Optimal Hydrothermal Energy Generation for Ghana

Christian John Etwire, Stephen B. Twum

Abstract: Power production and distribution in Ghana is ever more becoming erratic and expensive, both for the power producer and the consumer. It is in this regard that an investigation of hydrothermal power generation scheduling is undertaken for a major power producer in the country. The goal of the study was to determine an optimal power production schedule that meets daily load demands at minimum cost of production and also ascertain the marginal cost of producing electricity per day and therefore tariff rate. The problem was formulated as Mixed Integer Linear Programming (MILP) and the resulting model tested using real data obtained from a major power producer in Ghana. The test results show that daily load demands could be met at a minimum cost. Furthermore, the marginal cost of producing power obtained from the dual of the MILP model provided insight into the appropriate Tariff that is reasonable for the power producer to charge consumers.

Keywords: Mixed Integer Linear Programming, Power Generation Scheduling, Marginal Cost, Unit Commitment, Economic Dispatch, Margin cost and Branch and Bound.

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1 INTRODUCTION

Electricity is a key infrastructure for economic growth of any country. It is a dynamic energy that underpins a wide range of products and services and improves the quality of life, increases productivity and encourages entrepreneurial activity. A reliable and accessible electricity system is critical in enabling Ghana to meet its long-term economic development goals. In recent times, Ghana as a nation has been experiencing frequent power outages across the length and breadth of the country. This is characterized by the inability of the country's power sector to maintain its aging equipment as a result of financial constraint due to under-pricing, mismanagement and fluctuations in crude oil prices on the international market as well as the ever increasing electricity demand by residential and industrial users [4]. This study will however, help to improve the efficiency of power generation in the country. For instance, the results of the study could be used by the major power producer in the country to minimize the production cost of electricity while meeting daily load demands and thus ensure reliable and continuous supply of power. Furthermore, the results of the study could provide a basis to charge realistic tariff. Generation scheduling is an important daily activity for electric power generation companies. Since electricity cannot economically be stored, demand and supply have to be matched at all times. The goal of generation scheduling is to determine which generators must be used in which periods in order to generate enough power to satisfy demand requirements and various technological constraints at minimum operating cost [17]. Hydro-thermal power generation scheduling is a multifaceted problem consisting of Unit Commitment and Economic Dispatch problems.

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Unit Commitment refers to the problem of deciding on the startup and shutdown of the generators while Economic Dispatch refers the problem of deciding on the loading levels of each of the committed generators to generate enough power to satisfy load demand, budgetary and operational constraints at minimum production cost [12].Many of the research works in the area of power generation scheduling using optimization techniques focus on solution methods such as Dynamic Programming, Bender's Decomposition, Lagrangian Relaxation (LR) and Integer and Mixed Integer Linear programming depending on the nature of the problem [11]. Integer and Mixed Integer Programming have been widely applied to solve different optimization problems such as the hydrothermal coordination and unit commitment problems [7]. The commonly shared characteristic of these problems is that they have either continuous, discrete variables or mixed variables. Ni and Luh [10] looked at the price based unit commitment problem in a hydrothermal context. The problem was solved using Lagrangian Relaxation (LR), by relaxing the spot and reserve markets transactions constraints in order to obtain one subproblem for each generating unit. Tseng [16] presented a unified unit decommitment (taking off units) method for solving unit commitment problems. The test results showed that the proposed method was more reliable, efficient, and robust than the LR method. Arroyo and Conejo [7] presented a detailed formulation of start-up and shut-down power trajectories of thermal units using Mixed-Integer Linear Programming. Simulation results showed that the proposed formulation was accurate and computationally efficient. Xu et al. [19] dealt with a power portfolio optimization problem that considered thermal, pumped storage and hydro units on the generation side, and forwards and options on electricity on the contracts side. The problem was solved by a Lagrangian Relaxation method, relaxing the load obligation constraints for decoupling the problem into financial market subproblems and generation unit subproblems. Nadia et al [9] looked at Optimal Unit Commitment Using Equivalent Linear Minimum Up and Down Time Constraints. The results showed that the proposed model was efficient and effective. Ana and Pedroso [1] presented a new MILP-based approach for unit commitment in power production planning. Computational analysis showed that the iterative linear method was capable of reaching the optimum of the quadratic model using much less computational time than required for its quadratic programming solution. Morales-Espana et al [5] presented a

Mixed Integer Linear Programming (MILP) formulation of start-up and shut-down power trajectories of thermal units. Multiple start-up power-trajectories and costs were modeled according to how long the unit was offline. Computational results showed that the proposed MILP model was tighter (i.e., the relaxed solution was nearer to the optimal integer solution) and compact (i.e., it used fewer constraints, variables and nonzero elements in the constraint matrix). This study, which is application oriented, seeks to model as MILP and propose an efficient power production schedule for a major power producer in Ghana. In so doing a determination of which power generators should be working in which periods of the day to meet load demands at minimum cost of production will be made. The marginal cost of producing electricity in each period of the day will also be determined to provide basis for determination of Tariff levels. In the next section an overview of MILP is presented and followed by a description and modeling of the power production problem of interest. The results are presented and discussed while the final section concludes the discussion and outlines directions for further investigation.

