USING ADAPTATION - BASED TEACHING INTO TEACHING THE PARAMETRIC EQUATION OF

A STRAIGHT LINE IN A PLANE

Nguyen Phu Loc, Nguyen Thi Anh Hang

Abstract: The famous psychologist J. Piaget was a founder of cognitive constructivism. He stated that the knowledge of human beings is “constructed” through experience, but not from the information they are given. He asserted that the cognitive development of learner passed through two processes: assimilation and accommodation (commonly called adaptation). From the above idea of Piaget, N.P. Loc (2019) proposed a six – phase teaching model (called adaptation – based teaching, or ABT) as follows [5]: Phase 1: Recalling relevant known knowledge; Phase 2: Inputting information (or an action request) and assimilating; Phase 3: Creating cognitive conflict; Phase 4: Accommodating; Phase 5: Testing and evaluating; Phase 6: Correcting learning knowledge.

In this study, we wanted to verify the effectiveness of applying the ABT model of Nguyen Phu Loc (2019) to teaching a mathematics content in Geometry 10 of secondary schools in Vietnam.

1. INTRODUCTION

Constructivism is a theory that considers learning to be a positive process, where learners build and construct (internalize) new concepts, ideas and knowledge based on their present and past experiences. Learning is not a passive acquisition of learned knowledge, but a process whereby learners create their own knowledge (A Brief Critical dictionary of Education). There were two kinds of constructivism that educators in the world paid much attention to study. The first one was social constructivism; another one was cognitive constructivism which J. Piaget was considered as a founder [8]. Jean Piaget (1953) believed that the cognitive development of learner is an adaptive process which consists of assimilation and accommodation. Making use of an adaptive idea of Piaget, some educators suggested teaching models to activate students in learning new knowledge in which students self – constructed their understanding. Claudia J. Stanny [7] suggested that effective learning activities should include: activate prior knowledge; create surprise; apply and evaluate the new knowledge; include a reflective closing assignment.

Hartle et al (2012) argued that teaching methods based on adaptation have 4 characteristics: eliciting prior knowledge; creating cognitive dissonance; applying new knowledge with feedback; reflecting on learning [4]. Also approaching to an adaptive idea of Piaget, N.P. Loc (2019) proposed a six – phase teaching model (called adaptation – based teaching, or ABT) as follows [5]: Phase 1: Recalling relevant known knowledge; Phase 2: Inputting information (or an action request) and assimilating; Phase 3: Creating cognitive conflict; Phase 4: Accommodating; Phase 5: Testing and evaluating; Phase 6: Correcting learning knowledge.

In this study, we wanted to verify the effectiveness of applying the ABT model of Nguyen Phu Loc (2019) to teaching a mathematics content in Geometry 10 of secondary schools in Vietnam.

2. THE RESEARCH QUESTION

In the case of applying the ABT model into teaching the parametric equation of a straight line in a plane (Geometry 10 - Hinh học 10), two questions are:
1. How will the teaching process be implemented?
2. How effective will the teaching process be?

3. METHODOLOGY

- Design the lesson: Based on the content of the parametric equation of a straight line in a plane in “Geometry 10” textbook [3] for Vietnamese students, we design the lesson according to 6 phases of ABT.
- Field test: Implement the lesson in a “10” class in a high school in Vietnam and evaluate the learning results of students.

4. DESIGNING THE LESSON

Learning objectives
At the end of a teaching process, the students are able to write the parametric equation of a straight line in a plane if elements to determine a straight line are given.

Teaching facilities
- Learning sheets: 2 sheets (1;2) for supporting knowledge construction and 1 testing sheet (3).
Teaching method
- Students' learning activities: students worked in groups.

