(\{1,2\}\right)\right| = 2$$

$$\mu_{02}^{(1)} = \left|\Delta_1\left(\{3,4\}\right) \cap \Delta_0\left(\{1,2\}\right)\right| = 0$$

$$\mu_{12}^{(1)} = \left|\Delta_1\left(\{3,4\}\right) \cap \Delta_1\left(\{1,2\}\right)\right| = 4$$

$$\mu_{22}^{(1)} = \left|\Delta_1\left(\{3,4\}\right) \cap \Delta_2\left(\{1,2\}\right)\right| = 2$$

By definition 7 the intersection matrix $M_1 = \left(\mu_{ij}^{(1)}\right)_{i,j}$, associated with $\Delta_1\{1,2\}$ where $\mu_{ij}^{(1)}$ are the intersection numbers relative to $\Delta_1\{1,2\}$ is obtained as follows;

$$M_1 = \begin{bmatrix} \mu_{00}^{(1)} & \mu_{01}^{(1)} & \mu_{02}^{(1)} \\ \mu_{10}^{(1)} & \mu_{11}^{(1)} & \mu_{12}^{(1)} \\ \mu_{20}^{(1)} & \mu_{21}^{(1)} & \mu_{22}^{(1)} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 6 & 3 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

2.2Intersection matrix corresponding to $\Delta_2\{1,2\}$

From definition 6, the intersection numbers relative to the suborbit $\Delta_2\{1,2\}$ are defined by

$$\mu_{ij}^{(2)} = \left|\Delta_2(b) \cap \Delta_i\{1,2\}\right|, \qquad b \in \Delta_j\{1,2\},$$

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$$\mu_{20}^{(2)} = \left|\Delta_2\left(\{1,2\}\right) \cap \Delta_2\left(\{1,2\}\right)\right| = 3$$

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By definition 7 the intersection matrix $M_2 = \left(\mu_{ij}^{(2)}\right)_{i,j}$, associated with $\Delta_2\{1,2\}$ where $\mu_{ij}^{(2)}$ are the intersection numbers relative to $\Delta_2\{1,2\}$ is obtained as follows;

$$M_2 = \begin{bmatrix} \mu_{00}^{(2)} & \mu_{01}^{(2)} & \mu_{02}^{(2)} \\ \mu_{10}^{(2)} & \mu_{11}^{(2)} & \mu_{12}^{(2)} \\ \mu_{20}^{(2)} & \mu_{21}^{(2)} & \mu_{22}^{(2)} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 2 \\ 3 & 1 & 0 \end{bmatrix}$$

2.3 Properties of the intersection matrices associated with $\Delta_1\{1,2\}$ and $\Delta_2\{1,2\}$

The column sum of the intersection matrix associated with Δ_i is equal to the length of the suborbit Δ_i .We can see that the column sum of M_1 is 6 also the column sum of M_2 is 3 . M_1 and M_2 are square matrices of order 3

3 INTERSECTION MATRICES ASSOCIATED WITH THE ACTION OF $G = A_5$ ON $X^{(2)}$

3.1 Intersection matrix corresponding to $\Delta_1\{1,2\}$

we take *a* ={1,2} in $X^{(2)}$ and $G_{\{1,2\}}$ - orbits arranged as follows

$$\Delta_0\{1,2\} = \{1,2\}$$

$$\Delta_1\{1,2\} = \{\{1,3\},\{1,4\},\{1,5\},\{2,3\},\{2,4\},\{2,5\}\}$$

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We therefore arrange G_b - orbits as follows

$$\Delta_0\{1,3\} = \{1,3\}$$

$$\Delta_1\{1,3\} = \{\{1,2\},\{1,4\},\{1,5\},\{3,2\},\{3,4\},\{3,5\}\}$$

$$\Delta_2\{1,3\} = \{\{2,4\},\{2,5\},\{4,5\}\}$$

$$\Delta_0\{3,4\} = \{3,4\}$$

$$\Delta_1\{3,4\} = \{\{3,1\},\{3,2\},\{3,5\},\{4,1\},\{4,2\},\{4,5\}\}$$

$$\Delta_2\{3,4\} = \{\{2,1\},\{2,5\},\{1,5\}\}$$

From definition 1.1.6.1, the intersection numbers relative to the suborbit $\Delta_1\{1,2\}$ are defined by

$$\mu_{ij}^{(l)} = \left|\Delta_l(b) \cap \Delta_i\{1,2\}\right|, \qquad b \in \Delta_j\{1,2\},$$

