2D-FEM Electromagnetic Characterization Of Quasi-TEM Coupled Elliptical Microstrip Line For Microwave Applications

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Abstract: In this paper, we present analysis and study of quasi-transverse electromagnetic (quasi-TEM) coupled elliptical microstrip line (CEML), convenient for microwave and RF applications, by using finite element method (FEM) in two dimensions (2D) under FreeFEM environment. Firstly, we have rigorously determined the quasi-TEM electromagnetic parameters (EM) of the CEML line. For this type of structure, there are no analytical or numerical results found in the scientific literature. Accordingly, to validate our results, we have adapted our 2D-FEM program to study the general configuration of the coupled cylindrical microstrip line (CCML), and excellent agreement is found between our FEM results and those obtained with other techniques of the scientific literature. Later, to show the practicality and the usefulness of our 2D-FEM results, we have successfully used the found quasi-TEM parameters to design a new microwave directional 20-dB, 50-Ω elliptical microstrip coupler (EMC) operating at 2-GHz, with high-performance in terms of isolation (53 dB), directivity (34 dB) and reflection (46 dB), realized in Matlab environment, convenient for microwave measurement telecommunication systems and radar applications. Finally, to reinforce our work, the directional EMC coupler’s coupling response obtained under Matlab environment was validated by our results obtained under CST Microwave Studio Software.

Index Terms: 2D-FEM electromagnetic characterization, Quasi-TEM coupled elliptical microstrip line, Elliptical microstrip directional coupler.

1 INTRODUCTION

Recently, elliptical and cylindrical microstrip lines have found widespread applications in both engineering communities and telecommunication systems. The main advantage of these types of lines over the conventional planar microstrip lines is the excitation of conformal antennas and arrays mounted on elliptical and cylindrical objects. Furthermore, these lines have been widely used in the design of directional couplers, filters, transition adaptors, impedance transformers, and medical applications. This has brought about the need for developing computer-aided design (CAD) programs for determining the quasi-TEM parameters of such structures. However, the analysis of passive components on the elliptical and cylindrical substrate with non-homogeneous media is not an easy task [1]. Several investigations for the electromagnetic characteristics of quasi-transverse electromagnetic elliptical and cylindrical microstrip lines have been reported in the literature. Until now, the finite difference time method [1]-[4] and method of moments [4]-[8] have been the most frequently used methods to study non-planar elliptical and cylindrical microstrip lines. In all the previous works, the capacitance (C) and the inductance (L), per unit length, of the elliptical microstrip lines have not been investigated, and the available expressions for characteristic impedance and effective permittivity existed in the scientific literature [7] contain many intermediate parameters and are so lengthy and cumbersome. The FEM method is well known to deliver optimal results for a wide range of structure parameters and over a very large band [9]-[14]. In the previous works, based on the FEM in 2D, we have successfully analyzed and delivered exact formulas for different types of TEM and quasi-TEM, planar and non-planar structures, and we have well explicited this method in references [9]-[14]. It is founded on solving Laplace’s equation in a TEM or a quasi-TEM approximation. The primary objective of this paper is the electromagnetic characterization of the open inhomogeneous quasi-TEM coupled elliptical microstrip line basing on the 2D-FEM technique under FreeFEM [15] environment, and to use the found 2D-FEM results to design a new conformal quasi-TEM elliptical microstrip directional coupler of high-performance in terms of directivity, isolation, and with minimum losses, widely requested in high-frequency telecommunication measurement systems, and microwave and radar applications. In this work, firstly, we have focused on studying the effect of the geometrical parameters on the quasi-TEM parameters, such as the strip-angle (θ) and the inner-conductor semi-major axis ratio (a1/a2), on the variation of the even- and the odd-mode characteristic impedances (Z_{e}, Z_{o}), the even- and the odd-mode effective dielectric constants (\varepsilon_{eff}, \varepsilon_{d}), the coupling coefficient (K), and the [L] and [C] matrices of the CEML line where:

\[
[L] = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}, \quad [C] = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}
\]

We can calculate the elements of the [C] matrix by using:

\[ C_{11} = C_{22} = \frac{C_{e} + C_{o}}{2} \]

\[ C_{12} = C_{21} = \frac{C_{e} - C_{o}}{2} \]

Where C_{e} and C_{o} are the capacitance per unit length of the structure, for the even and the odd modes respectively.

The elements of the matrix [L] are calculated from the matrix [C] when the quasi-TEM structure is empty, i.e. for (\varepsilon_{s})=1. The [L] matrix can be calculated as:

\[
[L] = \mu_{0} \varepsilon_{0} [C]^{-1}
\]
In order to show the value and the practicability of our study, we have used the identified 2D-FEM results to design a new 50-Ω, 20-dB, 2-GHz elliptical microstrip directional coupler with high performances in terms of isolation and directivity. To reach this goal, we have successfully established a program under Matlab [16] environment to obtain the response of the directional EMC coupler. Furthermore, to reinforce our work, we have validated the coupler’s coupling response by using CST Microwave Studio software [17].

