Radiative Mhd Stagnation Point Flow Over A Chemical Reacting Porous Stretching Surface With Convective Thermal Boundary Condition

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Abstract: This paper investigates the effects of radiation and chemical reaction on magneto hydrodynamic stagnation point flow over a porous stretching surface with convective thermal boundary condition. The nonlinear and coupled governing differential equations were solved numerically using the fourth order Runge-Kutta shooting method. Numerical results for the skin friction coefficient, the rate of heat transfer represented by the local Nusselt number and the rate of mass transfer by the local Sherwood number were presented whilst the velocity, temperature and concentration profiles illustrated graphically and analyzed. The effects of the velocity ratio parameter, magnetic field parameter, suction parameter, Prandtl number, radiation parameter, Schmidt number, reaction rate parameter and Biot number on the flow field were discussed.

Keywords: Schmidt number, radiation, Magnetohydrodynamic, convective, Lorentz force, concentration and reaction.

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1INTRODUCTION

MHD flows through a porous stretching surface are of great significance to engineers and scientists in the field of meteorology, cosmic fluid dynamics, astrophysics and geophysics. In addition to this, the influence of radiation and convective heat exchange on chemically reacting fluids, arise in many heat and mas transfer processes with applications in many branches of science and engineering. Such applications include cooling of solar collectors, recovery of petroleum resources, building thermal insulation, design of geothermal systems, heat exchanger design, hot metal rolling, wire drawing, manufacturing of ceramics or glassware and polymer extrusion processes. Due to these numerous applications, some investigations have been conducted to study the effects of radiation and chemical reaction on magneto hydrodynamics convective flow towards a stretching porous surface. Pioneering work on two-dimensional stagnation point flow problem was first studied by Hiemenz [1] who used the similarity transformations approach to reduce the Navier–Stokes equations to non-linear ordinary differential equations. Makinde and Charles [2] conducted a computational dynamics on the hydrodynamic stagnation point flow towards a stretching sheet and concluded that the cooling rate of a stretching sheet in an electrically conducting fluid, subject to a magnetic field could be controlled and a final product with desired characteristics could be achieved. Ishak et al. [3] also studied numerically a steady two-dimensional MHD stagnation point flow towards a stretching sheet with variable surface temperature. They found that the heat transfer rate at the surface increased with the magnetic parameter when the free stream velocity exceeded the stretching velocity. Arthur and Seini [4] analyzed MHD thermal stagnation point flow towards a stretching porous surface. The effects of chemical reaction on free convection MHD flow through porous medium bounded by vertical surface with slip flow region was analyzed by Senapati *et al* [5]. Alireza et al. [6] presented an analytical solution for MHD stagnation point flow and heat transfer over a permeable stretching sheet with chemical reaction. Seini [7] investigated the flow over an unsteady stretching surface with chemical reaction and non-uniform heat source. Srinivas and Muthuraj [8] considered Effects of thermal radiation and space porosity on MHD mixed convection flow in a vertical channel using homotopy analysis method. Hossain and Takhar [9] studied the effect of radiation using the Rosseland diffusion approximation, which led to non-similar solutions for the forced and free convection flow of an optically dense fluid from vertical surfaces with constant free stream velocity and surface temperature. Vyas and Ranjan [10] discussed the dissipative MHD boundary- layer flow in a porous medium over a sheet stretching nonlinearly in the presence of radiation. Pop *et al*., [11] analyzed the radiative effects on the steady two-dimensional stagnation-point flow of an incompressible fluid over a stretching sheet. The effects of thermal radiation on MHD stagnation point flow past a stretching sheet with heat generation was studied by Zhu et al., [12]. Arthur and Seini [13] studied hydromagnetic stagnation point flow over a porous stretching surface in the presence of radiation and viscous dissipation. Kesavaiah *et al* [14] investigated the effects of radiation absorption, chemical reaction and magnetic field on the free convection and mass transfer flow through porous medium with constant suction and constant heat flux. Sahin and Chamkha [15] analyzed the effects of radiation and chemical reaction on steady mixed convective heat and mass transfer flow of an optically thin gray gas over an infinite vertical porous plate with constant suction in presence of transverse magnetic field, and they found that the velocity is reduced considerably with a rise in the conduction-radiation parameter. Seini and Makinde [16] studied the MHD boundary layer flow due to exponential stretching surface with radiation and chemical reaction. Ahmed and Kalita [17] analytical and numerical study for MHD radiating flow over an infinite vertical surface bounded by a porous medium in presence of chemical reaction. Recently, Etwire *et al.* [18] investigated the MHD boundary layer stagnation point flow with radiation and chemical reaction towards a heated shrinking porous surface. This paper investigates the effects of radiation and chemical reaction on magneto hydrodynamic stagnation point flow over a porous stretching surface with convective thermal boundary condition. Section 2 presents the mathematical model of the

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problem. The numerical procedure is outlined in section 3 whilst results and discussions are presented in section 4. Section 5 presents some useful conclusions.