Hence we find the intersection numbers relative to $\Delta_1\{1,2\}$ as follows

$$\mu_{00}^{(1)} = \left|\Delta_1(\{1,2\}) \cap \Delta_0(\{1,2\})\right| = 0$$

$$\mu_{10}^{(1)} = \left|\Delta_1(\{1,2\}) \cap \Delta_1(\{1,2\})\right| = 6$$

$$\mu_{20}^{(1)} = \left|\Delta_1(\{1,2\}) \cap \Delta_2(\{1,2\})\right| = 0$$

$$\mu_{01}^{(1)} = \left|\Delta_1(\{1,3\}) \cap \Delta_0(\{1,2\})\right| = 1$$

$$\mu_{11}^{(1)} = \left|\Delta_1(\{1,3\}) \cap \Delta_1(\{1,2\})\right| = 3$$

$$\mu_{21}^{(1)} = \left|\Delta_1(\{1,3\}) \cap \Delta_2(\{1,2\})\right| = 2$$

$$\mu_{02}^{(1)} = \left|\Delta_1(\{3,4\}) \cap \Delta_0(\{1,2\})\right| = 0$$

$$\mu_{12}^{(1)} = \left|\Delta_1(\{3,4\}) \cap \Delta_1(\{1,2\})\right| = 4$$

$$\mu_{22}^{(1)} = \left|\Delta_1(\{3,4\}) \cap \Delta_2(\{1,2\})\right| = 2$$

By definition 1.1.6.2 the intersection matrix $M_1 = \left(\mu_{ij}^{(1)}\right)_{i,j}$, associated with $\Delta_1\{1,2\}$ where $\mu_{ij}^{(1)}$ are the intersection numbers relative to $\Delta_1\{1,2\}$ is obtained as follows;

$$M_1 = \begin{bmatrix} \mu_{00}^{(1)} & \mu_{01}^{(1)} & \mu_{02}^{(1)} \\ \mu_{10}^{(1)} & \mu_{11}^{(1)} & \mu_{12}^{(1)} \\ \mu_{20}^{(1)} & \mu_{21}^{(1)} & \mu_{22}^{(1)} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 6 & 3 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

3.2 Intersection matrix corresponding to $\Delta_2\{1,2\}$

From definition 1.1.6.1, the intersection numbers relative to the suborbit $\Delta_2\{1,2\}$ are defined by

$$\mu_{ij}^{(2)} = \left|\Delta_2(b) \cap \Delta_i\{1,2\}\right|, \qquad b \in \Delta_j\{1,2\},$$

We therefore find the intersection numbers relative to $\Delta_2\{1,2\}$ as follows

$$\mu_{00}^{(2)} = \left|\Delta_2(\{1,2\}) \cap \Delta_0(\{1,2\})\right| = 0$$

$$\mu_{10}^{(2)} = \left|\Delta_2(\{1,2\}) \cap \Delta_1(\{1,2\})\right| = 0$$

$$\mu_{20}^{(2)} = \left|\Delta_2(\{1,2\}) \cap \Delta_2(\{1,2\})\right| = 3$$

$$\mu_{01}^{(2)} = \left|\Delta_2(\{1,3\}) \cap \Delta_0(\{1,2\})\right| = 0$$

$$\mu_{11}^{(2)} = \left|\Delta_2(\{1,3\}) \cap \Delta_1(\{1,2\})\right| = 2$$

$$\mu_{21}^{(2)} = \left|\Delta_2(\{1,3\}) \cap \Delta_2(\{1,2\})\right| = 1$$

$$\mu_{02}^{(2)} = \left|\Delta_2\left(\{3,4\}\right) \cap \Delta_0\left(\{1,2\}\right)\right| = 1$$

$$\mu_{12}^{(2)} = \left|\Delta_2\left(\{3,4\}\right) \cap \Delta_1\left(\{1,2\}\right)\right| = 2$$

$$\mu_{22}^{(2)} = \left|\Delta_2\left(\{3,4\}\right) \cap \Delta_2\left(\{1,2\}\right)\right| = 0$$

By definition 1.1.6.2 the intersection matrix $M_2 = \left(\mu_{ij}^{(2)}\right)_{i,j}$, associated with $\Delta_2\{1,2\}$ where $\mu_{ij}^{(2)}$ are the intersection numbers relative to $\Delta_2\{1,2\}$ is obtained as follows;

$$M_2 = \begin{bmatrix} \mu_{00}^{(2)} & \mu_{01}^{(2)} & \mu_{02}^{(2)} \\ \mu_{10}^{(2)} & \mu_{11}^{(2)} & \mu_{12}^{(2)} \\ \mu_{20}^{(2)} & \mu_{21}^{(2)} & \mu_{22}^{(2)} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 2 \\ 3 & 1 & 0 \end{bmatrix}$$