2 OVERVIEW OF MIXED INTEGER LINEAR PROGRAMMING

- The general problem

A MILP consists of a blend of Linear and Integer Programming in which some of the decision variables are restricted to integer and the rest to continuous values [11], or some to zeros or one values [18]. The general form LP model is stated as:

Optimize $f(x) = \sum_{i=1}^{n} c_i x_i$

Subject to:

$\sum_{i=1}^{n} a_{ij} x_j = b_i \qquad p+1 \le i \le k$

$\sum_{i=1}^{n} a_{ij} x_j \ge b_i \qquad k+1 \le i \le m$

$x_i \ge 0 x_j$integer for some $j \in (1, \ldots, n)$ (1)

Where: $f(x)$ is an objective function and x_i $(1 \le j \le n)$ is the jth decision variable. The parameters of the model are c_i, a_{ij} and b_i which respectively are the ith cost coefficient, the ith technological coefficient of the jth variable and the ith right hand side coefficient$(i = 1, \ldots m, j = 1, \ldots n)$. The variable x_i can assume any real non-negative value and can be discrete or continuous for some of them. A matrix version of (1) in standard form [11]:

$Optimize\ c^T x$

Subject to

$Ax = b$

$x \ge 0$

$x \epsilon \mathbb{Z}^p \times \mathbb{R}^{n-p}$ (2)

Where c and x are respectively n parameter and variable vectors. A is mxn matrix of the technological coefficients and b is an m parameter vector. P (P > 0) of the variable in the vector x is required to be integer.

3 THE DUAL MILP

Duality in MILP is essential in ascertaining the robustness of LP model as well as the marginal cost of production [14]. Associated with every linear program, is a corresponding dual linear program. Both programs are constructed from the same underlying cost and constraint coefficients but in such a way that if one of these problems is one of minimization the other is one of maximization. The optimal values of the corresponding objective functions are equal if finite [14]. The variables of the dual problem are the prices associated with the constraints of the original (primal) problem. The dual variables (shadow prices) constitute marginal cost in economics. Marginal cost is the change in the total cost that arises when the quantity produced change by a unit. That is, it is the change in the objective value of the optimal solution of an optimization problem obtained by relaxing the constraint by one unit [13]. In this regard the dual of the primal problem (the original problem) was constructed. The dual variables associated with the dual model constitute the marginal costs of producing electricity. Consider the primal problem in (2) above. The associated dual problem is:

$min\ b^T y$

Subject to

$A^T y \ge C$ 3)

$y \ge 0$

Where y is an $m \times 1$ vector called the dual variables (marginal costs).

4 SOLUTION METHOD

Once the MILP problem was successfully formulated, the integer restrictions on the problem were relaxed initially and an optimal solution of the resulting LP problem obtained using the Simplex Algorithm. There are metaheuristics and exact algorithms that can be used to solve integer programming problems [14]. There are a wide variety of metaheuristics and a number of properties along which to classify them [2]. Some of the metaheuristics are classified according to search strategy, Single solution approaches and population-based searches [2]. Search strategy is an improvement on simple local search algorithms; Metaheuristics of this type include simulated annealing, tabu search, iterated local search, variable neighborhood search, evolutionary computation, and genetic algorithms [15]. Single solution approaches focus on modifying and improving a single candidate solution; single solution metaheuristics include simulated annealing, iterated local search, variable neighborhood search, and guided local search [15]. Population-based approaches maintain and improve multiple candidate solutions, often using population characteristics to guide the search; population based metaheuristics include evolutionary computation, genetic algorithms, and particle swarm optimization [15]. The exact algorithms are Cutting plane, Branch and Bound and Branch and Cut methods [14]. The Cutting plane method works by solving a non-integer linear program, the linear relaxation of the given integer program. A cut is added to the relaxed linear program if the current solution is non-integer. This process is repeated until an optimal integer solution is found [3].The Branch and Bound method enumerates all candidate solutions systematically, discarding subsets of fruitless candidates by using upper and lower estimated bounds of the quantity being optimized. Branch and Cut involves running a branch and bound algorithm and using cutting planes to tighten the linear programming relaxations [6]. There is no a known algorithm that is superior when it comes to the solution of the integer programming problem. They are selected based on the nature of the problem and the software the researcher wishes to use. Therefore the Branch-and-Bound method due to Land and Doig [14] which is implemented in LP Solve IDE V5.5.2.0 by Henri Gourvest was used to obtain the required integer solutions.