2 Problem formulation

Consider a steady laminar two-dimensional flow of an incompressible and electrically conducting fluid towards the stagnation point on a porous stretching surface in the presence of radiation and magnetic field of strength, B_o, applied in the positive y direction as shown in Figure 1. The tangential velocity U_w and the free stream velocity $U\infty$ are assumed to vary proportional to the distance x, from the stagnation point so that $U_w(x)=bx$ and $U_\infty(x)=ax$, where a and b are constants. The induced magnetic field due to the motion of the electrically conducting fluid and the pressure gradient are neglected. The wall temperature is maintained at the prescribed constant value T_w.

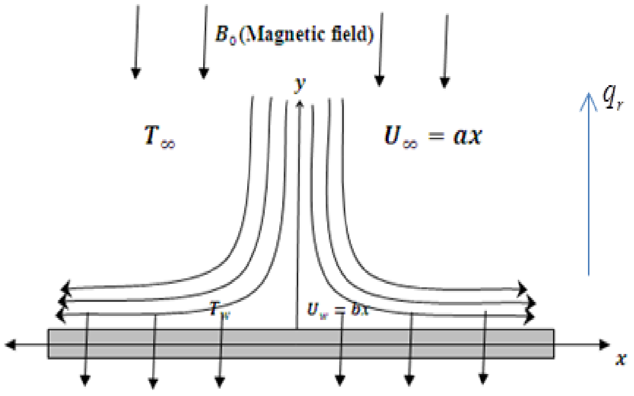

Figure 1: Schematic Diagram of the Problem

The boundary layer equations for a steady incompressible viscous hydrodynamic fluid are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_\circ^2}{\rho}u+a^2x \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2-\frac{\alpha}{\kappa}\frac{\partial q_r}{\partial y}+\frac{\sigma B_\circ^2}{\rho c_p}u^2 \tag{3}$$

$$u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=D_m\frac{\partial^2 c}{\partial y^2}-\gamma(C-C_\infty) \tag{4}$$

Boundary conditions:

$u(x,0)=bx$, $v(x,0)=-v$, $T(x,0)=T_w$, $C(x,0)=C_W$,
$u(x,\infty)=ax$, $T(x,\infty)=T_\infty$, $C(x,\infty)=C_\infty$

$$k=\frac{\partial T}{\partial y}(x,0)=h_f[T_f-T(x,0)] \tag{5}$$

where υ is the kinematic viscosity, σ is the electrical conductivity, α is the thermal diffusivity, κ is the thermal conductivity, ρ is the fluid density, c_p is the specific heat capacity at constant pressure and q_r is the radiative heat flux, C_W is the wall surface concentration and C_∞ is the concentration of the fluid outside the boundary layer $(C_W > C_\infty)$, T_W is the heated wall surface temperature and T_∞ is the temperature of the fluid outside the boundary layer ($T_W > T_\infty$).

3 Numerical Procedure

Using the Rosseland approximation for radiation, Ibrahim and Makinde (2011) simplified the heat flux as

$$q_r=-\frac{4\sigma^*}{3K'}\frac{\partial T^4}{\partial y} \tag{6}$$

where K' and σ^* are the Stefan-Boltzmann constant and the mean absorption coefficient respectively. We assume that the temperature differences within the flow such as the term T^4 may be expressed as a linear function of temperature. Hence, expanding T^4 in a Taylor series about T_∞ and neglecting higher order terms, we get;

$$T^4\cong 4T_\infty^3T-3T_\infty^4 \tag{7}$$

Introducing a similarity variables η, a dimensionless stream function $f(\eta)$, temperature $\theta(\eta)$ and concentration $\phi(\eta)$ as

$$\eta=y\sqrt{\frac{b}{\upsilon}},u=xbf'(\eta),v=-\sqrt{\upsilon b}\,f(\eta),\quad \theta(\eta)=\frac{T-T_\infty}{T_S-T_\infty}$$
$$,\ \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty} \tag{8}$$

Equations (1) - (7) reduce to:

$$f'''+ff'-(M+f')f'=-\lambda^2 \tag{9}$$

$$\left(1+\frac{3}{4}Ra\right)\theta''+\Pr f\theta'+Br\left(Mf'^2+f''^2\right)=0 \tag{10}$$

$$\phi''+Scf\phi'-Scf'\phi-Sc\beta\phi=0 \tag{11}$$

The associated boundary conditions then become:

$$f'(0)=1,\ f(0)=f_w,\theta(0)=1,\ \phi(0)=1,$$
$$\theta'(0)=-Bi_x[1-\theta(0)]\quad f'(\infty)=\lambda,\theta(\infty)=0,$$
$$\phi(\infty)=0 \tag{12}$$

In the above equations, primes denote the order of differentiation with respect to the similarity variable, η, $\lambda = \frac{a}{b}$ is the velocity ratio parameter ,□□ $\beta = \frac{\gamma}{b}$ is the reaction rate parameter, $\Pr = \frac{\upsilon}{\alpha}$ is the Prandtl number, $Ra = \frac{4\sigma^{*} T^{3}_{\infty}}{\kappa K'}$ is the thermal radiation parameter, $M = \frac{\sigma B_0^2}{\rho b}$ is the local magnetic field parameter, $f_w = -\, {}^{v}\!\big/\!{}_{\sqrt{b\upsilon}}$ is the suction parameter, $Br = \frac{\mu (bx)^2}{\kappa (T_w - T_\infty)}$ is the Brinkmann number, $Sc = \frac{\nu}{D}$ is the Schmidt number and $Bi_x = \frac{h_f}{k}\sqrt{\frac{\nu x}{U_\infty}}$ is the Biot number. The coupled nonlinear equations (9), (10) and (11) with the boundary conditions in equation (12) were solved numerically using the fourth-order Runge–Kutta method with a shooting technique and results presented in tables and graphically. A step size of 0.001 was used to obtain the numerical solution with seven-decimal place accuracy as the criterion of convergence.

