Two Point Acceptance Sampling Plan By Variable Using Range

S.Geetha, S.Pavithra, Dr. S.Geetha, S.Pavithra

Abstract: Product control is a methodology in statistical quality control in which the decision is made on the finished products, the decision is about accepting or rejecting the lot. Sampling inspection by variable is one of the product control technique, in which the samples drawn from the lot is subjected to detect defects and the decision of acceptance or rejection of the lot is based on the quality characteristic of the product. These characteristic is based on the assumption that the functional form of probability distribution must be known, the upper or lower specification should be specified and the quality characteristic is assessable on continuous scale. Further, the variable sampling plan is divided into known standard deviation and unknown standard deviation. In many industrial situations it is hard to calculate standard deviation, which makes variable sampling plan impractical. In this paper single sampling plan by variable using range is discussed by specifying two points on OC curve. Also plan parameters are determined for the specified requirements in terms of producers and consumer risk.

KeyWord: Acceptance sampling plans, Consumer's risk, Normal distribution, Producer's risk, Range, single sampling inspection, Variable sampling plan.

1. INTRODUCTION
Acceptance sampling plan are used as a statistical tools in industries for taking decision about the disposition of lots. The suitable sampling plans are used to make decision either accept or reject the lot. The sampling plans are classified into two types, sampling inspection by attributes and sampling inspection by variable. The sampling inspection by attribute checks whether an item is conforming or non-conforming. Attribute sampling is easy to supervise and perform. In sampling inspection by variable; the quality characteristic measurement is recorded for each item; the quality characteristic is assessable on the continuous scale and their functional form of the probability distribution should be known. Variable sampling plan generates more information about each item of the lot. The smaller sample of variable sampling gives same protection as compared to the larger sample of attribute sampling. (see Montgomery[1] and Bowker and Goode[2]). The variable sampling is generally classified into known standard deviation and unknown standard deviation which has been studied by many researchers. Some of their works on sampling inspection by variables are seen in Hamaker [3], Schilling [4], Liberman and Resnikoff [5], Owen [6,7]. In some industrial situation, the range can be used. The studies based on range are also found in the Literature of acceptance sampling J.Gulde [8], W.Grant Ireson and J. Resnikoff [9], Daly,J.F[10], Patnaik P.B[11], Pearson E.S[12], Cadwell.J.H[13], are some reference which deals with variable sampling inspection using Range. The calculation of the standard deviation is tedious in many industrial situations. Many standard procedure and estimation process is needed in the case of standard deviation but the same precision can be obtained by using range which is easy to calculate. J.Gulde [8] presented a detailed work related to acceptance sampling by variable using the range. In this paper, a study on single sampling plans by variables using range is formulated. A procedure for determining the parameters for specified requirements are given in terms of consumer's and producer's risk.

2. Single sampling inspection by variable using Range
The following assumption defines the single sampling inspection by variables.

1. The quality characteristic is assessable on continuous scale and the functional form of the probability distribution of quality characteristic should be known.

2. Every unit should have one-sided specification say, upper specification limit U or lower specification limit L. An item is defective if X>U (or X<L), otherwise the item is non-defective.

The operating procedure for variable sampling plan using range is as follows

Step 1. Draw a random sample from the lot and observe the measurement for each item of the random sample.

Step 2. Calculate the Mean $\bar{X}$ and Mean Range $\bar{R}$ of the sample.

Step 3. If $\bar{X} + k\bar{R}/d_2 \leq U$ or $(\bar{X} + k\bar{R}/d_2 \geq L)$, the lot is accepted, otherwise the lot is rejected.

The single sampling inspection plan by variable using range is labeled by three parameters namely number of subgroups m, acceptability constant k and the sample size n. The expression for n,m and k are derived by specifying two point, namely $(p_1, 1 - \alpha)$ and $(p_2, \beta)$. Where $p_1$ and $p_2$ are termed as Acceptable Quality Level(AQL) and Limiting Quality Level (LQL) with respect to the consumers and producers risk.

3.OPERATING CHARACTERISTIC FUNCTION
A important measure of the performance of a sampling plan using variable is its operating characteristic (OC) function; it is function of non-conforming units of the proportion p and it provides a probability of acceptance of lot $P_a(p)$. The plot of p against $P_a(p)$ results in a curve called the operating characteristic curve. For a given U (upper specification limit) when range is known, p and $P_a(p)$ are defined by

$$p = P(X > U \mid \mu)$$

(1)

$$P_a(p) = P(\bar{X} + k\bar{R}/d_2 \leq U \mid \mu)$$

(2)
Acceptance Quality Level(AQL) and Limiting Quality Level(LQL), using (1), are defined by

\[
p_1 = P(X > U | \mu_1) \]

\[
p_2 = P(X > U | \mu_2) \]

where \(\mu_1\) and \(\mu_2\) are the means of the distribution which results in AQL and LQL, respectively.

Assume that \(X\) is given by normal distribution with parameter \(\mu\) and \(\sigma^2\). Then, its Probability density function and cumulative distribution function are respectively, given by

\[
f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x-\mu}{\sigma} \right)^2} \]

(4)

The AQL and LQL for standard normal variate is given as

\[
p = P(Z > k^*_p) \]

(5)

\[
p_1 = P(Z > k^*_p) \]

(6)

\[
p_2 = P(Z > k^*_p) \]

(7)

where \(z = (X - \mu) / \sigma\), \(k^*_p = (U - \mu) / \sigma\)

\(k^*_p = (U - \mu_1) / \sigma\), \(k^*_p = (U - \mu_2) / \sigma\)

The producers risk \(\alpha\) and the consumers risk \(\beta\) are defined as

\[
\alpha = P(X + kR / d_2 \leq U | \mu = \mu_1) \]

(8)

\[1 - \beta = P(X + kR / d_2 \leq U | \mu = \mu_2) \]

(9)

4. Designing single sampling inspection plan by variable using Range

In many industrial practice the variable sampling plan using standard deviation is more complicated. In such situation Range is used instead of standard deviation. Gulde [8] has discussed the distribution of range as \(x + kR / d_2\). The distribution is very close to normal, as the sample taken from the population is assumed to be normal.

