# Analytical Determining Of The Steinmetz Equivalent Diagram Elements Of Single-Phase Transformer

T. Aly Saandy, M. Rakotomalala, Said Mze, A. F. Toro, A. Jaomiary

**Abstract**: This article presents to an analytical calculation methodology of the Steinmetz Equivalent Diagram Elements applied to the prediction of Eddy current loss in a single-phase transformer. Based on the electrical circuit theory, the active and reactive powers consumed by the core are expressed analytically in function of the electromagnetic parameters as resistivity, permeability and the geometrical dimensions of the core. The proposed modeling approach is established with the duality parallel series. The equivalent diagram elements, empirically determined by Steinmetz, are analytically expressed using the expressions of the no loaded transformer consumptions. To verify the relevance of the model, validations both by simulations with different powers and measurements were carried out to determine the resistance and reactance of the core. The obtained results are in good agreement with the theoretical approach and the practical results.

**Index Terms**: Black box theory, Eddy current, Iron loss, Modeling, Single-phase Transformer, Steinmetz Equivalent circuit.

## 1 INTRODUCTION

THE transformer is one of the important elements constituting the electrical systems. In order to predict, the electrical chain performance, a relevant model of transformer is required. Therefore, different transformer models have been established by the electrical engineers since the invention of AC current by Tesla in the late 19th century. So far, the equivalent circuit of a single-phase transformer shows a central branch formed by a resistance in parallel with an inductance. This inductance is associated to the magnetizing and the resistance to the Eddy current. This resistance, known as iron resistance, corresponds to the active power empirically expressed by Steinmetz [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12] and [13].


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This model, preached by any teacher of electrical engineering to his students on the various levels, was given like postulate. This article proposes an analytical justification to the equivalent diagram elements of single-phase transformer current, particularly to those of the central branch. The article is organized in three main sections. Section II begins by the calculation of active power consumed in a parallelepiped electromagnetic domain subjected to a variable flow. The result is applied to a shell form single-phase transformer. With open load test, the power consumption linked to the leakage inductance and resistance of the transformer winding is neglected in front of the consumption of the central branch [11]. But it is not the case for a loaded transformer. The methods to determine resistances of the reels are indisputable. This article gives the expressions of the elements of the central branch in the model of Steinmetz starting from the powers and their site. It studies also the variation relation of the iron resistance and the iron reactance compared to the computed values with 220 V. Section 2 is dedicated to this calculation. Discussions and a comparison of results are described in Section 3. The final section is devoted to the article conclusion.

## 2 METHODOLOGY AND USED MATERIALS

The theoretical methodology of the synthesis of black box like receiver is developed in the present section. The investigation is performed based on the constituting material characteristics.

### 2.1 Study of Black Box Supplied to a Voltage Source

The proposed modeling approach is based on the synthesis of the black box consumption in receiver mode, applied to the core of shell form single phase transformer and its windings as illustrated in Fig. 1 below.

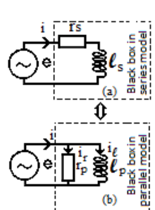

**Fig.1 (1).** *Temporal electric model of a black box supplied to an alternative source*

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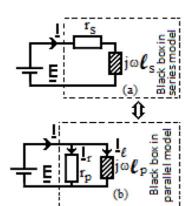

**Fig.1 (2).** *Complex electric model of a black box supplied to an alternative source*

The analytical calculation of the active loss can be performed in the winding resistances crossed by currents and the core crossed by the magnetic flux. The internal consumption of the transformer is made up of:

❖ for the resistances

$P_R = RI^2$ ; (1)

❖ for the core [1]

$P_{Fe} = KVs^2f^2B^2$. (2)

The active power consumed by the black box is given by the law of Joule. It has as instantaneous expression.