4 Results and Discussions

4.1 Numerical results

To determine the accuracy of the results, a comparison between the present numerical solution and the work of Arthur and Seini [13] for the skin friction coefficient *(-f ″* (*0*)) and rate of heat transfer (-θ′ (0)) represented by the local Nusselt number are presented in Table 1. It was observed that the results were consistent with that available in the literature.

Table 1.*Comparison of skin friction coefficient, -f ″* (*0*) and rate of heat transfer -θ′ (0)

Arthur and Seini [13] **λ**	**-f ″ (0)**	**-θ′(0)**	**Present Work** **-f ″ (0) -θ′(0)**
0	**1.10000**	**0.40375**	1.10000
0.5	**0.76409**	**0.53760**	0.40375
2.0	**-1.97723**	**0.68112**	0.76409
			0.53760
			-1.97723
			0.68112

The result of varying parameter values on the skin friction coefficient *(-f″*(*0*)), the rate of heat transfer (-θ′(0)) represented by the local Nusselt number and the rate of mass transfer (-$\phi'(0)$) by the local Sherwood number at the surface are presented in Table 2. It is observed that increasing the velocity ratio parameter decreases the skin friction coefficient and enhances the rate of heat and mas transfers on the surface. Also, increasing the magnetic field intensity tends to increase the skin friction due to the presence of the Lorentz force induced by the magnetic field in the flow. The Lorentz force retards the fluid flow which results in a decrease of the rate at which heat and mass are transferred. Moreover, increasing the Prandtl, radiation, Biot and Brinkman numbers have no effect on the skin friction coefficient and rate of mass transfer but increases the rate of heat transfer due to the dominance effects of fluid momentum over thermal diffusion, radiative heating and viscous dissipation. Furthermore, increasing the Schmidt and reaction rate numbers kept the skin friction and rate of heat transfer constant whiles the rate of mass transfer is increased at the surface of the plate. However, the skin friction, the rates of heat and mas transfers are enhanced at the surface of the plate as the suction parameter is increased.

4.2. Graphical Results

4.2.1. Effects of Parameter Variation on the Velocity Profiles

Figures 1 –3 present the velocity profiles for varying parameters. Generally, the velocity of the fluid is lowest at the surface of the plate and increases parabolically to the free stream value satisfying the far field boundary condition. In Figure 2, it is observed that increasing the velocity ratio parameter increases the velocity boundary layer. Furthermore, it is observed in Figures 2-3 that increasing the magnetic field intensity and suction parameter reduce the velocity profile throughout the boundary layer due to the presence of a resistive force called the Lorentz force.

Table 2. Numerical results for varying parameter values

λ	M	Pr	fw	Ra	Br	Sc	Bi_x	β	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0	0.1	0.71	0.1	0.1	0.1	0.24	0.1	0.1	1.10000	0.07332	0.42427
0.5	0.1	0.71	0.1	0.1	0.1	0.24	0.1	0.1	0.76409	0.08089	0.54062
1.5	0.1	0.71	0.1	0.1	0.1	0.24	0.1	0.1	-0.84872	0.08140	0.72095
1.0	1.0	0.71	0.1	0.1	0.1	0.24	0.1	0.1	1.45591	0.06256	0.38696
1.0	3.0	0.71	0.1	0.1	0.1	0.24	0.1	0.1	2.04507	0.04305	0.31602
1.0	5.0	0.71	0.1	0.1	0.1	0.24	0.1	0.1	2.49562	0.02727	0.31353
1.0	0.1	0.72	0.1	0.1	0.1	0.24	0.1	0.1	1.07283	0.07507	0.44592
1.0	0.1	3.60	0.1	0.1	0.1	0.24	0.1	0.1	1.07283	0.09097	0.44592
1.0	0.1	7.20	0.1	0.1	0.1	0.24	0.1	0.1	1.07283	0.09424	0.44592
1.0	0.1	0.71	1.0	0.1	0.1	0.24	0.1	0.1	1.59101	0.08329	0.55077
1.0	0.1	0.71	3.0	0.1	0.1	0.24	0.1	0.1	3.09751	0.08929	0.89519
1.0	0.1	0.71	5.0	0.1	0.1	0.24	0.1	0.1	4.78959	0.09114	1.31694
1.0	0.1	0.71	0.1	1.0	0.1	0.24	0.1	0.1	1.07283	0.06822	0.44592
1.0	0.1	0.71	0.1	2.0	0.1	0.24	0.1	0.1	1.07283	0.06361	0.44592
1.0	0.1	0.71	0.1	3.0	0.1	0.24	0.1	0.1	1.07283	0.06073	0.44592
1.0	0.1	0.71	0.1	0.1	1.0	0.24	0.1	0.1	1.07283	0.00611	0.44592
1.0	0.1	0.71	0.1	0.1	2.0	0.24	0.1	0.1	1.07283	0.07028	0.44592
1.0	0.1	0.71	0.1	0.1	3.0	0.24	0.1	0.1	1.07283	0.14667	0.44592
1.0	0.1	0.71	0.1	0.1	0.1	0.62	0.1	0.1	1.07283	0.07486	0.82467
1.0	0.1	0.71	0.1	0.1	0.1	1.78	0.1	0.1	1.07283	0.07486	1.58934
1.0	0.1	0.71	0.1	0.1	0.1	2.62	0.1	0.1	1.07283	0.07486	2.00592
1.0	0.1	0.71	0.1	0.1	0.1	0.24	1.0	0.1	1.07283	0.29066	0.44592
1.0	0.1	0.71	0.1	0.1	0.1	0.24	5.0	0.1	1.07283	0.39081	0.44592
1.0	0.1	0.71	0.1	0.1	0.1	0.24	7.0	0.1	1.07283	0.40067	0.44592
1.0	0.1	0.71	0.1	0.1	0.1	0.24	0.1	2.0	1.07283	0.07486	0.85060
1.0	0.1	0.71	0.1	0.1	0.1	0.24	0.1	5.0	1.07283	0.07486	1.21711
1.0	0.1	0.71	0.1	0.1	0.1	0.24	0.1	7.5	1.07283	0.07486	1.44899