5 THE POWER GENERATION PROBLEM

The power generation firm operates eight power plants comprising two Hydroelectric (H_i, i=1, 2) and six Thermal (T_i, i=1,…, 6). These plants are committed to meeting the daily electricity load demands at some daily operational cost. The eight power plants together have twenty-four generators, ten (10) of which are for hydroelectric power generation and 14 for thermal. Each generator has to work between a minimum and a maximum level. There is an hourly cost of running each generator at its minimum level. In addition there is an extra hourly cost for each megawatt (MW) of power generated above the minimum level. Startup of a generator also involved cost. In addition to meeting the estimated daily electricity load demands, there must be sufficient generators working at any time of the day to make it possible to meet an increase in load. This increase would have to be met by the generators already operating within their permitted limits. There must be enough reserve (spinning reserve) to cater for unexpected increase in load demands or breakdown of any generator. The desire of the firm is to meet the daily load demands of consumers at minimum cost of operation of the power plants. The details of load demands and the key parameters of the problem described are presented in Tables 1 and 2 (with costs in Ghana Cedis (Gh¢)). Table 1 shows the periods (in hourly interval) and their corresponding load demands (LD) and spinning reserve (SR). Table 2 also shows the plants (P), the distribution of generators (DG), the maximum (max) and minimum (min) power generating levels, the cost per minimum level of operation of the generators (CM), the cost per hour of generating above minimum the level (CA) and the startup costs (SC). The interval [1, 2) am is the period starting from 1 am and ending before 2 am.

Table 1: Daily electricity load demands

Period (am, pm)	LD (MW)	SR (MW)
[1, 2)	1519	94
[2, 3)	1409	123
[3, 4)	1473	140
[4, 5)	1480	133
[5, 6)	1505	108
[6, 7)	1584	69
[7, 8)	1505	148
[8, 9)	1552	137
[9, 10)	1589	100
[10, 11)	1616	73
[11, 12)	1622	67
[12, 1)	1639	50
[1, 2)	1617	55
[2, 3)	1634	55
[3, 4)	1644	159
[4,5)	1659	144
[5, 6)	1653	150
[6, 7)	1637	166
[7, 8)	1775	64
[8, 9)	1785	54
[9, 10)	1777	62
[10, 11)	1738	101
[11, 12)	1703	136
[12, 1)	1602	87

Table 2 key parameters of the problem

P	DG	Max Level (MW)	Min Level (MW)	CM GH¢	CA GH¢	SC GH¢
H1	6	150	125	4360.29	34.05	5743.92
H2	4	40	30	2399.24	69.80	2769.58
T1	3	110	100	23145.46	229.24	25238.99
T2	2	110	100	34582.95	345.62	38576.57
T3	1	126	110	24234.50	220.22	28737.02
T4	6	36	30	10047.74	302.59	11209.11
T5	1	49.5	45	9469.86	212.30	11614.91
T6	1	126	110	62508.10	564.31	72367.49

6 FORMULATION OF THE MODEL

The modeling process required identification of the decision variables and parameters of the problem and the relevant constraints. These were subsequently posed symbolically and the necessary relations derived to obtain the objective function and the constraint equations and inequalities.

- *Decision variables*

The following decision variables are associated with the problem;

x_{ij} : Power output in MW from each generator of plant j, $(j = 1, \dots, 8)$ in period $i\ (i = 1, \dots, 24)$

y_{ij} : Excess power output in MW from each generator of plant j, $j = 1, \dots, 8$ in period $i, (i = 1, \dots, 24)$ above minimum level.

n_{ij} : Number of generators of plant j working in period i.

z_{ij} : Number of generators of plant j to start in period *i*.

- *Parameters*

The following parameters are associated with the model;

D_i : Demand of power in MW in period *i*

R_i : Reserve margin of power in MW in period *i*

M_j : Maximum level of power output of each generator of plant *j*

m_j : Minimum level of power output of each generator of plant *j*

C_j : Cost (in GH ¢) of running each generator of plant *j* at the minimum level.

C_j^I : Extra hourly cost (in GH ¢) of running each generator of plant *j* above minimum level.