❖ for the Fig. 1(1a) : $p = r_s i^2$ (3)

❖ for the Fig. 1(1b) : $p = r_p i_r^2$ (4)

The mean value of this active power and the reactive power are worth:

❖ for the Fig. 1(2a) $\begin{cases} P = r_s I^2 = r_s \dfrac{E^2}{r_s^2 + \omega^2 \ell_s^2} \\ Q = \omega \ell_s I^2 = \omega \ell_s \dfrac{E^2}{r_s^2 + \omega^2 \ell_s^2} \end{cases}$ (5)

❖ for the Fig. 1(2b) $\begin{cases} P = r_p I_r^2 = \dfrac{E^2}{r_p} \\ Q = \omega \ell_p I_\ell^2 = \dfrac{E^2}{\omega \ell_p} \end{cases}$ (6)

The powers are preserved for the two configurations.

$$\begin{cases} P = r_s \dfrac{E^2}{r_s^2 + \omega^2 \ell_s^2} = \dfrac{E^2}{r_p} \\ Q = \omega \ell_s \dfrac{E^2}{r_s^2 + \omega^2 \ell_s^2} = \dfrac{E^2}{\omega \ell_p} \end{cases} \quad (7)$$

Resistances $r_s$ and $r_p$ as well as inductances $\ell_s$ and $\ell_p$ are bound by the following relations.

$$\begin{cases} r_p = \dfrac{r_s^2 + \omega^2 \ell_s^2}{r_s} \\ \ell_p = \dfrac{r_s^2 + \omega^2 \ell_s^2}{\omega^2 \ell_s} \end{cases} \quad (8)$$

In equations (3), (4), (5) and (6), E, I, $I_r$ and $I_\ell$ indicate respectively the modules of the complex voltage $\underline{E}$ and currents $\underline{I}$., $\underline{I}_r$ and $\underline{I}_\ell$. E, I, $I_r$ and $I_\ell$ are equal to the effective values of e, i, $i_r$ and $i_\ell$.

## 2.2 Description of the Single Phase Transformer Under Study

Fig. 2 represents the photograph of the single phase transformer. Its structure including the geometrical dimensions is illustrated by Fig. 3.

***Fig. 2.*** *Photograph of the Single Phase Transformer Understudy*

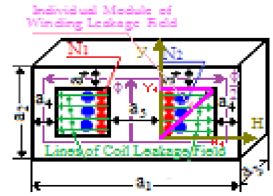


***Fig. 3:*** *Physical diagram of the studied transformer*

$a_1$ = 16.5 cm: external length of the core
$a_2$ = 16.5 cm: external height of the core
$a_3$ = 5.5 cm: external width of the core
$a_4$= 2 cm: width of the outer wings of the core
$a_5$ = 4 cm: width of the central wing of the core
$N_1$ = 220: whorls turn of the primary winding
$N_2$ = 115: whorls turn of the secondary winding
$\ell_f$ : mean length of the field route

The 3D structure diagram of the studied transformer shown in Fig. 4 is manly constituted by:

➢ a core obtained by stacking of $N_f$ layers in E and I form. The mean length $\ell_f$ of the field route in the core is determined by:

$$\ell_f = a_1 + 2a_2 - 2a_4 \quad (9)$$

➢ Two concentric coils with $N_1$ whorls for the primary winding and $N_2$ for the secondary.

## 2.3 Determination of the Equivalent Circuit of the Single Phase Transformer

### 2.3.1 Simplifying assumptions

The following simplifying assumptions are adopted:
❖ Horizontality of the lines of winding leakage field;
❖ Permanent mode at the industrial frequencies;
❖ Isotropy of the transformer core;
❖ Constancy of the iron conductivity;
❖ Absence of the skin effect in each layer used to form the transformer core;
❖ Active loss due to the Eddy current loss in the transformer core and Charge currents in the windings;
❖ Weakness of the hysteresis loss in front of the Eddy current loss;
❖ Possibility of passing from series configuration to parallel configuration and vice versa;
❖ Assimilation of the transformer core to a parallelepipedic domain.

### 2.3.2 Calculation of Leakage Inductances and Resistances of the Transformer Windings

The studied transformer has a central column and two external columns. The reels are concentric and uniformly set out again on the core**.** The leakage field lines pass horizontally from the central column to the external columns as in Fig.3. The module $H_k$ of the leakage magnetic field, created to a height y by the reel k traversed by the current $i_k$, is calculated with the theorem of Ampere as follows.

$$H_k = \frac{2N_k i_k}{(a_2 - 2a_4)(a_1 - a_5 - 2a_4)} y \quad (10)$$

The transformer single-phase with concentric reels reveals:

- the resistance of each reel,

$$R_k = \frac{\ell_{Cuk}}{\sigma_{Cu} S_{Cuk}} \quad (11)$$

where
$\ell_{Cuk}$ : overall length of the copper of the reel k in m;
$S_{Cuk}$ : section of the copper of the reel k in $m^2$;
$\sigma_{Cu}$ : conductivity of copper in Ωm.