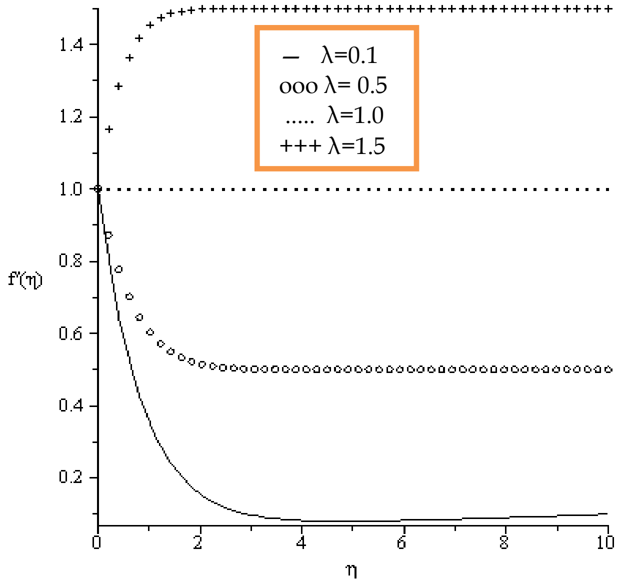

Figure 1.*Velocity Profile for Varying Velocity Ratio Parameter*

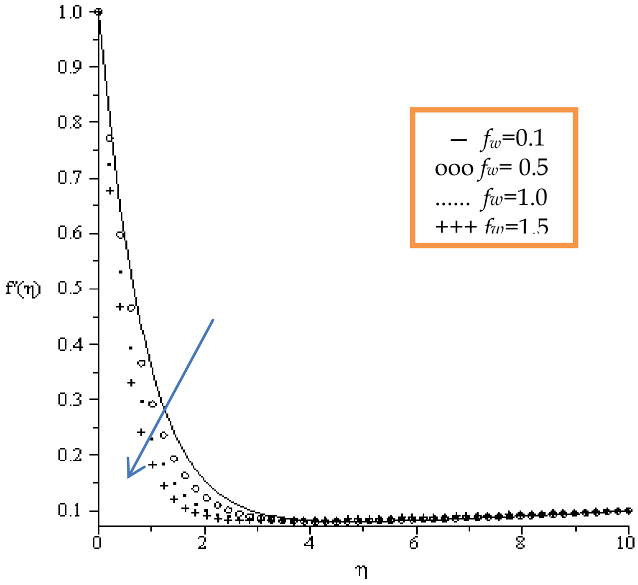

Figure 2. *Velocity Profiles for varying suction parameter*

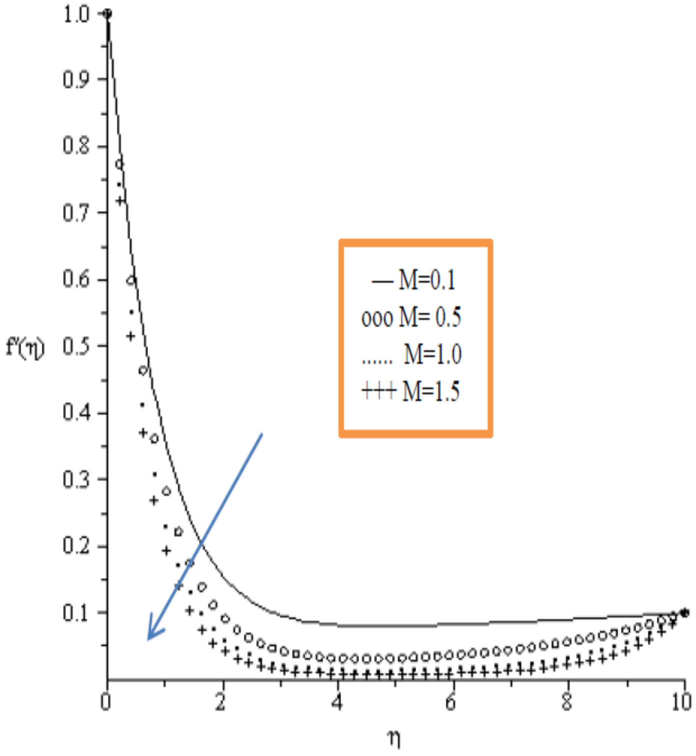

Figure 3 Velocity profiles for varying values of the Magnetic parameter

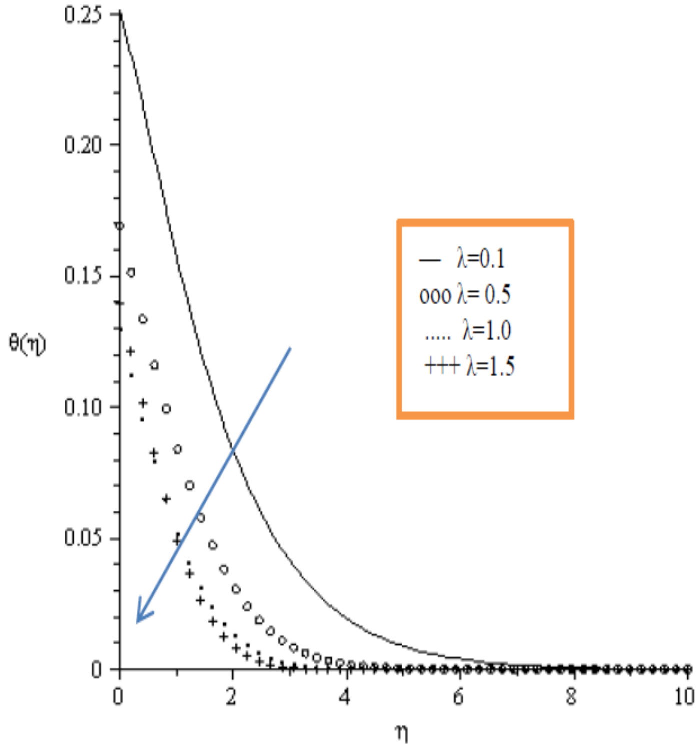

Figure 4.Temperature Profiles for increasing velocity ratio parameter

4.2.2. Effects of Parameter Variation of Temperature Profiles

Figures 4 – 11 present the temperature profiles for various parameter variations. Generally, the temperature of the fluid reaches its maximum at the surface of the plate and decreases exponentially to the free stream zero value away from the plate where it attain its minimum, satisfying the boundary condition. In figure 4, the thermal boundary layer thickness reduces as the velocity ratio parameter is increased. Also, it is noted in figures 5 and 6 that increasing the Suction parameter and Prandtl number reduce the thermal boundary layer thickness. However, in figure 7, the thermal boundary layer thickness is observed to increase as the Brinkman number is increased. This is due to viscous dissipation which enhances the heating of the fluid. Furthermore, from figures 8-11, increasing the Biot number, Magnetic field parameter, Radiation parameter and Schmidt number enhances the thermal boundary layer thickness.