Using the adaptation – based teaching model as follows:
- Phase 1: Recalling relevant known knowledge;
- Phase 2: Inputting information (or an action request) and assimilating;
- Phase 3: Creating cognitive conflict;
- Phase 4: Accommodating;
- Phase 5: Testing and evaluating;
- Phase 6: Correcting learned knowledge

Phase 1:
Before this lesson, relevant knowledge that students already knew: the directional vector of a straight line, how to determine a straight line, coordination expression of a vector, how to find coordinates of a vector, coordinates of two equal vectors.

In order to help students remember the above knowledge, the teacher offers them Learning Sheet 1 consisting of 4 questions; Students find answers in their group (see Table 1)

**Table 1: Learning Sheet 1 for recalling the relevant knowledge**

<table>
<thead>
<tr>
<th>Questions</th>
<th>Expected answers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Question 1. Given two straight lines parallel (d) and (d'), observing the picture and show directional vectors of them</td>
<td>(d) and (d') get $\vec{u};\vec{v}$ as their directional vectors</td>
</tr>
<tr>
<td>Question 2. Given $A(x_a; y_a), B(x_b; y_b)$. Find the coordinates of the vector $\overrightarrow{AB}$?</td>
<td></td>
</tr>
<tr>
<td>Question 3. Given $\overrightarrow{u}(x; y), \overrightarrow{v}(x'; y')$. In which conditions for $\overrightarrow{u} = \overrightarrow{v}$?</td>
<td></td>
</tr>
<tr>
<td>Question 4. Construct a straight line (d) such that (d) passes through $M$ and $\vec{u}$ is its directional vector.</td>
<td></td>
</tr>
</tbody>
</table>

Phase 2:

**Fig 1. For question 5-Phase 2**

Question 5. Observe the picture (see Fig 1), out of vectors $\overrightarrow{M_0A}; \overrightarrow{M_1B}; \overrightarrow{M_0C}; \overrightarrow{M_1D}$, which are the same direction as vector $\vec{u}$ - directional vector of (d)?

Expected answer
+ $\overrightarrow{M_0A}; \overrightarrow{M_1B}$ are the same direction as $\vec{u}$
+ $\overrightarrow{M_0C}; \overrightarrow{M_1D}$ are not the same direction as $\vec{u}$

Phase 3:

Question 6. Given a straight line (d) going through a point $M_0$ and getting $\vec{u}$ as a directional vector. Find conditions such that any point $M$ (d) is on (d)?

Expected answer

$M$ is on (d) if only if $\overrightarrow{M_0M}$ is the same direction as $\vec{u}$.

Teacher: Give students a problem: In plane Oxy, given a straight line $\Delta$ passing through $M_0(x_0; y_0)$ and getting $\vec{u}(a;b)$ as a directional vector. Find conditions $X$ and $Y$ such that $M(x, y)$ are on $\Delta$.

Students discuss in groups to find the answer.

Expected answer:

\[ M \in \Delta \iff \overrightarrow{M_0M} \text{ is the same direction as } \vec{u} \iff \overrightarrow{M_0M} = t\vec{u} \]

\[ \iff \begin{cases} x - x_0 = ta \\ y - y_0 = tb \end{cases} \iff \begin{cases} x = x_0 + ta \\ y = y_0 + tb \end{cases} \tag{1} \]

Teacher: Equation (1) is called the equation of the straight line $\Delta$, where $t$ is a parameter.

Teacher: Given a specific value of $t$, we can determine a specific point on $\Delta$.

Imbalance happens to students. The parametric equation of a straight line is a concept completely new to them. The problem is that in order to write the equation of a straight line, what elements do we need to have.

Phase 4:

Teacher: Equation (1) is completely determined if you know a point in a straight line and a vector of the direction of it.

Students realize that according to (1), we can write a parametric equation of a straight line $\Delta$ if we have two the following things: a point $M_0(x_0; y_0)$ in it and its directional vector $\vec{u}(a;b)$; in this case: $\begin{cases} x = x_0 + ta \\ y = y_0 + tb \end{cases}$ is the equation of $\Delta$.

