INTRODUCTION:
The word Cryptography comes from the Greek word Kryptos which means hidden and graphein means 'to write'. Cryptography is the branch in which we study some techniques to convert our original message to such a message which could not be easily understandable without providing some additional information or formula. Original message which is to be converted to hidden form is called as plain text and the converted form is called as cipher text. The conversion of plain text to cipher text is called as encryption and the conversion from cipher text to plain text is called as decryption. After demonetization in 2016 people prefers cashless transactions like ATM, PayTM, Mobile banking, internet banking, etc. Password or Code is necessary to operate all these facilities. In Mathematics there are some integral transforms like Laplace transform, Sumudu transform, Fourier transform etc. Integral transforms play an important role in the process of encryption and decryption. The main aim of this paper is to present method to create such a confidential text so that communication between two persons will be secured. Integral transforms play an important role in solving differential and integral equations and also in other sciences. In the process of cryptography there is a contribution of some integral transforms like Laplace transform, Sumudu transform, and Elzaki transform. There are various kinds of techniques for the process of encryption and decryption found in literature [2], [4], [8]. A new cryptographic scheme was developed by applying L.T. to hyperbolic sine and cosine functions [2], [4]. In this paper we have applied Laplace transform and Inverse Laplace transform to trigonometric sine function to derive such results regarding encryption decryption.

1. Some important definitions & theorems
Def. 1.1.1 [6] we define Laplace transform of $g(x)$ by
$$L[g(x)] = F(p) = \int_0^\infty e^{-px} g(x) \, dx \; , \; \text{Re} (p)>0$$

Where $e^{-px}$ the kernel of this transform and $p$ is the transform variable which is a complex number.
Def. 1.1.2: If $F(p)$ is the Laplace transform of $f(x)$ then the inverse Laplace transform of $F(p)$ is $f(x)$ and we write $L^{-1}[F(p)] = f(x)$.
Def. 1.1.3: [3] [The relation of congruent modulo n]
Let $n$ be a positive integer. Then an integer $a$ is congruent to an integer $b$ modulo $n$ if $n$ divides $a - b$. If $a$ is congruent to $b$ modulo $n$ then symbolically we write $a \equiv b \pmod{n}$. If $a$ is not congruent to $b$ modulo $n$ then we denote it as $a \not\equiv b \pmod{n}$.

Theorem 1.1.4[2] Let $H_0, H_1$
$$H_i, H_{i+1}, ..., \text{be coefficients of } t^2 \sinh 2t \text{ then given plaintext in terms of } H_i \; i=0, 1, 2, 3, 4,... \text{ under Laplace transform of } H^2 \sinh 2t \text{ can be converted to cipher text } H_i' = r_i \cdot 26k_i \; \text{for } i=0, 1, 2, 3, 4,... \text{ where } r_i = 2^{2i+1}(2i+2)(2i+3) \; \text{H}_i \; \text{for } i=0, 1, 2, 3, 4... \text{ and a key is given by } k_i = \frac{r_i - H_i}{26} \text{ for } i=0, 1, 2, 3, 4...$$

Theorem 1.1.5[2] The given cipher text in terms of $H_i'$ With a given key $k_i$ for $i=0, 1, 2, 3, 4,...$ can be converted to plain text $H_i$ under the inverse Laplace transform of
$$H_i' = 2^{2i+1}(2i+2)(2i+3)$$

2. Methodology for encryption & decryption
In this Paper we will use the method to convert the given plain text in to such a hidden text which could not Possible to crack without key by operating Laplace transforms. Suppose that we are given A B C D E F G H.........Z as a plain text. In the first step we have to give the following allotment to letters in the given plain text.
A $\rightarrow$ 0, B $\rightarrow$ 1, C $\rightarrow$ 2, D $\rightarrow$ 3, E $\rightarrow$ 4, F $\rightarrow$ 5, G $\rightarrow$ 6, H $\rightarrow$ 7, I $\rightarrow$ 8, J $\rightarrow$ 9, K $\rightarrow$ 10, L $\rightarrow$ 11, M $\rightarrow$ 12, N $\rightarrow$ 13, O $\rightarrow$ 14, P $\rightarrow$ 15, Q $\rightarrow$ 16, R $\rightarrow$ 1, S $\rightarrow$ 19, T $\rightarrow$ 20, V $\rightarrow$ 21, W $\rightarrow$ 22, X $\rightarrow$ 23, Y $\rightarrow$ 24, Z $\rightarrow$ 25
Consider the trigonometric sine series given by
$$\sin ny = ny - \frac{n^3y^3}{3!} + \frac{n^5y^5}{5!} - \frac{n^7y^7}{7!} + \cdots$$
$$\sinh ny = ny + \frac{n^3y^3}{3!} + \frac{n^5y^5}{5!} + \frac{n^7y^7}{7!} + \cdots$$
\begin{equation}
(1)
\end{equation}
Let $k_0, k_1, k_2, ... k_i$ be the coefficients of the $eq^n$ (1) We write
By operating L.T. to eq^n (2) we will obtain one equation having some new variable in denominator and some values in the numerator (we call them as resulting values say \( r_i \)) adjusting these resulting values such that \( r_i \equiv K_i \mod 26 \) for \( i = 0, 1, \ldots, j \) we obtain \( K_i' \) which is our required cipher text. As decryption is the reverse process of encryption we can obtain plain text by applying I.L.T. of \( L[\text{Kym} \sin n y] \). To determine cipher text by applying L.T. to trigonometric cosine function we may use the above method by considering some series of the form \( \text{Kym} \cos ny \).