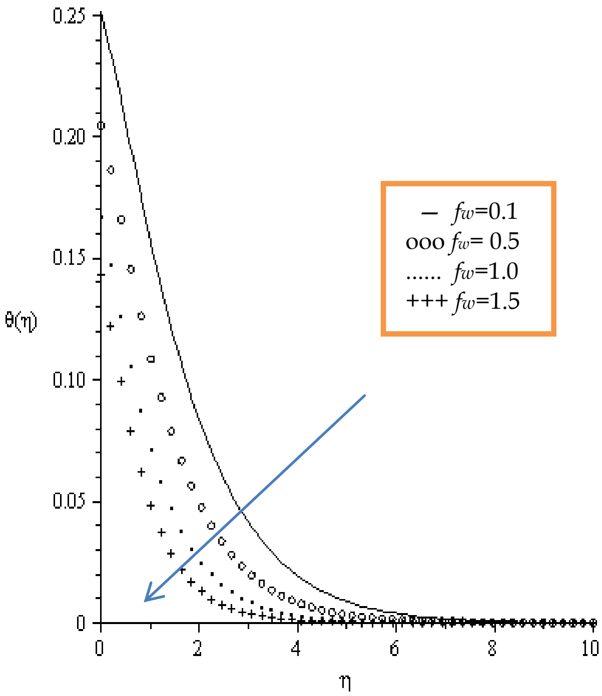

Figure 5. *Temperature Profiles for varying Suction Parameter*

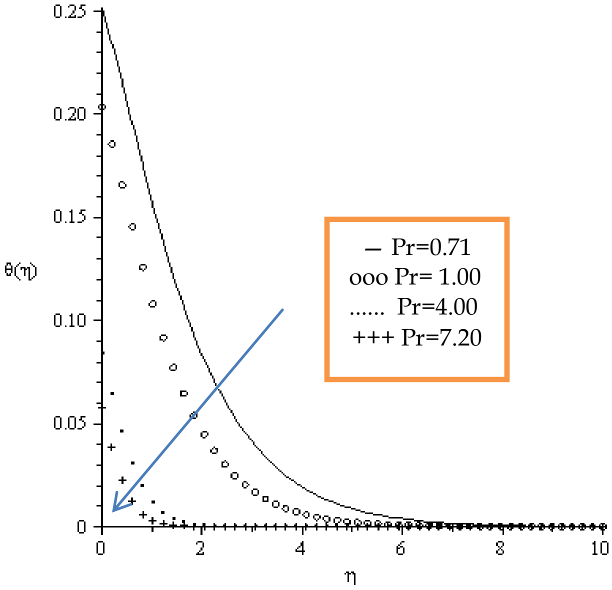

Figure 6. *Temperature Profiles for increasing Prandtl number*

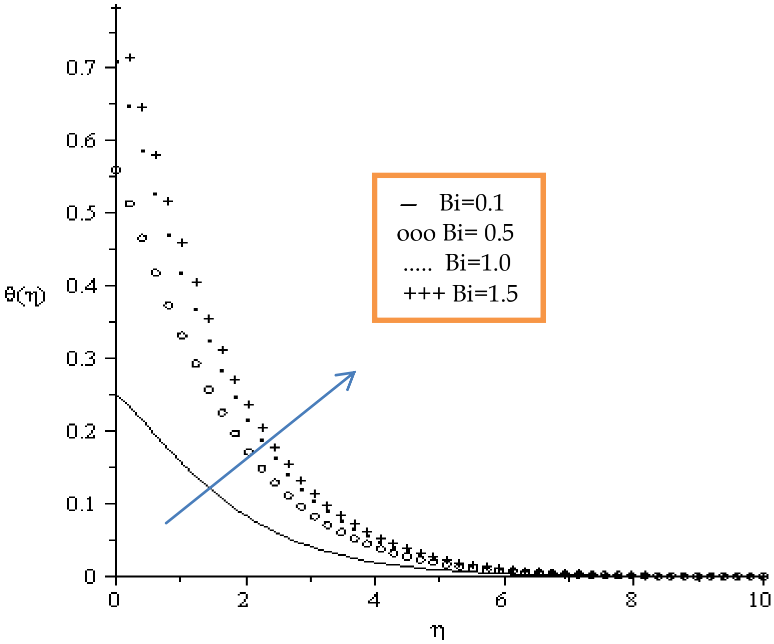

Figure 8.*Temperature Profiles for increasing Biot number*

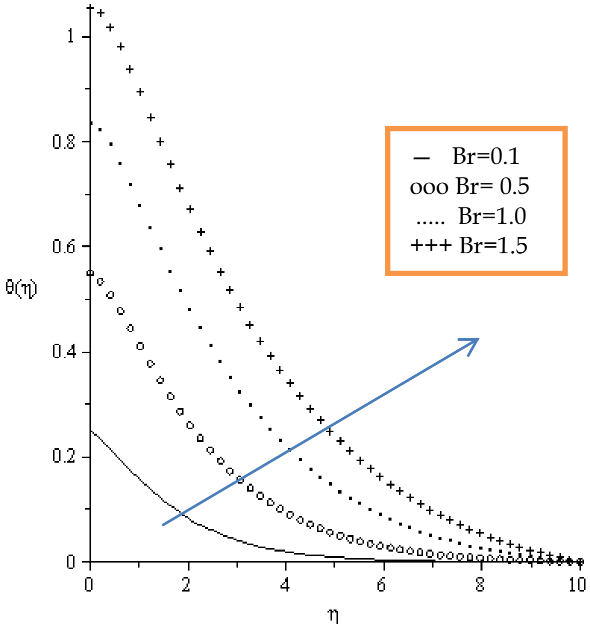

Figure 7. *Temperature Profiles for varying Brinkmann Number*

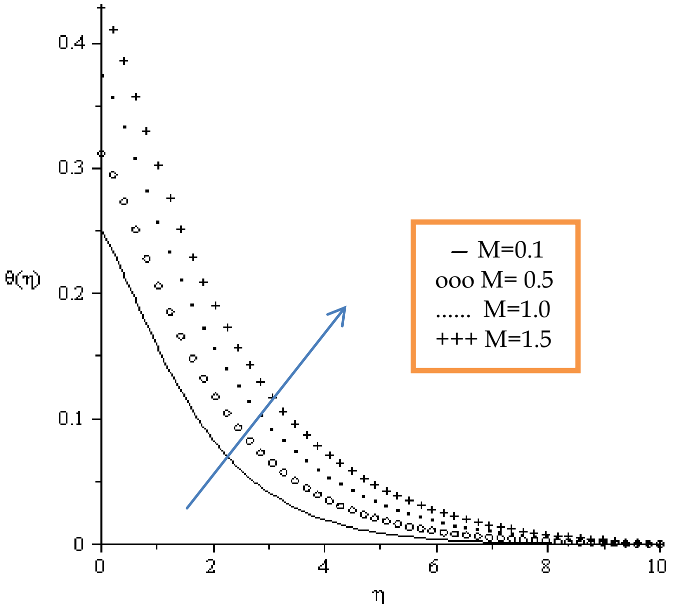

Figure 9. *Temperature Profiles for increasing Magnetic field parameter*

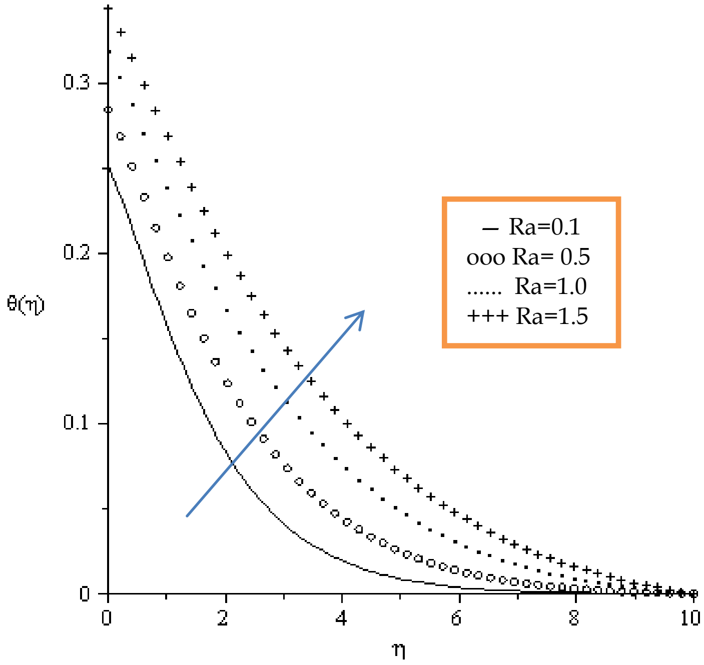

Figure 10. *Temperature Profiles for varying Radiation parameter*

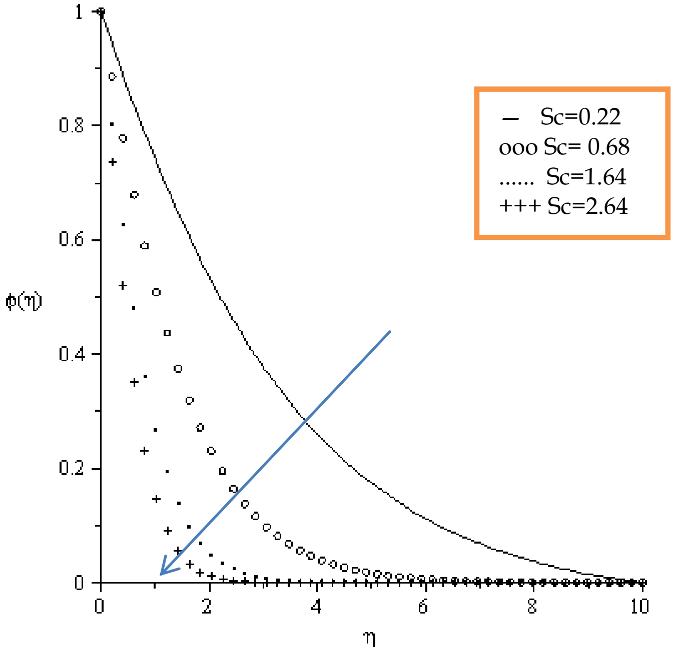

Figure 12. *Concentration Profiles for varying Schmidt number*