2 2D-FEM RESULTS

2.1 Presentation of the Quasi-TEM Coupled Elliptical Microstrip Line

In Fig. 1, we present the schematic representation of the quasi-TEM coupled elliptical microstrip line. The CEML line consists of two outer infinitely elliptical conducting arc-strips, with major and minor axes of \((a_2, b_2)\) and a conduction angle of \((\theta)\), and an inner central grounded elliptical conductor of dimensions \((a_1, b_1)\). The dimensional relationship between the two ellipses is \(a_1/a_2 = b_1/b_2\). The substrate fills the inside of the CEML line and has a relative dielectric constant \((\varepsilon_r)\) of 9.6. The ratio \(R = a_1/a_2 = b_1/b_2\) and the conduction strip-angle \(\theta\) are used to define and determine the EM-parameters of the CEML line. In Fig. 2, we present the equipotential lines of the structure for the even- and the odd-modes. For the even mode: the potential \(V = 1\) V on the two elliptical strips and \(V = 0\) on the central elliptical ground conductor. For the odd mode: \(V = 1\) V on one of the two elliptical conducting arc strips, \(V = -1\) V on each other, and \(V = 0\) on the central elliptical ground conductor.

![Fig. 1. This figure illustrates (a) a cross-sectional view of the coupled elliptical microstrip line with its (b) 3D representation.](image)

Fig. 2. Potential distribution for the CEML (a) even-mode with \(V_1=V_2=1\) V on the two conducting strips, with \(\theta=120^\circ\) and \(R=a_1/a_2=0.4\) (b) odd-mode with \(V_1=1\) V on one of the two elliptical strips and \(V_2=-1\) V on the other one, with \(\theta=80^\circ\) and \(R=a_1/a_2=0.7\).

2.2 2D-FEM Quasi-TEM Electromagnetic Parameters Results

For our studied structure, there are not analytical or numerical results existing in the scientific literature except the numerical results of the general configuration of the open coupled cylindrical microstrip line (CCML) presented in Fig. 3 studied before in reference 4. Accordingly, to validate our FEM results, we have first adapted our program to study the CCML structure with \(a_1/a_2=0.9\) and \(\varepsilon_r=9.6\) studied previously with other techniques. We show in Tables 1 and 2 the numerical 2D-FEM results for the CCML line compared against existing results found in the literature [4]. The results are very satisfactory and excellent agreement is found, which confirms the exactitude and the correctness of our 2D-FEM program.

| Table 1. Odd-mode characteristic impedance and effective permittivity of the cylindrical coupled microstrip line, compared with other techniques. |
|---|---|---|
| 178.854° | 31.62 | 31.65 | 31.67 |
| 43.72 | 43.72 | 43.76 |
| 174.268° | 5.518 | 5.485 | 5.486 |
| 5.823 | 5.722 | 5.722 |
| 122.675° | 50.71 | 50.9 | 51.01 |
| 6.382 | 6.369 | 6.372 |
This study is applicable in isolation. Thus, from these results, it is shown that the quasi-TEM CEML line, compared with other techniques, our design allows very high losses, high agility, and more. Our interesting results for even- and odd-mode characteristic impedances, even- and odd-mode effective dielectric constants, coupling coefficient, and [L] and [C] matrices of the CEML, as functions of the strip conduction angle (θ), with taking R as a parameter, are illustrated in Figs. 4-12. The graphs are realized under Origin 50 [18]. It is observed that all the calculated quasi-TEM parameters depend strongly on the variation of strip conduction-angle (θ) and the (R) ratio. These curves allow the design of quasi-TEM CEML lines having a characteristic impedance \( Z_0 = \sqrt{\mu / \varepsilon} \) varying between 2.97 and 88 Ω, and a coupling coefficient from 2.58 to 150 dB or more. For these dimensions, a very tight coupling was reached, of (-2.59 dB) at (R=0.1) and (θ) of 179°, which is a very interesting result. Another interesting result is, when the even- and the odd-mode effective permittivities are equal, the even- and odd-mode phase velocities became also equal, this structure will allow very high-performance in terms of low-losses, high-directivity, and good isolation. Thus, from these design-curves, we can find the geometrical parameters of any high-performance circuit, with equal even- and odd-mode phase velocities, using our quasi-TEM CEML line.

### Table 2. Even-mode characteristic impedance and effective permittivity of the cylindrical coupled microstrip line, compared with other techniques.