Teacher: Give students additional examples to illustrate how to write the parametric equation of a straight line.

Teacher: Distribute Learning Sheet 2 (see Table 2), ask students to work in groups (to help them construct a correct schema of a parametric equation of a straight line)

**Table 2: Learning Sheet 2**

<table>
<thead>
<tr>
<th>Questions</th>
<th>Expected answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Question 1: Given a straight line (d): $\begin{cases} x = 5 - t \ y = -2 + 3t \end{cases}$</td>
<td></td>
</tr>
<tr>
<td>a) Find one point on the line (d) and one directional vector of (d)?</td>
<td></td>
</tr>
<tr>
<td>b) Out of $A(4; -2), B(4; 1)$ which points are in (d)?</td>
<td></td>
</tr>
</tbody>
</table>
Question 2: In order to write the parametric equation of a straight line $\Delta$, what factors do we need to know?

Expected answer
If knowing one point on $\Delta$, And one directional vector of it.

Question 3: Write the equation of a straight line $\Delta$ in the case of:
a) $\Delta$ passing through $A(3;7)$ and getting $\overrightarrow{u}(-2;5)$ as a directional vector.
b) $\Delta$ gets $MN(4;3)$ as a directional vector.

Passing through $M(0;2), N(4;1)$

a) $\Delta : \begin{cases} x = 3 - 2t \\ y = 7 + 5t \end{cases}$
b) $\Delta : \begin{cases} x = 4t \\ y = -2 + 3t \end{cases}$

Question 4: Given a straight line $(d)$: $\begin{cases} x = 4 + 8t \\ y = 5 - 7t \end{cases}$ Write the parametric equation of $\Delta$ which passes through $A(1;-2)$ and is parallel to $(d)$.

Expected answer
A directional vector of $(d)$ is $\overrightarrow{u}_d(8;-7)$.

Because $(\Delta \parallel (d))$, $(\Delta)$ gets $\overrightarrow{u}_\Delta = \frac{\overrightarrow{u}_d}{||\overrightarrow{u}_d||} = (8;-7)$ as a directional vector.

Hence, $\Delta : \begin{cases} x = 1 + 8t \\ y = -2 - 7t \end{cases}$

Phase 5:
Teacher: Constructing multiple choice questions to test students’ understanding. Students: Writing their answers in Learning Sheet 3 (see Table 3).

Table 3: Learning sheet 3

<table>
<thead>
<tr>
<th>Question</th>
<th>Expected answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Given $\Delta: \begin{cases} x = 12 - 5t \ y = 3 + 6t \end{cases}$. A directional vector of $\Delta$ is:</td>
<td>B</td>
</tr>
<tr>
<td>A. $\overrightarrow{u}(12;13)$</td>
<td>B. $\overrightarrow{u}(-5;6)$</td>
</tr>
<tr>
<td>C. $\overrightarrow{u}(12;-5)$</td>
<td>D. $\overrightarrow{u}(6;5)$</td>
</tr>
<tr>
<td>2. The coordinates of a directional vector of a straight line which passes through two points $A(-3;2)$ and $B(1;4)$ are:</td>
<td>A</td>
</tr>
<tr>
<td>A. $(4;2)$</td>
<td>B. $(2;-1)$</td>
</tr>
<tr>
<td>C. $(1;-2)$</td>
<td>D. $(1;2)$</td>
</tr>
<tr>
<td>3. Given $\Delta: \begin{cases} x = 3 + 6t \end{cases}$. Out of the following points, which points are on $\Delta$?</td>
<td>D</td>
</tr>
<tr>
<td>A. $(7;5)$</td>
<td>B. $(20;9)$</td>
</tr>
<tr>
<td>C. $(12;0)$</td>
<td>D. $(2;15)$</td>
</tr>
<tr>
<td>4. The coordinates of a directional vector of a straight line parallel to Ox are:</td>
<td>C</td>
</tr>
<tr>
<td>A. $(0;1)$</td>
<td>B. $(0;-1)$</td>
</tr>
<tr>
<td>C. $(1;0)$</td>
<td>D. $(1;1)$</td>
</tr>
<tr>
<td>5. The parametric equation of the straight line passing through $A(3;-1)$ and $B(1;5)$ is:</td>
<td>D</td>
</tr>
</tbody>
</table>