3.3 Properties of the intersection matrices associated with $\Delta_1\{1,2\}$, and $\Delta_2\{1,2\}$.

The column sum of the intersection matrix associated with Δ_i is equal to the length of the suborbit Δ_i .We can see that the column sum of M_1 is 6 also the column sum of M_2 is 3 . M_1 and M_2 are square matrices of order 3

4 INTERSECTION MATRICES ASSOCIATED WITH THE ACTION OF G=D_5 ON X

By Definition 6, given an arrangement of the G_a-orbits , the G_b–orbits are arranged such that if b $\in$ X and $g(a) = b$ then,

$$g\left(\Delta_l(a)\right) = \Delta_i\left(g(b)\right) = \Delta_l(b)$$

4.1 Intersection matrix corresponding to $\Delta_1(1)$.

Taking $a = 1$ in X and G_1-orbits arranged as follows,

$$\Delta_0(1) = \{1\}.$$

$$\Delta_1(1) = \{2,5\},$$

$$\Delta_2(1) = \{3,4\},$$

We arrange the G_b- orbits as follows:

$$\Delta_0(2) = \{2\}.$$

$$\Delta_1(2) = \{1,3\},$$

$$\Delta_2(2) = \{4,5\},$$

$$\Delta_0(3) = \{3\}.$$

$$\Delta_1(3) = \{1,5\},$$

$$\Delta_2(3) = \{2,4\},$$

From definition 6, the intersection numbers relative to the suborbit $\Delta_1(1)$ are defined by

$$\mu_{ij}^{(l)} = \left|\Delta_l(b) \cap \Delta_i(1)\right|, \qquad b \in \Delta_j(1),$$

Hence we find the intersection numbers relative to $\Delta_1(1)$ as follows

$$\mu_{00}^{(1)} = \left|\Delta_1(1) \cap \Delta_0(1)\right| = 0$$

$$\mu_{10}^{(1)} = \left|\Delta_1(1) \cap \Delta_1(1)\right| = 2$$

$$\mu_{20}^{(1)} = \left|\Delta_1(1) \cap \Delta_2(1)\right| = 0$$

$$\mu_{01}^{(1)} = \left|\Delta_1(2) \cap \Delta_0(1)\right| = 1$$

$$\mu_{11}^{(1)} = \left|\Delta_1(2) \cap \Delta_1(1)\right| = 0$$

$$\mu_{21}^{(1)} = \left|\Delta_1(2) \cap \Delta_2(1)\right| = 1$$

$$\mu_{02}^{(1)} = \left|\Delta_1(3) \cap \Delta_0(1)\right| = 1$$

$$\mu_{12}^{(1)} = \left|\Delta_1(3) \cap \Delta_1(1)\right| = 1$$

$$\mu_{22}^{(1)} = \left|\Delta_1(3) \cap \Delta_2(1)\right| = 0$$

By definition 7 the intersection matrix $M_1 = \left(\mu_{ij}^{(1)}\right)_{i,j}$, associated with $\Delta_1\{1,2\}$ where $\mu_{ij}^{(1)}$ are the intersection numbers relative to $\Delta_1(1)$ is obtained as follows;

$$M_1 = \begin{bmatrix} \mu_{00}^{(1)} & \mu_{01}^{(1)} & \mu_{02}^{(1)} \\ \mu_{10}^{(1)} & \mu_{11}^{(1)} & \mu_{12}^{(1)} \\ \mu_{20}^{(1)} & \mu_{21}^{(1)} & \mu_{22}^{(1)} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

4.2 Intersection matrix corresponding to $\Delta_2(1)$

From definition 1.1.6.1, the intersection numbers relative to the suborbit $\Delta_2(1)$ are defined by

$$\mu_{ij}^{(2)} = \left|\Delta_2(b) \cap \Delta_i(1)\right|, \qquad b \in \Delta_j(1),$$

We therefore find the intersection numbers relative to $\Delta_2(1)$ as follows

$$\mu_{00}^{(2)} = \left|\Delta_2(1) \cap \Delta_0(1)\right| = 0$$

$$\mu_{10}^{(2)} = \left|\Delta_2(1) \cap \Delta_1(1)\right| = 0$$

$$\mu_{20}^{(2)} = \left|\Delta_2(1) \cap \Delta_2(1)\right| = 2$$

$$\mu_{01}^{(2)} = \left|\Delta_2(2) \cap \Delta_0(1)\right| = 0$$

$$\mu_{11}^{(2)} = \left|\Delta_2(2) \cap \Delta_1(1)\right| = 1$$

$$\mu_{21}^{(2)} = \left|\Delta_2(2) \cap \Delta_2(1)\right| = 1$$

$$\mu_{02}^{(2)} = \left|\Delta_2(3) \cap \Delta_0(1)\right| = 0$$

$$\mu_{12}^{(2)} = \left|\Delta_2(3) \cap \Delta_1(1)\right| = 1$$

$$\mu_{22}^{(2)} = \left|\Delta_2(3) \cap \Delta_2(1)\right| = 1$$

By definition 6 the intersection matrix $M_2 = \left(\mu_{ij}^{(2)}\right)_{i,j}$, associated with $\Delta_2(1)$ where $\mu_{ij}^{(2)}$ are the intersection numbers relative to $\Delta_2(1)$ is obtained as follows;