- the leakage inductance of each reel drawn from the magnetic energy stored in the domain occupied by the windings as below,

$$\begin{cases} W_{mgk} = \frac{1}{2}\mu_0 \underset{\text{central column-outercoluns}}{\oiiint} H_k^2 d\wp \\ \text{with} \quad d\wp = (a_1 - 2a_4 - a_5)(a_3 - 2a_4 - a_5)dy \end{cases} \quad (12)$$

The formulas (10) and (12) give the following magnetic energy expression:

$$\begin{aligned} W_{mgk} &= 2\mu_0 \frac{(a_3 - a_5 - 2a_4)N_k^2 i_k^2}{(a_2 - 2a_4)^2 (a_1 - a_5 - 2a_4)} \int_0^{Y_1} y^2 dy \text{ with } Y_1 = a_2 - 2a_4 \\ &= \frac{2}{3}\mu_0 \frac{(a_3 - a_5 - 2a_4)(a_2 - 2a_4)N_k^2 i_k^2}{(a_1 - a_5 - 2a_4)} \end{aligned} \quad (13)$$

As $W_{mgk} = \frac{1}{2}L_k i_k^2$, the leakage inductance of the reel k is worth:

$$L_k = \frac{4}{3}\mu_0 \frac{(a_3 - a_5 - 2a_4)(a_2 - 2a_4)}{(a_1 - a_5 - 2a_4)} N_k^2 \quad (14)$$

### 2.3.3 Calculation of the Transformer Core Eddy current Loss

The core counts $N_f$ layers, the total active power which it consumes has the expression below [1]:

$$P_{Fe} = \frac{\pi^2}{16\rho_{Fe}} N_f \frac{L_1^2}{L_1^2 + L_2^2} L_1 L_2 L_3 L_2^2 f^2 B^2 \quad (16)$$

The transformer iron loss is worth.

$$P_{Fe} = K.V.s^2 f^2 B^2 \quad (17)$$

where
- K : Eddy currents Steinmetz coefficient;
- B : effective value of induction in the plate ;
- V : volume of the domain through which passes the flow of the induction B;
- s : thickness of each sheet forming the domain;

The constant K in the formula (17) has the unit of electric conductivity. The identification between the formula (16) and formula (17) gives:

$$\begin{cases} V = N_f L_1 L_2 L_3 \\ s^2 = L_2^2 \\ K = \frac{\pi^2}{16\rho_{Fe}} \frac{L_1^2}{L_1^2 + L_2^2} \end{cases} \quad (18)$$

Thereafter, the width will be noted w and the thickness s. The attenuation factor of the Steinmetz coefficient becomes:

$$A = \frac{1}{1 + \frac{s^2}{w^2}} = \frac{1}{1 + x^2} \quad \text{with } x = \frac{s}{w} \quad (19)$$

where
- s : thickness of the sheet;
- w: width of the sheet surface crossed by the magnetic flow.

### 2.3.4 Electrical Diagram Associated with the Iron Losses

The elements associated with the Eddy current loss are calculated by considering the open secondary transformer. In this case the current of magnetization is equal to the primary current $\underline{I}_{10}$ of the transformer. With the weakness of the terminal voltage of the leakage reactance, the primary induced voltage $\underline{E}_1$ is supposed equal to the primary voltage $\underline{U}_{10}$. The transformer becomes a black box supplied to a source having a fem $e_{10}$ and current $i_{10}$ with effective values $E_1$ and $I_{10}$. It consumes the active power given by the formula (16) in the carcass and by the formula (1) in windings. But the latter is neglected. With an induction B of pulse ω=2πf,

- the primary induced voltage is worth;

$$E_1 = \omega N_1 B S_{Fe} = U_{10}, \quad (20)$$

- the primary current is drawn from Ampere law.

$$I_{10} = \frac{H_{Fe} L_{Fe}}{N_1} \quad (21).$$

When B in the formula (17) is replaced by its value drawn from the primary induced electromotive force in formula (20), and the frequency by its value in function of pulse, the iron loss is worth below.