Phase 6:
From the students ‘results of doing the test, the teacher finds students’ mistakes and errors and gives corrections to them. Comment Old knowledge is very important, is a basic element in teaching according to the approach of constructivist teaching, students can mobilize old knowledge in Phase 1 to give them a ready position in learning and at the same time, old knowledge is also a premise to create new knowledge.

+ The previous cognitive schemas of students only had: the concept of vector indicating the direction of the line, the coordinate expression of two vectors in the same direction. To develop a cognitive schema of parametric equations of straight lines requires students to use the two cognitive schemas above. + Through question 5 in Phase 2, students implicitly comment: If $A \in (d)$, $\overrightarrow{MN}$ is the same direction as the directional vector $\overrightarrow{u}$; if the point $A \notin (d)$ is $\overrightarrow{MN}$ is not the same direction as the directional vector $\overrightarrow{u}$. Question 6 clarifies the meaning of question 5 by students having to know how to analyze and synthesize to give answers to themselves. Through the guidance of teachers, students can build a formula to write parametric equations of a straight line when knowing the coordinates of the passing point and the coordinates of the directional vector of the straight line. At this time, students lose balance, cognitive conflict. The question is from equation (1) what can we know? What factors must we know in order to create a parametric equation for a straight line? + Students think, analyze, synthesize from that discovered from equation (1), we can show the coordinates of a directional vector ‘of the straight line, and the coordinates of the points in the straight line. To write a parametric equation of a straight line, you must know the coordinates of the passing point and the coordinates of a vector that indicates the direction of the straight line.

+ Through questions in Phase 4, students are exchanged to give their opinions in group discussion activities. After discussing, group members agree on opinions, present opinions of groups and listen to the opinions of other groups. Students comment that a straight line may have many different parametric equations, depending on the selection of the passing point and the directional vector. Students know fully and accurately about parametric equations of straight lines. At this time students adjust, change awareness, and build a new cognitive schema. + Phase 5, students apply a new cognitive schema to solve related exercises, make adjustments (if the new cognitive schema is
not correct). Students correct knowledge, engrave knowledge in an effective way.

Phase 6: through Phase 5, the teacher assesses the student's receptive level to make appropriate adjustments.

5 IMPLEMENTING THE LESSON AND RESULTS

Class to implement the lesson: Class 10A1, including 38 students, Thanh Dong High School, Kien Giang Province, Vietnam. Learning sheets: two for Phase 1, 2, 3, 4; one for Phase 5. Time: Starting from 14: 35 to 15: 05 (30 minutes) in March, 2019.

Groups of student:
Phase 1; 2; 3; 4: Students were divided into 6 large groups (including 4 groups of 6 and 2 groups of 7 students). They worked with learning sheets 1 and 2. Phase 5: In order to evaluate the learned results of students in a good way, the class was divided into 19 groups of 2, they answered questions in learning sheet 3.

The learning results of students

The student's results of answering questions in sheet 1

<table>
<thead>
<tr>
<th>Question</th>
<th>Right answer (The number of groups)</th>
<th>Wrong answer (The number of groups)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Question 1</td>
<td>6 (100%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Question 2</td>
<td>5 (83.3%)</td>
<td>1 (16.7%)</td>
</tr>
<tr>
<td>Question 3</td>
<td>6 (100%)</td>
<td>0(0%)</td>
</tr>
<tr>
<td>Question 4</td>
<td>6 (100%)</td>
<td>0(0%)</td>
</tr>
<tr>
<td>Question 5</td>
<td>5 (83.3%)</td>
<td>1 (16.7%)</td>
</tr>
<tr>
<td>Question 6</td>
<td>5 (83.3%)</td>
<td>1 (16.7%)</td>
</tr>
</tbody>
</table>