$$M_2 = \begin{bmatrix} \mu_{00}^{(2)} & \mu_{01}^{(2)} & \mu_{02}^{(2)} \\ \mu_{10}^{(2)} & \mu_{11}^{(2)} & \mu_{12}^{(2)} \\ \mu_{20}^{(2)} & \mu_{21}^{(2)} & \mu_{22}^{(2)} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

4.3 Properties of the intersection matrices associated with $\Delta_2(1)$, and $\Delta_2(1)$.

The column sum of the intersection matrix associated with Δ_i is equal to the length of the suborbit Δ_i .We can see that the column sum of M_1 is 6 also the column sum of M_2 is 3 . M_1 and M_2 are square matrices of order 3

5 Conclusion

We conclude thatIntersection matrices associated with the action of rank 3 on $X^{(2)}$ aresquare matrices of order 3x 3 and that the column sum of the intersection matrices associated with Δ_i is equal the length of the suborbit Δ_i .

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APPENDIX:

1. NOTATIONS

i). S_n, Symmetric group of degree *n* and order *n*!.

ii). $|G|$, The order of a group *G*.

iii). $X^{(2)}$, the set of unordered pairs from the set $X\{1,2,...,n\}$

iv). $\{t,q\}$, Unordered pair.

v). Δ_i, the *i*th orbit or suborbit.

vi). $\mu^{(l)}$, the intersection number relative to a suborbit $\Delta_l(a)$.

vii). M_l, intersection matrix of a suborbit $\Delta_l(a)$.

2. DEFINITION AND PRELIMINARY RESULTS

Definition 1

The alternating group A_n is the subgroup of S_n comprising of all even permutations.Its order is $\frac{n!}{2}$.

Definition 2

Let *X* be a finite set $\{1,2,\ldots n\}$, then a symmetric group of regular *n*-gon is called a dihedral group denoted by Δ_n.

Definition 3

When *G* act on a set *X*, *X* is divided into disjoined equivalence classes of the action called orbits. The orbits containing *X* is called the orbit of *x*, denoted by $\mathrm{Orb}_G(x)$.

Definition 4

Let *G* be transitive on *X* and let G_x be the stabilizer of a point $x \in X$. The orbits $\Delta_0, \Delta_1, \Delta_2, ...\Delta_{r-1}$ of G_x on *X* are the suborbits of *G*.

Definition 5

The rank *r* of *G* is the number of the suborbits of *G* while the lengths of the suborbits of *G* are known as the subdgrees of *G*.

Note: The cardinalities of the suborbits Δ_i and *r* are independent of the choice of $x \in X$.

INTERSECTION NUMBERS AND INTERSECTION MATRICES

Definition 6

Let *G* be a finite group acting on a finite set *X* and $\Delta_l(a)$ be the *l*th G_a-orbit for $a \in X$ and for a given arrangement of the G_a-orbits. The G_b-orbit, $b \in X$, are also arranged such the $g(a)=b$, then $g(\Delta_l(a)) = \Delta_l(g(a)) = \Delta_l(b)$. The intersection numbers relative to a suborbit $\Delta_l(a)$ are defined by

$$\mu_{ij}^{(l)} = |\Delta_l(b) \cap \Delta_i(a)|, \quad (b \in \Delta_j(a))$$

Definition 7

If the rank of *G* is *r*, then the $r \times r$ matrix $M_l = \left(\mu_{ij}^{(l)}\right)_{i,j}$ is called the intersection matrix of $\Delta_l\left(a\right)$.

Theorem 1 [Higman, [11]]

If $\left|\Delta_i\right| = n_i$ and $\Delta_i^* = \Delta_{i*}$ is the suborbit paired with Δ_i , then

a) $\mu_{i0}^{(l)} = \begin{cases} n_i & \text{if } i = l \\ 0 & \text{if } i \neq l \end{cases}$

b) $\mu_{i0}^{(l)} = \begin{cases} 1 & \text{if } j = l^* \\ 0 & \text{if } j \neq l^* \end{cases}$

c) $n_j \mu_{ij}^{(l)} = n_i \mu_{ji}^{\left(l^*\right)}$ and $n_i \mu_{l^* i}^{(j)} = n_j \mu_{i^* j}^{(l)} = n_l \mu_{j^* l}^{(i)}$