$$P_{Fe} = KV\left(\frac{s}{2\pi N_1 S_{Fe}}\right)^2 E_1^2 = \frac{E_1^2}{\frac{4\pi^2 N_1^2 S_{Fe}^2}{KVs^2}} = \frac{U_{10}^2}{\frac{4\pi^2 N_1^2 S_{Fe}^2}{KVs^2}} \quad (22)$$

The iron volume is worth:

$$\begin{cases} V = L_{Fe} S_{Fe} \\ \text{with} \\ L_{Fe} = a_1 + a_2 - 2a_4 - a_5 \\ S_{Fe} = a_3 a_5 \end{cases} \quad (23)$$

where

V: iron volume in $m^3$;

$L_{Fe}$: length of the parallelepipedic domain equivalent to transformer core in m;

$S_{Fe}$: surface crossed by the magnetic flux of the parallelepipedic domain equivalent to the transformer core in $m^2$;

The Eddy current loss becomes:

$$P_{Fe} = \frac{E_1^2}{\dfrac{64 N_1^2 \rho_{Fe} S_{Fe}}{As^2 L_{Fe}}} \quad (24)$$

where

- ♣ $E_1$: induced primary voltage in volt given by Faraday's law;
- ♣ $\rho_{Fe}$ : iron resistivity in Ωm;
- ♣ A: attenuation of Steinmetz coefficient;
- ♣ s : thickness of each sheet forming the domain;

The expression $\frac{64 N_1^2 \rho_{Fe} S_{Fe}}{As^2 L_{Fe}}$ has ohm as unit. It is the iron resistance and can be noted:

$$R_{Fe} = \frac{64 N_1^2 \rho_{Fe} S_{Fe}}{As^2 L_{Fe}} \quad (25)$$

The active puissance is expressed as below:

$$P_{Fe} = \frac{E_1^2}{R_{Fe}} \quad (26)$$

The expression (26) gives a parallel configuration of the elements representing the central branch of the transformer. A reactive power, usually noted $Q_m$ or $Q_\mu$, because it symbolized magnetizing, is associated with this active power of the transformer. As it is associated with consumption in iron as well as the active loss, it can be noted $Q_{Fe}$. Thus, the complex apparent primary power is written:

$$\underline{S}_{10} = \underline{E}_1 \underline{I}_{10}^* = P_{Fe} + jQ_{Fe} \quad (27)$$

The combination of the formulas (27), (26), (25), (24) and (21) gives the expression of the following reactive loss.

$$Q_{Fe} = \sqrt{(E_1 I_1)^2 - \frac{E_1^4}{R_{Fe}^2}} = \frac{E_1^2}{\omega N_1^2 \mu \dfrac{S_{Fe}}{L_{Fe}}} \frac{\sqrt{R_{Fe}^2 - \left(\omega N_1^2 \mu \dfrac{S_{Fe}}{L_{Fe}}\right)^2}}{R_{Fe}} \quad (28)$$

As in a parallel configuration, the reactive loss is expressed,

$$Q_{Fe} = \frac{E_1^2}{X_{Fe}} \quad (29)$$

The identification between the formulas (28) and (29), gives the iron reactance $X_{Fe}$ and inductance $L_{Fe}$ below.

$$\begin{cases} X_{Fe} = \omega N_1^2 \mu \dfrac{S_{Fe}}{L_{Fe}} \dfrac{R_{Fe}}{\sqrt{R_{Fe}^2 - \left(\omega N_1^2 \mu \dfrac{S_{Fe}}{L_{Fe}}\right)^2}} \quad \text{(a)} \\ L_{Fe} = N_1^2 \mu \dfrac{S_{Fe}}{L_{Fe}} \dfrac{R_{Fe}}{\sqrt{R_{Fe}^2 - \left(\omega N_1^2 \mu \dfrac{S_{Fe}}{L_{Fe}}\right)^2}} \quad \text{(b)} \end{cases} \quad (30)$$

The magnetic circuit, face to the magnetic flux, is equivalent to a resistance in parallel to an inductance. This result explains the empirical model of Steinmetz whose diagram follows.

