Performance Analysis of Reliability Growth Models using Supervised Learning Techniques

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Abstract— Software reliability is one of a number of aspects of computer software which can be taken into consideration when determining the quality of the software. Building good reliability models is one of the key problems in the field of software reliability. A good software reliability model should give good predictions of future failure behavior, compute useful quantities and be widely applicable. Software Reliability Growth Models (SRGMs) are very important for estimating and predicting software reliability. An ideal SRGM should provide consistently accurate reliability estimation and prediction across different projects. However, that there is no single such model which can obtain accurate results for different cases. The reason is that the performance of SRGMs highly depends on the assumptions on the failure behavior and the application data-sets. In other words, many models may be shown to perform well with one failure data-set, but bad with the other data-set.

Thus, combining some individual SRGMs than single model is helpful to obtain a more accurate estimation and prediction. SRGM parameters are estimated using the least square estimation (LSE) or Maximum Likelihood Estimation (MLE). Several combinational methods of SRGMs have been proposed to improve the reliability estimation and prediction accuracy. The AdaBoosting algorithm is one of the most popular machine learning algorithms. An AdaBoosting based Combinational Model (ACM) is used to combine the several models. The key idea of this approach is that we select several SRGMs as the weak predictors and use an AdaBoosting algorithm to determine the weights of these models for obtaining the final linear combinational model. In this paper, the Fitness and Prediction of various Software Reliability Growth Models (SRGMs) can be compared with AdaBoosting based Combinational Model (ACM) with the help of Maximum likelihood estimation to estimate the model parameters.

Index Terms— Software Reliability, Software Reliability Growth Models (SRGMs), AdaBoosting Algorithm, Least Square Estimation, Maximum Likelihood Estimation.

1 INTRODUCTION

SOFTWARE Reliability Engineering is defined as quantitative study of the operational behavior of software-based systems with respect to user requirements concerning reliability. The demand for software systems has recently increased very rapidly. The reliability of software systems has become a critical issue in the software systems industry. With the 90’s of the previous century, computer software systems have become the major source of reported failures in many systems. Software is considered reliable if anyone can depend on it and use it in critical systems. The importance of software reliability will increase in the years to come, specifically in the fields of aerospace industry, satellites, and medicine applications. The process of software reliability starts with software testing and gathering of test results, after that, the phase of building a reliability model.

In general, the concept of reliability can be defined as “the probability that a system will perform its intended function during a period of running time without any failure”. IEEE defines software reliability as “the probability of failure-free software operations for a specified period of time in a specified environment”. In other words, software reliability can be viewed as the analysis of its failures, their causes and effects. Software reliability is a key characteristic of product quality. Most often, specific criteria and performance measures are placed into reliability analysis, and if the performance is below a certain level, failure occurred. Mathematically, the reliability function R(t) is the probability that a system will be successfully operating without failure in the interval from time 0 to time t [1].

\[ R(t) = P(T>t) \]

T is a random variable representing the failure time or time-to-failure (i.e. the expected value of the lifetime before a failure occurs). R(t) is the probability that the system’s lifetime is larger than t, the probability that the system will survive beyond time t, or the probability that the system will fail after time t. From above Equation, we can conclude that failure probability F(t), unreliability function of T is:

\[ F(t) = 1 - R(t) = P(T\leq t) \]

If the time-to-failure random variable T has a density function f(t), then the reliability can be measured as following

\[ R(t) = \int_{-\infty}^{t} f(x) \, dx \]

Where f(x) represents the density function for the random variable T. Consequently, the three functions R(t), F(t) and f(t) are closely related to one another.
2 Supervised Learning Techniques

2.1 Bootstrap

The Bootstrap procedure is a general purpose sample-based statistical method which consists of drawing randomly with replacement from a set of data points. It has the purpose of assessing the statistical accuracy of some estimate, say \( S(Z) \), over a training set \( Z = \{ z_1, z_2, \ldots, z_N \} \), with \( z_i = (x_i, y_i) \). To check the accuracy, the measure is applied over the \( B \) sampled versions of the training set. The Bootstrap algorithm[10] is as follows:

**Bootstrap Algorithm**

**Input:**
Training set \( Z = \{ z_1, z_2, \ldots, z_N \} \), with \( z_i = (x_i, y_i) \).

**B**, number of sampled versions of the training set.

**Output:**
\( S(Z) \), statistical estimate and its accuracy.

**Step 1:**
for \( n=1 \) to \( B \)

a) Draw, with replacement, \( L < N \) samples from the training set \( Z \), obtaining the \( n \)th sample \( Z^n \).

b) For each sample \( Z^n \), estimate a statistic \( S(Z^n) \).

**Step 2:**
Produce the bootstrap estimate \( S(Z) \), using \( S(Z^n) \) with \( n = \{1, \ldots, B \} \).

**Step 3:**
Compute the accuracy of the estimate, using the variance or some other criterion.

We start with the training set \( Z \), obtaining several versions \( Z^n \) (bootstrap samples). For each sampled version, we compute the desired statistical measure \( S(Z^n) \).

2.2 Bagging

The Bagging technique[10] consists of Bootstrap aggregation. Let us consider a training set \( Z = \{ z_1, z_2, \ldots, z_N \} \), with \( z_i = (x_i, y_i) \) for which we intend to fit a regression model, obtaining a prediction \( f(x) \) at input \( x \). Bagging averages this prediction over a collection of bootstrap samples, thereby reducing its variance.