C_j^{II}: Cost of starting up each generator of plant *j*.

L_i : Length of each period *i*.

- *Objective function*

The operating cost of power generation depends on one hand the cost of running a generator at the minimum level, the extra hourly cost of running a generator above the minimum level and the start up cost (which are fixed parameters) and on the other hand on the number of generators running in a given period, their power output (which are variables) and the length of the period. These considerations constitute the objective function:

$$Cost = \sum_{i=1}^{24}\sum_{j=1}^{8}\left(C_j L_i n_{ij} + C_j^I L_i y_{ij} + C_j^{II} z_{ij}\right) \quad (4)$$

- *Constraints of the problem*

The constraints associated with the power scheduling problem are: generation, demand, reserve margin and start-up. *Generation constraint:* the power output of each generator must lie within the minimum and maximum production levels. This is given by:

$$m_j n_{ij} \leq x_{ij} \leq M_j n_{ij} \quad (5)$$

The power output in MW from each generator of plant *j* in period *i* is equal to the sum of the minimum power output and the excess power output of that generator. This is given by:

$$x_{ij} = m_j n_{ij} + y_{ij} \quad (6)$$

The excess power output from each generator of plant *j* in period *i* should at most be equal to the difference between the maximum and minimum power output of that generator in that period. This is given by:

$$y_{ij} \leq (M_j - m_j) n_{ij} \quad i = 1, \dots, 24 \quad (7)$$

Demand constraint: the sum of power to be generated by each generator of plant *j* in period *i* should at least meet the demand for that period. This is given by:

$$\sum_{j=1}^{8} x_{ij} \geq D_i$$

Which is equivalent to:

$$\sum_{j=1}^{8}\left(m_j n_{ij} + y_{ij}\right) \geq D_i \quad (8)$$

Reserve margin constraint: the maximum power output of the generators in period i should at least be equal to the sum of the demand and reserve load requirement in that period. This is given by:

$$\sum_{j=1}^{8} M_j n_{ij} \geq (D_i + R_i) \quad (9)$$

Start -up generator constraint: the number of generators of plant *j* to start up in period *i* is at least equal to the difference between the number of generators of plant j to commit and the number of generators of plant j already working. This is given by:

$$z_{ij} \geq n_{ij} - n_{(i-1)j} \quad (10)$$

Non-negativity constraint: it is required that

$$n_{ij}, x_{ij}, y_{ij}, z_{ij} \geq 0 \quad (11)$$

The resulting MILP is to minimize Z as given in (4) subject to the constraints (5) to (12)

7 RESULTS AND DISCUSSION

7.1 Results

The output of the optimization algorithm (using LPsolve version: 5.5.2) are presented in Tables 3(a) to (d) below. In each Table, the first column indicates the production periods; the second the power plants (PP) to commit to power generation; the third the number of generators of a power plant to be working (GW) in any period is recorded in the third column: the fourth the number of generators to start up (GS) in any period (Zero entry in the fourth column means no new generator should be added to those already working whiles nonzero entry indicates the number of generators that should be added to those already working): the fifth the total power outputs (TPO) from the committed generators and the fifth the load demands (LD) in any period.

Table 3(a) Operating Generators and output levels 1

Period (am)	PP	GW	GS	TPO (MW)	LD (MW)
[1, 2)	H1	6	0	889	1519
	H2	4	0	120	
	T1	3	0	300	
	T2	1	0	100	
	T3	1	0	110	
	T4	0	0	0	
	T5	0	0	0	
	T6	0	0	0	
[2, 3)	H1	6	0	860	1409
	H2	4	0	120	
	T1	3	0	300	
	T2	1	0	100	
	T3	1	0	110	
	T4	0	0	0	
	T5	0	0	0	
	T6	0	0	0	
[3, 4)	H1	6	0	843	1473
	H2	4	0	120	
	T1	3	0	300	
	T2	1	0	100	
	T3	1	0	110	
	T4	0	0	0	
	T5	0	0	0	
	T6	0	0	0	
[4, 5)	H1	6	0	850	1480
	H2	4	0	120	
	T1	3	0	300	
	T2	1	0	100	
	T3	1	0	110	
	T4	0	0	0	
	T5	0	0	0	
	T6	0	0	0	
[5, 6)	H1	6	0	875	1505
	H2	4	0	120	
	T1	3	0	300	
	T2	1	0	100	
	T3	1	0	110	
	T4	0	0	0	
	T5	0	0	0	
	T6	0	0	0	
[6, 7)	H1	6	0	900	1584
	H2	4	0	129	
	T1	3	0	300	
	T2	1	0	100	
	T3	1	0	110	
	T4	0	0	0	
	T5	1	1	45	
	T6	0	0	0	