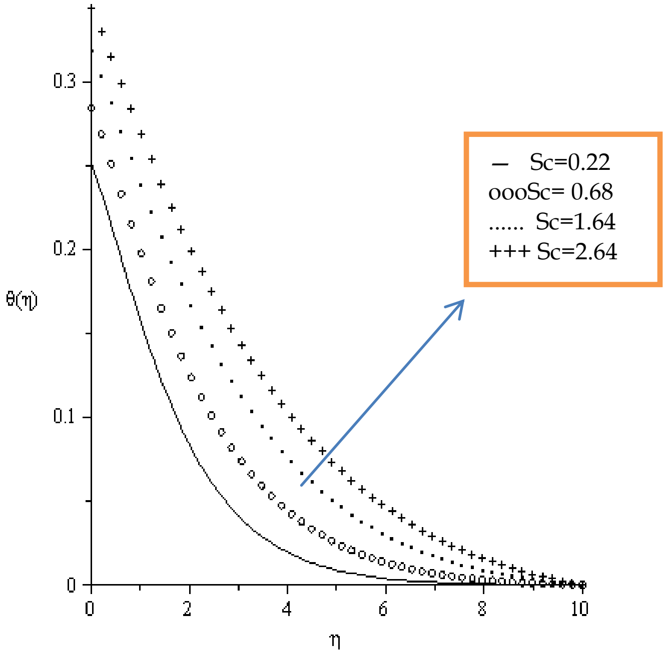

Figure 11. *Temperature Profiles for varying Schmidt number*

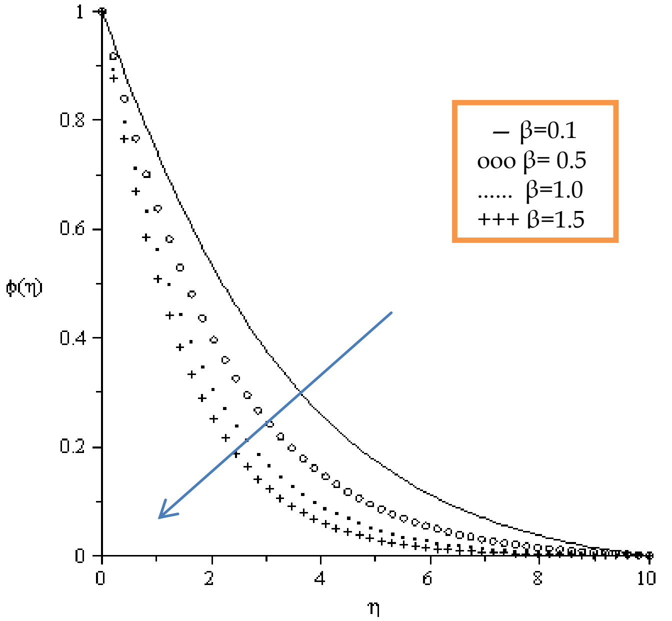

Figure 13. *Concentration Profiles for increasing reaction rate parameter*

4.2.3. Effects of Parameter Variation of Concentration Profiles

Figures 12 – 15 present the temperature profiles for various parameter variations. Generally, the species concentration in the fluid had a maximum value at the plate surface and decreased exponentially to the free stream zero value away from the plate. The concentration boundary layer thickness was observed to reduce as the Schmidt number Reaction rate parameter and Suction parameter were increased whilst it was observed to increase as the Magnetic field parameter was increased.