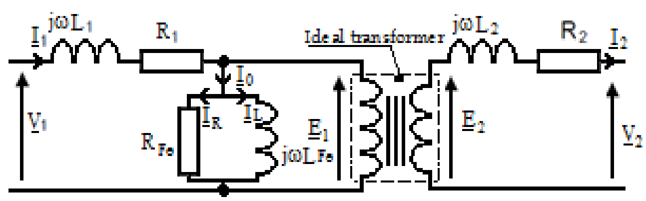


***Fig. 4:*** *Steinmetz Equivalent Diagram of the studied transformer*

$\underline{V}_1$: Complex Primary Supply Voltage;.
$\underline{I}_1$: Complex Primary Supply Current;
$\omega L_1$: Primary Leakage Reactance;
$R_1$: Primary Winding Resistance;
$R_{Fe}$: Primary Referred Iron Resistance;
$\omega L_{Fe}$: Primary Referred Iron Reactance;
$\underline{E}_1$: Complex Primary Induced Voltage;.
$\underline{V}_2$: Complex Secondary Supply Voltage;.
$\underline{I}_2$: Complex Secondary Supply Current;
$\omega L_2$: Secondary Leakage Reactance;
$R_2$: Secondary Winding Resistance;
$\underline{E}_2$: Complex Secondary Induced Voltage;.
$\underline{I}_0$: Complex Primary Referred Iron Current;
$\underline{I}_L$: component of $\underline{I}_0$ passing in $L_{Fe}$;
$\underline{I}_R$: component of $\underline{I}_0$ passing in $R_{Fe}$;
ω: pulse of exciting flow Φ wich induce $\underline{E}_1$ and $\underline{E}_2$.

### 2.3.5 Simplified Electrical Diagram of a Single Phase Transformer

The ideal transformer converts rating $E_1 I_1$ into rating $E_2 I_2$ without internal loss. The voltages $E_1$ and $E_2$ are induced by same flow Φ. The transformer is governed by the two electric equations and two induced voltages below.

$$\begin{cases} \underline{V}_1 = R_1\underline{I}_1 + j\omega L_1\underline{I}_1 + \underline{E}_1 & \text{(a) Receiver Mode} \\ \underline{V}_2 = -R_2\underline{I}_2 - j\omega L_2\underline{I}_2 + \underline{E}_2 & \text{(b) Generator Mode} \end{cases} \quad (31)$$

$$\begin{cases} \underline{E}_1 = j\omega N_1\phi \\ \underline{E}_2 = j\omega N_2\phi \end{cases} \quad (32)$$

The turn ratio m is calculated from the expression (32) as follows.

$$m = \frac{\underline{E}_2}{\underline{E}_1} = \frac{N_2}{N_1} \quad (33)$$

If the equation (30b) is divided by m, the group of formulas (30) becomes :

$$\begin{cases} \underline{V}_1 = R_1\underline{I}_1 + j\omega L_1\underline{I}_1 + \underline{E}_1 & \text{(a) Receiver Mode} \\ \dfrac{\underline{V}_2}{m} = -\dfrac{R_2\underline{I}_2}{m} - j\dfrac{\omega L_2\underline{I}_2}{m} + \dfrac{\underline{E}_2}{m} & \text{(b) Generator Mode} \end{cases} \quad (34)$$

Let $\begin{cases} \underline{V}_1 = R_1\underline{I}_1 + j\omega L_1\underline{I}_1 + \underline{E}_1 & \text{(a) Receiver Mode} \\ \dfrac{\underline{V}_2}{m} = -\dfrac{R_2}{m^2}m\underline{I}_2 - j\omega\dfrac{L_2}{m^2}m\underline{I}_2 + \underline{E}_1 & \text{(b) Generator Mode} \end{cases}$ (35).

The induced voltage $\underline{E}_1$ is the common element of the equations (35a) and (35b). It is also the central branch voltage when it is traversed by the magnetizing current $\underline{I}_0$ and the ideal transformer induced voltage. The division of the equation (31a) by m referres all quantities in secondary side of the transformer to the primary side. These referred quantities become:

$$\begin{cases} V_2^{'} = \dfrac{\underline{V}_2}{m} : \text{Referred secondary voltage} \\ I_2^{'} = m\underline{I}_2 : \text{Referred secondary current} \\ R_2^{'} = \dfrac{R_2}{m^2} : \text{Referred secondary resistance} \\ L_2^{'} = \dfrac{L_2}{m^2} : \text{Referred secondary inductance} \end{cases} \quad (36).$$

$\underline{E}_1$ and ideal transformer form a footbridge between the primary quantities and the secondary quantities. The diagram in Fig. 4 is simplified and been referred to the primary as shows the Fig.5 below.