From Table 3 (a), at period [1, 2) am; six, four and three generators from H1, H2 and T1 respectively and a generator each from T2 and T3 should be committed to power generation. Their respective outputs; 889 MW, 120 MW, 300 MW, 100 MW and 110 MW satisfy exactly the load demand at that period. Six generators from T4 and a generator each from T2, T5 and T6 should be on standby for emergency use. Zero entries in column four mean no new generator from those plants should be added to those already working. At period [2, 3) am; six, four and three generators from H1, H2 and T1 respectively and a generator each from T2 and T3 should be committed to power generation. Their respective outputs; 860 MW, 120 MW, 300 MW, 100 MW and 110 MW satisfy exactly the load demand at that period. Six generators from T4 and a generator each from T2, T5 and T6 should be on standby for emergency use. At period [3, 3) am; six, four and three generators from H1, H2 and T1 respectively and a generator each from T2 and T3 should be committed to power generation. Their respective outputs; 843 MW, 120 MW, 300 MW, 100 MW and 110 MW satisfy exactly the load demand at that period. Six generators from T4 and a generator each from T2, T5 and T6 should be on standby for emergency use. At period [4, 5) am; six, four and three generators from H1, H2 and T1 respectively and a generator each from T2 and T3 should be committed to power generation. Their respective outputs; 850 MW, 120 MW, 300 MW, 100 MW and 110 MW satisfy exactly the load demand at that period. Six generators from T4 and a generator each from T2, T5 and T6 should be on standby for emergency use. At period [5, 6) am; six, four and three generators from H1, H2 and T1 respectively and a generator each from T2 and T3 should be committed to power generation. Their respective outputs; 875 MW, 120 MW, 300 MW, 100 MW and 110 MW satisfy exactly the load demand at that period. Six generators from T4 and a generator each from T2, T5 and T6 should be on standby for emergency use. At period [6, 7) am; six, four and three generators from A1, A2 and T1 respectively and a generator each from T2, T3 and T5 should be committed to power generation. Their respective outputs; 843 MW, 120 MW, 300 MW, 100 MW, 110 MW and 45 MW satisfy exactly the load demand at that period. Six generators from T4 and a generator each from T2 and T6 should be on standby for emergency use. One recorded in column four against T5 indicates that a new generator from that plant has to be added to those already working. Similar interpretations follow for the outputs displayed in Tables 3 (b) to (d) below.

Table 3(b) Operating Generators and output levels 2

Period (am)	PP	GW	GS	TPO (MW)	LD (MW)
[7, 8)	H1	6	0	830	1505
	H2	4	0	120	
	T1	3	0	300	
	T2	1	0	100	
	T3	1	0	110	
	T4	0	0	0	
	T5	1	0	45	
	T6	0	0	0	
[8, 9)	H1	6	0	847	1552
	H2	4	0	120	
	T1	3	0	300	
	T2	1	0	100	
	T3	1	0	110	
	T4	1	1	30	
	T5	1	0	45	
	T6	0	0	0	
[9, 10)	H1	6	0	884	1589
	H1	4	0	120	
	T1	3	0	300	
	T2	1	0	100	
	T3	1	0	110	
	T4	1	0	30	
	T5	1	0	45	
	T6	0	0	0	
[10, 11)	H1	6	0	900	1616
	H2	4	0	131	
	T1	3	0	300	
	T2	1	0	100	
	T3	1	0	110	
	T4	1	0	30	
	T5	1	0	45	
	T6	0	0	0	
[11, 12)	H1	6	0	900	1622
	H2	4	0	137	
	T1	3	0	300	
	T2	1	0	100	
	T3	1	0	110	
	T4	1	0	30	
	T5	1	0	45	
	T6	0	0	0	
[12, 1)	H1	6	0	900	1639
	H2	4	0	154	
	T1	3	0	300	
	T2	1	0	100	
	T3	1	0	110	
	T4	1	0	30	
	T5	1	0	45	
	T6	0	0	0	