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<tr>
<td>122.675°</td>
<td>66.39</td>
<td>66.46</td>
<td>66.55</td>
</tr>
<tr>
<td>122.922°</td>
<td>6.922</td>
<td>6.921</td>
<td>6.926</td>
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Fig. 3. Cross-sectional view of the general configuration of the coupled cylindrical microstrip line (CCML). \( w = \theta_a f_2, \quad s = \theta_f f_2 \) and \( n = r_2 + 1 \).

Fig. 4. Even-mode characteristic impedance of coupled elliptical microstrip line as a function of (θ), with taking the (R) ratio as a parameter.

Fig. 5. Odd-mode characteristic impedance of coupled elliptical microstrip line as a function of (θ), with taking the (R) ratio as a parameter.

Fig. 6. Even- and odd-mode characteristic impedances of coupled elliptical microstrip line as a function of (θ), with taking the (R) ratio as a parameter.
Fig. 7. Even- and odd-mode effective dielectric constants of coupled elliptical microstrip line as a function of (θ), with taking the (R) ratio as a parameter.

Fig. 8. Coupling coefficient of coupled elliptical microstrip line as a function of (θ), with taking the (R) ratio as a parameter.

Fig. 9. Proper inductance of coupled elliptical microstrip line as a function of (θ), with taking the (R) ratio as a parameter.

Fig. 10. Mutual inductance of coupled elliptical microstrip line as a function of (θ), with taking the (R) ratio as a parameter.

Fig. 11. Proper capacitance of coupled elliptical microstrip line as a function of (θ), with taking the (R) ratio as a parameter.

Fig. 12. Coupling capacitance of coupled elliptical microstrip line as a function of (θ), with taking the (R) ratio as a parameter.

3 50 Ω MICROWAVE ELLIPTICAL MICROSTRIP DIRECTIONAL COUPLER DESIGN

In order to show practicability and the usefulness of our analysis, the found results were used to design a new (20-dB) quasi-TEM directional EMC coupler, with equal even- and odd-mode phase velocities, adapted to (50 Ω) and operating at (2 GHz), convenient for microwave measurement systems and telecommunication applications.
To reach this objective, we have proposed an EMC directional coupler having \( b_1=1\text{mm}, a_2/b_2=3, a_1/a_2=0.2, a_1/b_1=3, b_1/b_2=0.2, \theta = 30^\circ \). The features of the coupler obtained from the FEM-analysis results include a dielectric constant \((\varepsilon_r)\) of 9.6, a length \( l = \lambda/4 = c/(4f(\sqrt{\varepsilon_{\text{eff}} + \varepsilon_{\text{effo}}})/2) = 12.145\text{mm} \), even- and odd- mode characteristic impedances \( Z_{\text{ee}} \) and \( Z_{\text{oo}} \), of 55.42\(\Omega\) and 45.41\(\Omega\), respectively, coupler’s characteristic impedance \( Z_c = \sqrt{Z_{\text{ee}}Z_{\text{oo}}} = 50.16\Omega \), even and odd-mode effective dielectric constants \( \varepsilon_{\text{eff}} \) and \( \varepsilon_{\text{effo}} \) of 9.596 and 9.47, coupling coefficient of
\[
k = 20\log_{10}\left(\frac{Z_{\text{ee}} - Z_{\text{oo}}}{Z_{\text{ee}} + Z_{\text{oo}}}\right) = -20.06\text{dB} \]
and \([L] \) and \([C] \) matrices calculated are as follows:
\[
[L] = \begin{bmatrix} 516.6 & 52.96 \\ 52.96 & 516.6 \end{bmatrix} \text{ (nH / m)}
\]
\[
[C] = \begin{bmatrix} 206.12 & -19.78 \\ -19.78 & 206.12 \end{bmatrix} \text{ (pF / m)}
\]

For a quarter wavelength \( l \) of 12.145 mm of the directional coupler, we present in Fig. 13 the coupler’s response realized in Matlab environment. The obtained Scattering \((S)\) parameters results show that the quasi-TEM EMC coupler has excellent performances, directivity \((S_{11})\) is of (34 dB), isolation \((S_{13})\) is of (53 dB), and reflection is of (46 dB). Accordingly, this type of coupler has very good characteristics in terms of isolation, directivity, and provide minimum losses. Furthermore, to reinforce our work, we have designed this structure (Fig. 14) in CST Microwave Studio environment. The found 2D-FEM characterization results represent a considerable improvement in the coupled elliptical microstrip line designs, and are completely compatible with the trends and requirements of microwave non-planar components mounted on non-planar shaped substrates. Our perspective is to develop new accurate closed-form formulas for all the EM-parameters of the CEML line, which are very required and well-suited in the design of non-planar microwave and RF circuits, convenient for high-frequency and telecommunication applications.

**REFERENCES**


[16] www.mathworks.com