The distribution of \(x + kR / d_2 \leq U\) is a linear combination of the independent variables, one variable is distributed normally and other variable is close to normal which has been discussed in Pearson.E.S [14]. The average \(x\) and mean range \(R\) are used as the base for acceptance or rejection. The mean range is range of the entire sample n, or it may be divided into equal subgroups 'm'. Larger the subgroup size \(m_1\), the larger is the mean range. \(d_2\) is used to compensate the value, it is calculated from the distribution of ranges of sample of the size which is drawn from normal distribution and its standard deviation is \(\sigma \sqrt{\frac{1}{n} + \frac{k^2}{md_2}}\). Let \(m\) be the number of subgroups, \(n\) is the sample size.

In this case, the determination of \(m, n\) and \(k\) is based two points \(p_1\) and \(p_2\), were the subgroup size \(m_1\) is assumed and the \(k_2\) and \(d_2\) varies based on \(m_1\). The \(k_2\) and \(d_2\) for various subgroup size given in Table 1.

<table>
<thead>
<tr>
<th>Subgroup size (m_1)</th>
<th>(d_2)</th>
<th>(k_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.1838</td>
<td>0.8525</td>
</tr>
<tr>
<td>3</td>
<td>1.69257</td>
<td>0.8884</td>
</tr>
<tr>
<td>4</td>
<td>2.05875</td>
<td>0.8798</td>
</tr>
<tr>
<td>5</td>
<td>2.32593</td>
<td>0.8641</td>
</tr>
<tr>
<td>6</td>
<td>2.53441</td>
<td>0.8480</td>
</tr>
<tr>
<td>7</td>
<td>2.70436</td>
<td>0.833</td>
</tr>
<tr>
<td>8</td>
<td>2.84720</td>
<td>0.820</td>
</tr>
<tr>
<td>9</td>
<td>2.97003</td>
<td>0.808</td>
</tr>
<tr>
<td>10</td>
<td>3.07751</td>
<td>0.797</td>
</tr>
</tbody>
</table>

It is known that \(k^*_p = \frac{U - \mu}{\sigma}\), for the specified \(p_1\) and \(p_2\)

\[
\mu_1 = U - k^*_p \sigma \]

(10)

\[
\mu_2 = U - k^*_p \sigma \]

The operating characteristic function \(P_a(p)\) for a range plan is expressed as

\[
P_a(p) = P(Y \leq U / P) = P(Y \leq U / P) \]

(11)

The \(\alpha\) and \(\beta\) are the consumer and producer risk for AQL and LQL. \(k_\alpha\) and \(k_\beta\) are the normal deviates of \(\alpha\) and \(\beta\) respectively defined by

\[
k_\alpha = \frac{U - (\mu_1 + kR / d_2)}{\sigma \sqrt{\frac{1}{n} + \frac{k^2}{md_2}}} \]

(12)

\[
k_\beta = \frac{U - (\mu_2 + kR / d_2)}{\sigma \sqrt{\frac{1}{n} + \frac{k^2}{md_2}}} \]

(13)

On substituting the expressions (10) for \(\mu_1\) and \(\mu_2\), (12) would result in the following

\[
k^*_p = k + k_\alpha \sqrt{\frac{1}{n} + \frac{k^2}{md_2}} \]

(13)

\[
k^*_p = k - k_\beta \sqrt{\frac{1}{n} + \frac{k^2}{md_2}} \]
The mathematical expression for \( n, m \) and \( k \) of a range plan for specified requirements in terms of point \((p_1, 1 - \alpha)\) and \((p_2, \beta)\) are given as

\[
m = \left( \frac{k_a^2 k_{md}^2}{md} \right)^{1/2} m_1\]

\[(14)\]

\[
k = \frac{k_a}{k_p} + \frac{k_m}{k_p} + \frac{k_d}{k_p}
\]

\[(15)\]

\[
n = m^* m_1
\]

\[(16)\]

5. NUMERICAL EXAMPLE

Suppose that the subgroup size \((m_1)\) is taken as 3 and the \(d_2 = 1.69257\) and \(k_2 = 0.8884\) are based on subgroup size \((m_1)\). The producer and consumer protection \((p_1 = 0.01, \alpha = 0.05)\) and \((p_2 = 0.06, \beta = 0.10)\), corresponding to \(p_1\) and \(p_2\), the values of \(k_{p_1}^*\) and \(k_{p_2}^*\) are determined, respectively as 3.6970 and 1.7828. The normal deviates \(k_\alpha\) and \(k_\beta\) are obtained as 1.64485 and 1.28155 for specified values of \(\alpha\) and \(\beta\). Substituting these values in \(14), (15)\) and \(16)\) the parameters of the desired plan are determined as \(m=19, k=1.8927\) and the sample size \(n=57\) is obtained.

6. CONCLUSION

Sampling inspection by variables is the procedure used for taking decision about the lot based on the samples taken from the lot. The difficulty in variable sampling is that the calculation of standard deviation which make the variable sampling complicate for practitioners. Hence it is necessary to develop the sampling plan based on the other measure of dispersion. Range is the one that replace standard deviation which makes the practitioners to use in inspection plans because of their easy calculation. In this paper sampling inspection plan by variables using range is developed. A procedure is developed for determining the parameter of this plan when requirements for producer's and consumer's protection are specified.

REFERENCES