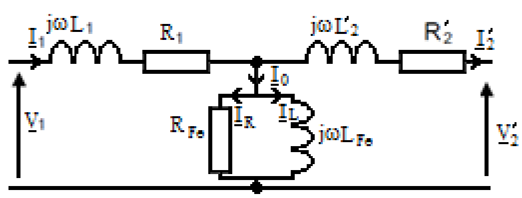


***Fig.5:*** *Simplified Steinmetz Equivalent Diagram of a transformer*

Note: The simplified diagram can be referred to the secondary by multiplying the equation (31a) by m. The common element becomes $\underline{E}_2$. The quantities $R_{Fe}$ and $X_{Fe}$ must be drawn from the expressions of the powers according to $E_2$.

### 2.4 Experimental checking of the model

The experimental setup corresponding to the performed open circuit and short-circuit test with the shell form single-phase transformer of 1 kVA to check the model is illustrated in Fig. 6 and Fig.7.

#### 2.4.1 Open circuit test

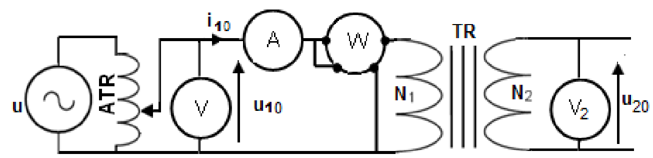


***Fig. 6:*** *Wiring diagram for open circuit test*

u: alternative voltage source.
ATR: auto-transformer supplied with u.
TR: tested transformer
$u_{10}$: primary no-load voltage.
$V_1$: voltmeter measuring the effective value of the primary voltage $u_{10}$
$i_{10}$: primary no-load current.
A: ammeter measuring the effective value of the current $i_{10}$.
W: wattmeter measuring the no-load power $P_{10}$ of TR.
$U_{20}$: secondary no-load voltage.
$V_2$: voltmeter measuring the effective value of the secondary voltage $u_{20}$

From the open-circuit test, it is possible to obtain:

- ❖ the turn ratio;

$$m = \frac{U_{20}}{U_{10}} \quad (37)$$

- ❖ the primary iron resistance;

$$R_{Fe} = \frac{U_{10}^2}{P_{10}} \quad (38)$$

- ❖ the iron reactance $X_{Fe}$;

$$X_{Fe} = \frac{U_{10}^2}{Q_{10}} = \frac{U_{10}^2}{\sqrt{(U_{10}I_{10})^2 - P_{10}^2}} \quad (39)$$

- ❖ the effective value B of induction in the core;

$$B = \frac{U_{10}}{\omega N_1 S_{Fe}} \quad (40)$$

- ❖ the effective value H of the magnetic field in the core;

$$H = \frac{N_1 I_{10}}{\ell_f} \quad (41)$$

- ❖ the magnetic permeability of the core.

$$\mu = \frac{B}{H} = \frac{\ell_f U_{10}}{\omega N_1 S_{Fe}} \tag{42}$$

### 2.4.2 Short circuit test

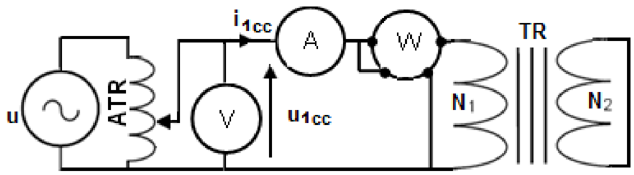


***Fig. 7:*** *Wiring diagram for short circuit test*

$u_{1cc}$ : primary short-circuit voltage.
$i_{1cc}$ : primary short-circuit current
W: wattmeter measuring the short-circuit power $P_{1cc}$ of TR.
The short circuit test gives :

- ❖ the short circuit resistance;

$$R_{1cc} = \frac{P_{1cc}}{I_{1cc}^2} \tag{43}$$

- ❖ the short circuit reactance;

$$X_{1cc} = \frac{Q_{1cc}}{I_{1cc}^2} = \frac{\sqrt{(U_{1cc} I_{1cc})^2 - P_{1cc}^2}}{I_{1cc}^2} \tag{44}$$

The following elements are deduced from the no-load test and short-circuit test.