3. Example Let us consider one plain text given below

R    E    L    I    A    N    C    E

and by methodology 1.1.6 we will convert this plain text to cipher text. First suppose by our allotment given plain text be equivalent to 17 4 11 8 0 13 2 4

Suppose that \( K_0 = 17, K_1 = 4, K_2 = 11, K_3 = 8, K_4 = 0, K_5 = 13, K_6 = 2, K_7 = 4 \) be the coefficients of the eq^n (1)

Case 1. In this case by taking \( m=1&n=1 \) equation (1) becomes

\[
\text{Kysiny} = 17y^2 - \frac{4}{13}y^4 + \frac{11}{15}y^6 + \frac{8}{17}y^8 + \frac{0}{19}y^{10} + \frac{13}{11}y^{12} + \frac{2}{13}y^{14} - \frac{4}{15}y^{16}
\]

Applying Laplace transform to both sides we have

\[
L[\text{Kysiny}] = L[17y^2] - \frac{4}{13}L[y^4] + \frac{11}{15}L[y^6] - \frac{8}{17}L[y^8] - \frac{0}{19}L[y^{10}] - \frac{13}{11}L[y^{12}] + \frac{2}{13}L[y^{14}] - \frac{4}{15}L[y^{16}]
\]

\[
L[\text{Kysiny}] = \left[ \frac{34}{p^3} - \frac{16}{p^5} + \frac{66}{p^7} - \frac{64}{p^9} + \frac{0}{p^{11}} - \frac{15}{p^{13}} + \frac{156}{p^{15}} + \frac{28}{p^{17}} - \frac{64}{p^{19}} \right]
\]

Let us assume that \( r_0 = 34, r_1 = -16, r_2 = 66, r_3 = -64, r_4 = 0, r_5 = -156, r_6 = 28, r_7 = -64 \) and we will obtain \( k_1 \) such that

\[
r_1 \equiv k_1 \mod 26 \text{ as follows}
\]

34 ≡ 8 mod 26, -16 ≡ 10 mod 26, 66 ≡ 14 mod 26, -64 ≡ 14 mod 26, 0 ≡ 0 mod 26, -156 ≡ 14 mod 26

28 mod 2 = 14, 64 mod 2 = 14, 64 mod 2

Let \( K_0' = 8, K_1' = 10, K_2' = 14, K_3' = 14, K_4' = 0, K_5' = 0, K_6' = 2, K_7' = 14 \) Thus we have converted given plain text

R    E    L    I    A    N    C    E    to cipher text i.e. in secret form as

B    M

Hence in general we have

**Theorem (3.1)**

Under the Laplace transform of \( \text{Kysiny} \) can be converted to cipher text \( K_1', K_2', \ldots, K_7' \) where \( r_1 = (1)^{2i}26c_i \) and key is given by

\[
C_i = \frac{r_i - K_i'}{26} \text{ for } i = 0, 1, 2, 3, 4, \ldots, j
\]

We will now convert the obtained cipher text in to plain text by applying the same methodology.

Applying Inverse Laplace transform to equation (2) we have

\[
L^{-1}[L\{\text{Kysiny}\}] = L^{-1}\left(\frac{34}{p^3} - 16L^{-1}\left(\frac{1}{p^5}\right) + 66L^{-1}\left(\frac{1}{p^7}\right) - 64L^{-1}\left(\frac{1}{p^9}\right) + 0L^{-1}\left(\frac{1}{p^{11}}\right) - 156L^{-1}\left(\frac{1}{p^{13}}\right) + 28L^{-1}\left(\frac{1}{p^{15}}\right) - 64L^{-1}\left(\frac{1}{p^{17}}\right)\right]
\]

\[
= 17 4 11 8 0 13 2 4 \text{ i.e. R E L I A N C E}
\]

From the above table (4.1) we may generalize the result on encryption given below.