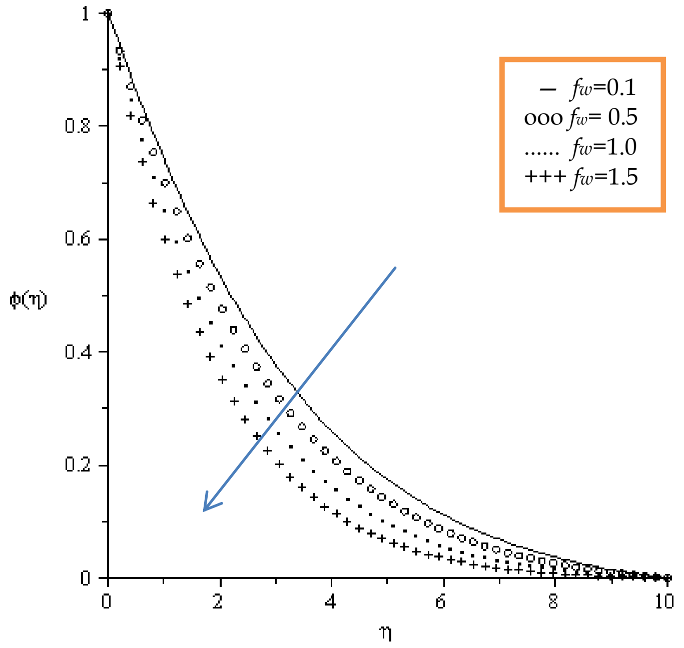

Figure 14. *Concentration Profiles for varying Suction Parameter*

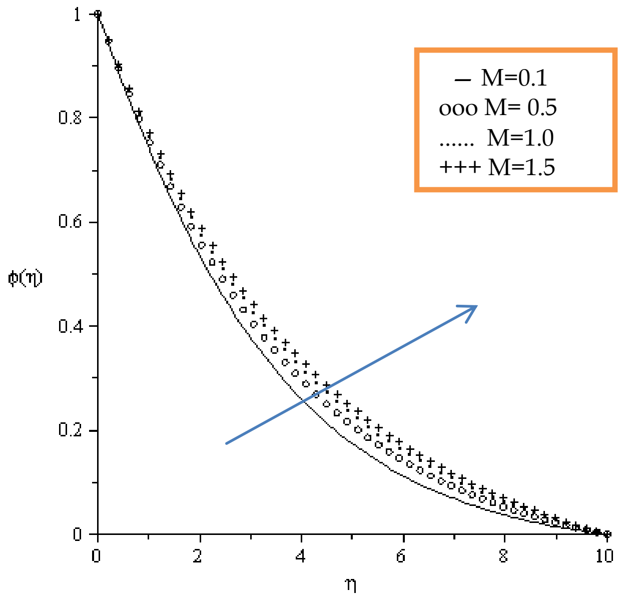

Figure 15. *Centration Profiles for increasing Magnetic field parameter*

5 Conclusions

In this study, the effects of radiation and chemical reaction on magneto hydrodynamic stagnation point flow over a porous stretching surface with convective thermal boundary condition was investigated. The nonlinear and coupled governing differential equations were solved numerically using the fourth order Runge-Kutta shooting method. Numerical results were presented whilst the velocity, temperature and concentration profiles illustrated graphically and analyzed. The following conclusions can be made.

- The velocity boundary layer thickness increases with increasing velocity ratio parameter. It however decreases with increasing suction and magnetic field parameters.
- The thermal boundary layer thickness increases with increasing Brinkmann number, Biot number, magnetic field parameter, radiation parameter and Schmidt number. It however decreases with increasing velocity ratio parameter, suction parameter and Prandtl number.
- The concentration boundary layer thickness increases with increasing magnetics field parameter. It however decreases with increasing Schmidt number and reaction rate parameter.
- The skin friction, rate of heat transfer and rate of mas transfer are enhanced at the surface of the plate as the suction parameter is increased.

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