- ❖ the primary winding resistance;

$$R_1 = \frac{R_{1cc}}{2} \tag{45}$$

- ❖ the primary winding reactance

$$X_{\sigma 1} = \frac{X_{1cc}}{2} \tag{46}$$

- ❖ the referred resistance of the secondary winding

$$R_2^{'} = \frac{R_{1cc}}{2} \tag{47}$$

- ❖ the referred reactance of the secondary winding;

$$X_2^{'} = \frac{X_{1cc}}{2} \tag{48}$$

## 3 Results and Discussions

### 3.1 Theoretical point of view

The single-phase transformer elements of the Steinmetz model equivalent diagram are given by considering two domains:

- ❖ the copper domain revealing the resistances and the reactances of the windings leakage;
- ❖ the iron domain revealing the resistance and the reactance of iron because of the magnetization.

The elements of the copper domain are calculated according to the usual formulas (11) and (14). The elements of the iron domain are calculated by the formulas (25) and (30). The iron resistance of the formula (24) depends on the geometry and the resistivity of core sheet. On the other hand, the iron reactance of the formula (30) depends on the geometry, the resistivity and the magnetic state of core as well as on the frequency. As these elements are calculable starting from the geometrical and electric data of materials entering in the transformer construction, the equivalent diagram elements of the Steinmetz model of a single-phase transformer are predictable in a given frequency.

### 3.2 Practical point of view

The results presented in this section are obtained with the transformer photographed in Fig. 3. Each element constituting the layers is suitably compared to the basic layer. It is worth noting that the transformer characteristic B(H) is not available in our laboratory. Therefore, the transformer core experimental characterization was carried out. As results, we reconstructed via vector fitting the curve plotted in Fig. 8. The red line of the curve plotted in Fig. 8 represents the considered transformer core smoothed characteristic B(H) and the crosses in blue the results of measurement in no load test.

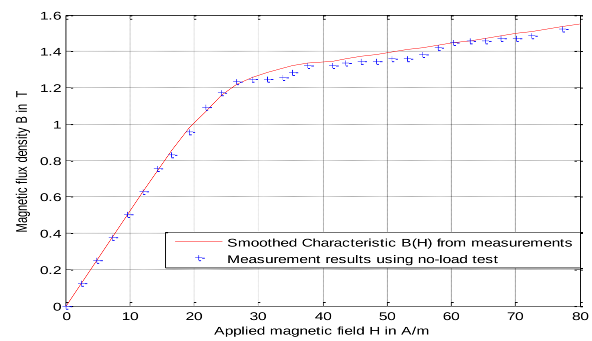


***Fig. 8:*** *Characteristics B(H) of the transformer core*

This characteristic is necessary for computing the iron loss and the reactance value of the transformer. These results are deduced from the open circuit test of which curves are depicted in Fig. 9 below.

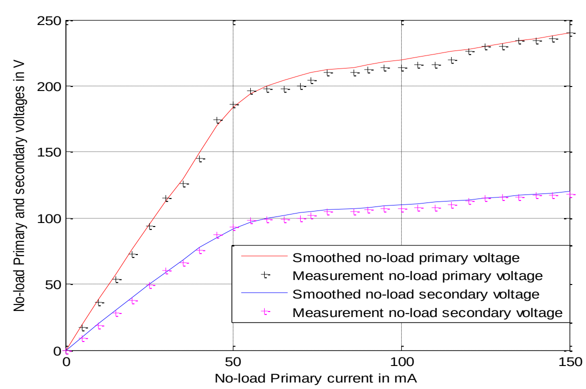


***Fig. 9 :*** *Open-circuit test characteristic of the transformer under study*

In this open-circuit characteristic, the magenta and blue curves represent respectively the primary and secondary voltage calculated using the smoothed characteristic B(H). The measured values which are represented by the data plotted in crosses are close to the smoothed values. The formula (25) gives to the iron resistance a constant value. The active power must vary linearly according to the square of the applied voltage. However, the curve in Fig.10 shows a takeoff of this power compared to the line which form the experimental points. This result comes from the diminution of the core resistivity under the magnetic induction effect.