Table 3(c) Operating Generators and output levels 3

Period (pm)	PP	GW	GS	TPO (MW)	LD (MW)
[1, 2)	H1	6	0	900	1617
	H2	4	0	132	
	T1	3	0	300	
	T2	1	0	100	
	T3	1	0	110	
	T4	1	0	30	
	T5	1	0	45	
	T6	0	0	0	
[2, 3)	H1	6	0	900	1634
	H2	4	0	149	
	T1	3	0	300	
	T2	1	0	100	
	T3	1	0	110	
	T4	1	0	30	
	T5	1	0	45	
	T6	0	0	0	
[3, 4)	H1	6	0	849	1644
	H2	4	0	120	
	T1	3	0	300	
	T2	1	0	100	
	T3	1	0	110	
	T4	4	3	120	
	T5	1	0	45	
	T6	0	0	0	
[4, 5)	H1	6	0	864	1659
	H2	4	0	120	
	T1	3	0	300	
	T2	1	0	100	
	T3	1	0	110	
	T4	4	0	120	
	T5	1	0	45	
	T6	0	0	0	
[5, 6)	H1	6	0	858	1653
	H2	4	0	120	
	T1	3	0	300	
	T2	1	0	100	
	T3	1	0	110	
	T4	4	0	120	
	T5	1	0	45	
	T6	0	0	0	
[6, 7)	H1	6	0	842	1637
	H2	4	0	120	
	T1	3	0	300	
	T2	1	0	100	
	T3	1	0	110	
	T4	4	0	120	
	T5	1	0	45	
	T6	0	0	0	

Table 3(d) Operating Generators and output levels 4

Period (pm)	PP	GW	GS	TPO (MW)	LD (MW)
[7, 8)	H1	6	0	900	1775
	H2	4	0	160	
	T1	3	0	300	
	T2	1	0	100	
	T3	1	0	115.5	
	T4	5	1	150	
	T5	1	0	49.5	
	T6	0	0	0	
[8, 9)	H1	6	0	900	1785
	H2	4	0	160	
	T1	3	0	300	
	T2	1	0	100	
	T3	1	0	125.5	
	T4	5	0	150	
	T5	1	0	49.5	
	T6	0	0	0	
[9, 10)	H1	6	0	900	1777
	H2	4	0	160	
	T1	3	0	300	
	T2	1	0	100	
	T3	1	0	117.5	
	T4	5	0	150	
	T5	1	0	49.5	
	T6	0	0	0	
[10, 11)	H1	6	0	900	1738
	H2	4	0	133	
	T1	3	0	300	
	T2	1	0	100	
	T3	1	0	110	
	T4	5	0	150	
	T5	1	0	45	
	T6	0	0	0	
[11, 12)	H1	6	0	878	1703
	H2	4	0	120	
	T1	3	0	300	
	T2	1	0	100	
	T3	1	0	110	
	T4	5	0	150	
	T5	1	0	45	
	T6	0	0	0	
[12, 1)	H1	6	0	897	1602
	H2	4	0	120	
	T1	3	0	300	
	T2	1	0	100	
	T3	1	0	110	
	T4	1	0	30	
	T5	1	0	45	
	T6	0	0	0	