Table 4 showed us some information on the learning results of students as follows:
Question 1: All groups answer correctly because this is the content of knowledge they have learned in the previous section, so it is easy for students to give the correct answer.
Question 2: Most groups complete the answer well, except group 1 gets confused with formula.
Questions 3, 4: Groups complete the answer.
Question 5: 5 groups have successfully completed their answers because of this content they have learned in the previous section. As for group 6, who responded that \( \overrightarrow{OC} \), \( \overrightarrow{OD} \) were their directions different from \( \overrightarrow{u} \), they forgot the concept of two vectors not in the same direction.

The student's results of answering questions in sheet 2:

<table>
<thead>
<tr>
<th>Question</th>
<th>Right answer (The number of groups)</th>
<th>Wrong answer (The number of groups)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Question 1</td>
<td>6 (100%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Question 2</td>
<td>6 (100%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Question 3</td>
<td>6 (100%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Question 3b</td>
<td>5 (83.3%)</td>
<td>1 (16.7%)</td>
</tr>
<tr>
<td>Question 4</td>
<td>5 (83.3%)</td>
<td>1 (16.7%)</td>
</tr>
</tbody>
</table>

From Table 5, we had the following comments:
Question 1: All groups answered correctly. Students could read information from parametric equations of straight lines. They knew how to find a point in a straight line, whether a point belongs to a line or not. Question 2: The groups answered correctly because the students have just discussed, along with the guidance of teachers to create new knowledge for themselves. Question 3: All groups answer the first idea correctly because the students have just built a theorem, so it is easy for them to solve this problem. For the second idea, there are 5 groups that answered correctly; one group answered the parameter equation incorrectly, here while learning this error, students often made mistakes. They often did it quickly, so taking the coordinates was wrong between the points and the directional vector. And they did not check again when doing homework. To overcome this error, teachers should ask students to find the passing point and directional vector of the given line. Question 4: The groups found the passing point and the coordinates of the directional vector, and wrote the correct equation.

The student's results of answering questions in sheet 3

<table>
<thead>
<tr>
<th>Question</th>
<th>Right answer (The number of groups)</th>
<th>Wrong answer (The number of groups)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Question 1, 2, 3, 5</td>
<td>19 (100%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Question 4</td>
<td>15 (78.9%)</td>
<td>4 (21.1%)</td>
</tr>
<tr>
<td>Question 6</td>
<td>13 (68.4%)</td>
<td>6 (31.1%)</td>
</tr>
</tbody>
</table>

It can be see data in Table 6 that Questions 1, 2, 3, 5: All the groups answered correctly because they have just built their own cognitive schema of the parametric equation of a straight line. This question only needed to apply the knowledge we have just built. Question 4: All 14 groups got it right, students could remember the coordinates of the unit vector on the axis \( O_x \) and the unit vector on the axis \( O_y \). The remaining 5 groups are confused about the unit vector coordinates on the axis \( O_x \) and \( O_y \). Question 6: 10 teams got it right, based on the vector that students did not choose B, D or they changed the coordinates of point A to find the correct answer.

Through the statistics Table of learning sheet 3, we found out that teaching according to the constructivist approach to the theorem of parametric equations of the straight line, students
understood the lesson and knew how to apply the newly created knowledge to solve the problem.

Conclusion
Through the results of the pedagogical experiment presented above, we see that: If applying ABT to teaching, the teacher provides good opportunities for students to self-construct their knowledge. Students are active in a learning and teaching process; they carry out the actions of thinking such as: analysing; comparing, abstracting, generalising in order to enhance their knowledge. We believe that the ABT model can be used widely in teaching mathematics in particular, and in teaching science in common.

REFERENCES