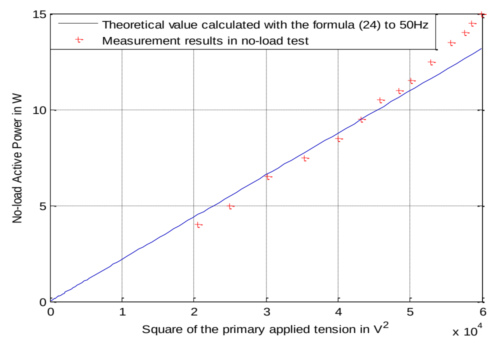


***Fig. 10:*** *Active Power in function of Square of Primary Applied Voltage*

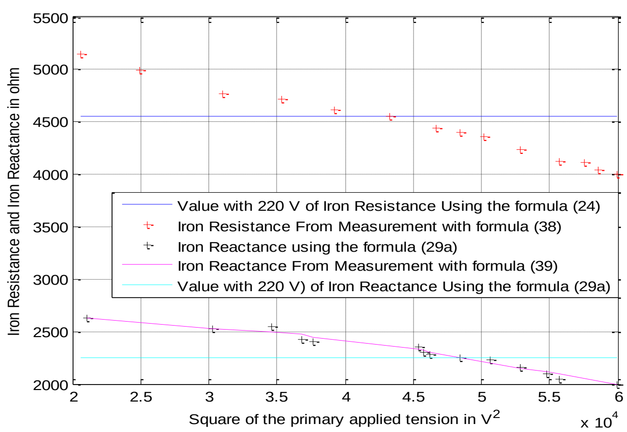


***Fig. 11:*** *Iron Resistance and Iron Reactance in function of Square of Primary Applied Voltage*

The characteristic values with 220 V of the central branch elements are:

$R_{Fe} = 4558\ \Omega$

$X_{Fe} = 2250\ \Omega$

These values show well that the no loaded transformer is an inductive device.

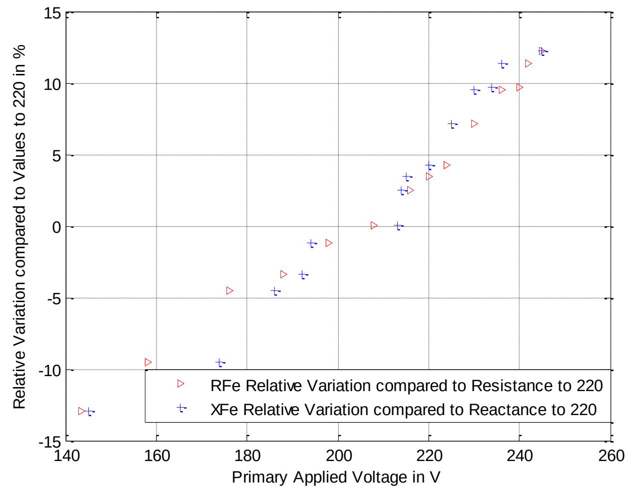


***Fig. 12:*** *Relative Variation compared to Values to 220 V of Iron Resistance and Reactance of the transformer under study*

The relative variations lie between -15% and 15%. This result shows the importance of the magnetic state of the core role in the behavior of the electrons in this one. Hysteresis could also influence the results.

## 4 Conclusion and Future Work

A modeling method of the Steinmetz equivalent diagram elements corresponding to the Eddy current losses from a single-phase transformer is proposed. The analytical calculations were carried out based on the transformer shell form. The obtained results can be employed for the calculation of the active loss in an electromagnetic domain of other forms such core form and toroidal form. However, the results do not include the hysteresis loss. The characteristic of alloy constituting the core is not given. Collaboration with manufacturers of sheets for the electric machines to have fuller information is necessary. A good correlation between the practical and theoretical results was verified with the proposed approach. The iron resistance and reactance are clarified analytically according to the geometrical and electromagnetic data of the core. Consequently, using the different powers in no load and short-circuit test, the article expressed the elements empirically given by Steinmetz. Finally, all the elements of the Steinmetz equivalent diagram can be calculated analytically. In the future, we expect to extend the proposed model to the larger fields of applications as electrical rotating machines modeling.

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