7.2 Discussion

It is evident from the results that from period [1, 2) am, to meet the load demands at the subsequent periods, the outputs of the generators already working should be adjusted (upward or downward) or more generators should be committed to or decommited from the production line. For instance, at periods [2, 3) am and [3, 4) am, the outputs of H1 should be reduced by 29 MW and 17 MW respectively to be commensurate with the reduction in their load demands (see Table 3 (a)). At periods [4, 5) am and [5, 6) am, the outputs of H1 should be increased by 7 MW and 25 MW respectively to match the increase in load demands during those periods. At period [6, 7) am, a generator from T5 with an output of 45 MW should be committed to the production line and the outputs of H1 and H2 should be increased by 25 MW and 9 MW respectively resulting in an increment of 79 MW over the preceding period to match the increase in load demand. At period [7, 8) am, the outputs of H1and H2 should be reduced by 70 MW and 9 MW respectively to reflect the reduction in load demand (see Table 3 (b)). At period [8, 9) am, a generator from T4 with an output of 30 MW should be committed to the production line and the output of H1 should be increased by 17 MW resulting in an increase of 47 MW over the previous period to match the increase in load demand. At period [9, 10) am, the output of H1 should be increased by 37 MW to be commensurate with the increase in load demand. At period [10, 11) am, the outputs of H1 and H2 should be increased by 16 MW and 11 MW respectively to reflect the increase in load demand. At periods [11am, 12 pm) and [12, 1) pm, the outputs of H2 should be increased by 6 MW and 17 MW respectively to be commensurate with the increase in their load demands. At period [1, 2) pm, the output of H2 should be reduced by 22MW to be commensurate with the reduction in load demand (see Table 3 (c)). At period [2, 3) pm, the output of H2 should be increased by 17 MW to reflect the increase in load demand. At period [3, 4) pm, the outputs of H1 and H2 should be reduced by 51 MW and 17 MW respectively and three generators from T4 with total output of 90 MW should be committed to the production line, resulting in an increase of 22 MW over the preceding period to be commensurate with the increase in load demand. At period [4, 5) pm, the output of H1 should be increased by 15 MW to be commensurate with the increase in load demand. At periods [5, 6) pm and [6, 7) pm, the outputs of H1 should be reduced by 6 MW and 16 MW respectively to reflect the reduction in their load demands. At periods [7, 8) pm, the outputs of H1, H2, T3 and T5 should be increased by 58 MW, 40 MW, 5.5 MW and 4.5MW respectively to be commensurate with the increase in load demand (see Table 3 (d)). At period [8, 9) pm, the output of T3 should be increased by 10MW to be commensurate with the increase in load demand. At period [9, 10) pm, the output of T3 should be reduced by 8 MW to match the reduction in load demand. At period [10, 11) pm, the outputs of H2, T3 and T5 should be reduced by 27 MW, 7.5 MW and 4.5 MW respectively to be commensurate with the reduction in load demand. At period [11 pm, 12 am), the outputs of H1 and T1 should be reduced by 22 MW and 13 MW respectively to reflect the reduction in load demand. At period [12, 1) am, the output of H1 should be increased by 19 MW and four generators from T5 with total output of 120 MW should be decommited from the production line, resulting in a reduction of 101 MW from the previous period to be commensurate with the reduction in load demand. It is observed that the electricity load demands at all the periods were satisfied exactly. No generator from T6 has to be operating in any of the production periods. This means that the T6 plant needs not be committed under normal operation except under emergency. From periods: [1, 2) am to [5, 6) am, five power plants namely H1, H2, T1, T2 and T3 should be used in power generation. At periods [6, 7) am and [7, 8) am, six power plants namely H1, H2, T1, T2, T3 and T5 should be used in power generation. From periods [8, 9) am to [12, 1) am, seven power plants namely H1, H2, T1, T2, T3, T4 and T5 should be used in power generation. The variation in the number of power plants used in power generation is as a result of the variation in the load demands. Thus, the higher the load demand, the higher the number of power plants used in power generation. The hydroelectric and thermal power plants, H1 and T1 are required to contribute substantial amounts of power to the power produced in every period of the day and thus are critical to the power generation operation. Their absence in the production cycle will create a serious power deficit in the country. The two plants must therefore be diligently managed and maintained to ensure continuous power supply. All the generators from the plants H1, H2, T1, T3 and T5 should be used in power generation in all the production periods. This is due to the low extra hourly cost for each megawatt of power generated above the minimum level. However, not all the generators from the plants T2 and T4 should be used in power generation as a result of the high extra hourly cost for each megawatt of power generated above the minimum level. One out of the two generators from plant T2 should be used in power generation in all the periods. One out of the six generators from T4 should be used in power generation at periods [12 am, 3 pm). Four and five generators respectively from T4 should be used in power generation at periods [3, 7) pm and [7 pm, 12 am) respectively. The generators from the plants T1, T2 and T4 used in power generation should be operating at the minimum level in all the production periods. Also the generators from the plant H1 used in power generation should be operating a little above the minimum level at periods [1, 6) am, [7, 10) am, [3, 7) pm and [11 pm, 1 am) and at the maximum level for the rest of the periods. Furthermore, the generators from the plant H2 used in power generation should be operating at the maximum level at periods [7, 10) pm, a little above the minimum at periods [6, 7) am, [10 am, 3 pm) and [10, 11) pm and at minimum level for the rest of the periods. Moreover, the generators from the plant T5 used in power production should be operating at the maximum level at periods [7, 10) pm and at minimum level for the rest of the periods. Finally, the generators from the plant T3 used in power production should be operating a little above the minimum level at periods [7, 10) pm and at minimum level for the rest of the periods. The variations in the outputs of the generators across the periods are as a result of the fluctuations in the daily load demands. The optimal cost from run of the optimization algorithm using the original data was Gh¢4,806,855.99. This indicates the minimum cost to the firm to invest in meeting the daily electricity load demand. However, the firm currently spends $2 million (Gh¢7,105,000) daily to generate power to meet the national demand (Fletcher, 2012). If the firm should adopt this model, they will be able to save Gh¢2,298,144.01 from their daily production.

8 MARGINAL COST OF PRODUCING ELECTRICITY

The marginal costs (MC) associated with each of the production periods and the ranges of the load demands for which they are valid are presented in Table 5 below. In the Table, the first column indicates the production periods. The marginal costs associated with each of the production periods are recorded in the second column. The third and fourth columns record the minimum and maximum ranges of the load demands for which the marginal costs are valid.