**Theorem (3.2)**

The given cipher text \( K_i' \) with a given key \( C_i \) can be converted to plain text \( K_1, K_2, \ldots, K_7 \) under the I.L.T. of \( L[\text{Kysiny}] \) as follows

\[
L^{-1}[L\{\text{Kysiny}\}] = L^{-1}\left(\sum \frac{(-1)^{j}}{p^{2j+2}}\right) \text{ where } K_j = (1)^{2i}26c_j \text{ and key is given by}
\]

\[
C_i = \frac{r_i - K_i'}{26} \text{ for } i = 0, 1, 2, 3, 4, \ldots, j
\]

We will now convert the obtained cipher text in to plain text by applying the same methodology.

Applying Inverse Laplace transform to the above equation we have

\[
L[\text{Kysiny}] = L[17L\{y^2\}] - \frac{8}{17}L\{y^4\} + \frac{6}{17}L\{y^6\} - \frac{2}{17}L\{y^8\} + \frac{9}{17}L\{y^{10}\} - \frac{21}{17}L\{y^{12}\} - \frac{31}{17}L\{y^{14}\} - \frac{31}{17}L\{y^{16}\}
\]

Operating Laplace transform to the above equation we have

\[
L[\text{Kysiny}] = \left[ \frac{68}{p^3} - \frac{32}{p^5} + \frac{192}{p^7} - \frac{1024}{p^9} + \frac{5120}{p^{11}} - \frac{24576}{p^{13}} + \frac{114688}{p^{15}} \right]
\]

\[
= \frac{524288}{p^{17}}
\]

Let us assume that \( r_0 = 68, r_1 = -32, r_2 = 192, r_3 = -1024, r_4 = 5120, r_5 = -24576, r_6 = 114688, r_7 = -524288 \) and we will obtain \( k_i' \) such that

\[
r_1 \equiv k_i' \mod 26 \text{ as follows}
\]

68 ≡ -10 mod 26

-32 ≡ -6 mod 26

192 ≡ 10 mod 26

Thus we have converted given plain text

R    E    L    I    A    N    C    E    to cipher text i.e. in secret form as

| \( i \) | \( K_i \) | \( r_i \) | \( C_i \) | \( K_i' \) = \( n_i = 26C_i \)
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From the above table we may generalize the result on encryption and decryption given below.

**Theorem (3.3)** Let $K_0, K_1, \ldots, K_n$ be coefficients of $Ky.\sin y$. Then the given plain text in terms of $K_i$ can be transformed to cipher text $K_i' = r_i - 26C_i$ by applying Laplace transform to $Ky.\sin y$ where

$$r_i = (-1)^i2^{2i+1}(2i + 2)K_i$$

and key is given by $C_i = \frac{r_i - K_i'}{26}$ for $i = 0, 1, 2, 3, 4, \ldots$. 

Theorem (3.4) The given cipher text $K_0', K_1', \ldots, K_n'$ with a given key $C_i$ can be converted to plain text $K_i$ under the inverse Laplace transform of $L[Ky.\sin y]$ where

$$K_i = (-1)^i\left[\frac{26C_i + K_i'}{2^6(2i + 2)}\right]\text{ Where } i = 0, 1, 2, 3, \ldots, j$$

From the above two cases and derived theorems (2.1) to (2.4) we make more generalization regarding encryption and decryption as follows.

**Theorem (3.5)** Let $K_0, K_1, \ldots, K_n$ be coefficients of $y^{m} \sin ny$. Then the given plaintext in terms of $K_i$ under the Laplace transform of $K^{y^{m}} \sin ny$ can be converted to cipher text $K_i' = r_i - 26C_i$ where

$$r_i = (-1)^in^{2i+1}(2i + 2)(2i + 3) \ldots \ldots (2i + m + 1)K_i$$

and key is given by $C_i = \frac{r_i - K_i'}{26}$ for $i = 0, 1, 2, 3, 4, \ldots, j$.

Theorem (3.6) The given cipher text $K_0', K_1', \ldots, K_n'$ with a given key $C_i$ can be converted to plain text $K_i$ under the inverse Laplace transform of $L[Ky^{m} \sin ny]$ where

$$K_i = (-1)^i\left[\frac{26C_i + K_i'}{2^6n^{2i+1}(2i + 2)(2i + 3) \ldots \ldots (2i + m + 1)}\right]\text{ Where } i = 0, 1, 2, 3, \ldots, j$$

4. Conclusions

From the above theory part proved it is clear that by changing values of $m$ & $n$ we will obtain different cipher texts for given plain text which cannot be easily cracked without providing some additional information. Thus Laplace transform with cryptography plays an important role in communication security.

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5. References