Table 5 Marginal cost for producing electricity using the original data

Period (am, pm)	MC (Gh¢)	Min (MW)	LD (MW)	Max (MW)
[1, 2)	34.05	1380.00	1519	1530.00
[2, 3)	34.05	1380.00	1409	1530.00
[3, 4)	34.05	1380.00	1473	1530.00
[4, 5)	34.05	1380.00	1480	1530.00
[5, 6)	34.05	1380.00	1505	1530.00
[6, 7)	69.80	1575.00	1584	1615.00
[7, 8)	34.05	1425.00	1505	1575.00
[8, 9)	34.05	1455.00	1552	1605.00
[9, 10)	34.05	1455.00	1589	1605.00
[10, 11)	69.80	1605.00	1616	1645.00
[11, 12)	69.80	1605.00	1622	1645.00
[12, 1)	69.80	1605.00	1639	1645.00
[1, 2)	69.80	1605.00	1617	1645.00
[2, 3)	69.80	1605.00	1634	1645.00
[3, 4)	34.05	1545.00	1644	1695.00
[4, 5)	34.05	1545.00	1659	1695.00
[5, 6)	34.05	1545.00	1653	1695.00
[6, 7)	34.05	1545.00	1637	1695.00
[7, 8)	220.22	1769.50	1775	1785.50
[8, 9)	220.22	1769.50	1785	1785.50
[9, 10)	220.22	1769.50	1777	1785.50
[10, 11)	69.80	1725.00	1738	1765.00
[11, 12)	34.05	1575.00	1703	1725.00
[12, 1)	34.05	1455.00	1602	1605.00

The marginal cost of producing electricity for the periods [1, 10) am, [3, 7) pm and [11 pm, 1 am) is Gh¢34.046. This marginal cost is the same as that of firm's tariff for H1. The marginal costs for the remaining periods are as displayed in Table 4. The marginal costs of Gh¢69.80 and Gh¢220.22 correspond to that of H2 and T3 respectively. The range of feasibility for periods [1, 6) am is (1380 MW, 1530 MW). This means that the firm can increase or decrease the load demand within the specified range and the marginal cost will remain Gh¢34.046. If the change in load demand is outside the range of feasibility, then the model has to be re-formulated and re-run to obtain new marginal cost. A unit increase in load demand increases the total production cost by Gh¢34.046 while a decrease in load demand decreases the total production cost by Gh¢34.046. Similar interpretations follow for the rest of the periods. The marginal cost of producing electricity increases as load demand increases and vice-versa. This accounts for the fluctuations in the marginal costs across the period. The average marginal cost for producing electricity in a day is Gh¢67.75. This marginal cost indicates the appropriate tariff that is reasonable for the firm to charge consumers for a megawatt of power.

9 CONCLUSIONS

The power generation scheduling problem of a firm in Ghana has been formulated as Mixed Integer Linear Programming and the resulting model tested using real data obtained from a major power producer in Ghana. The test results showed that daily load demands could be met at a minimum cost. Furthermore, the marginal cost of production of power obtained from the dual of the MILP model provided insight into the appropriate Tariff that is reasonable for the power producer to charge consumers. The daily electricity load demands were satisfied exactly at all times. No generator from T6 should be operating in any of the production periods except under emergency. The hydroelectric and thermal power plants, H1 and T1 are required to contribute substantial amounts of power to the power produced in every period of the day and thus are critical to the power generation operation All the generators from the plants H1, H2, T1, T3 and T5 should be used in power generation in all the production periods while not all the generators from the plants T2 and T4 should be used in power generation. The generators from the plants H1, H2 and T5 used in power generation should be operating at the maximum level for just few of the periods and either at minimum level or a little above the minimum for the rest of the periods. The generators from the plant T3 used in power generation should either be operating at the minimum level or a little above the minimum level. The generators from the plants T1, T2 and T4 used in power generation should be operating at the minimum level in all the production periods. The optimal cost the model provided was Gh¢4,806,855.99 which is far lower than the $2 million (Gh¢7,105,000) the firm currently invests daily to generate power to meet the load demands. The firm could therefore save Gh¢2,298,144.01 from their daily production, if they should adopt the proposed MILP model.

RECOMMENDATIONS

This is just the preliminary results of the model. A subsequent paper will consider post optimal analysis of the model, since the parameters of the model are usually approximations of their exact value. The analysis of their sensitivity to slight variations in their values is crucial towards finding an implementable solution.

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