For classification purposes, the Bagging algorithm is as follows:

**Bagging Algorithm for Classification**

**Input:**
\( Z = \{ z_1, z_2, \ldots, z_N \} \), with \( z_i = (x_i, y_i) \) as training set.

**B**, number of sampled versions of the training set.

**Output:**
\( H(x) \), a classifier suited for the training set.

**Step 1:**
for \( n=1 \) to \( B \)

a) Draw, with replacement, \( L < N \) samples from the training set \( Z \), obtaining the \( n \)th sample \( Z^n \).

b) For each sample \( Z^n \), learn classifier \( H_n \).

**Step 2:**
Produce final classifier as a vote of \( H_n \) with \( n = \{1, \ldots, B \} \)

\[ H(x) = \text{sign} \left( \sum_{n=1}^{B} H_n(x) \right) \]

As compared to the process of learning a classifier in a conventional way, that is, strictly from the training set, the Bagging approach increases classifier stability and reduces variance.

2.3 Boosting

The Boosting[10] procedure is similar to Bootstrap and Bagging. The first was proposed in 1989 by Schapire and is as follows:

**Boosting Algorithm for Classification**

**Input:**
\( Z = \{ z_1, z_2, \ldots, z_N \} \), with \( z_i = (x_i, y_i) \) as training set.

**B**, number of sampled versions of the training set.

**Output:**
\( H(x) \), a classifier suited for the training set.

**Step 1:**
Randomly select, without replacement, \( L1 < N \) samples from \( Z \) to obtain \( Z^n \); train weak learner \( H_1 \) on \( Z^n \).

**Step 2:**
Select \( L2 < N \) samples from \( Z \) with half of the samples misclassified by \( H_1 \) to obtain \( Z^n \); train weak learner \( H_2 \) on it.

**Step 3:**
Select all samples from \( Z \) that \( H_1 \) and \( H_2 \) disagree on; train weak learner \( H_3 \), using three samples.

**Step 4:**
Produce final classifier as a vote of the three weak learners

\[ H(x) = \text{sign} \left( \sum_{n=1}^{3} H_n(x) \right) \]

2.4 AdaBoosting

A very promising well known used boosting algorithm is AdaBoost [2]. The idea behind adaptive boosting is to weight the data instead of (randomly) sampling it and discarding it. In recent years, ensemble techniques, namely boosting algorithms, have been a focus of research. The AdaBoost algorithm is a well-known method to build ensembles of classifiers with very good performance. It has been shown empirically that AdaBoost with decision trees has excellent performance, being considered the best off-the-shelf classification algorithm.

AdaBoosting is a commonly used machine learning algorithm which can combine several weak predictors into a single strong predictor for highly improving the estimation and prediction accuracy. It has been applied with great success to several benchmark Machine Learning (ML) problems using rather simple learning algorithms, in particular decision trees.

**AdaBoosting Algorithm**

AdaBoosting is a commonly used machine learning algorithm for constructing a strong classifier \( f(x) \) as linear combination of weak classifiers \( h_t(x) \).

\[ f(x) = \sum_{t=1}^{T} \alpha_t h_t(x) \]

AdaBoosting calls a weak classifier \( h_t(x) \) repeatedly in a series of rounds \( t=1, 2, \ldots, T \). For each call a distribution of weights \( \alpha_t \) is updated in the data-set for the classification.
algorithm takes as input a training set \((x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\) where \(x_i \in X\), \(y_i \in \{-1, +1\}\) where each \(x_i\) belongs to some domain or instance space \(X\) and each label \(y_i\) is in some label set \(Y\). One of the main ideas of the algorithm is to maintain a distribution or set of weights over the training set. 

**Given**

\[(x_1, y_1), \ldots, (x_n, y_n)\] where \(x_i \in X\), \(y_i \in \{-1, +1\}\]

**Initialize**

weights \(D_0(i) = 1/n\)

**Iterate** \(t=1, \ldots, T:\)

Train weak learner using distribution \(D_t\)

Get weak classifier: \(h_t : X \rightarrow \mathbb{R}\)

Choose \(\alpha_t \in \mathbb{R}\)

Update to \(D_{t+1}(i)\) from \(D_t(i)\)

\[
D_{t+1}(i) = \frac{D_t(i) \exp(-\alpha_t y_i h_t(x_i))}{Z_t}
\]

Where \(Z_t\) is a normalization factor (chosen so that \(D_{t+1}\) will be a distribution), and \(\alpha_t\)

\[
\alpha_t = \frac{1}{2} \ln \left( \frac{1 - e_t}{e_t} \right) > 0
\]

**Output**

Final Classifier

\[
H(x) = \text{sign}\left( \sum_{t=1}^{T} \alpha_t h_t(x) \right)
\]

### 3 Existing System

#### 3.1 Software Reliability Growth Models

A software reliability growth model (SRGM) describes the mathematical relationship of finding and removing faults to improve software reliability. A SRGM performs curve fitting of observed failure data by a pre-specified model formula, where the parameters of the model are found by statistical techniques like maximum likelihood method. The model then estimates reliability or predicts future reliability by different forms of extrapolation.

After the first software reliability growth model was proposed by Jelinski and Moranda in 1972, there have been numerous reliability growth models following it. These models come under different classes, e.g. exponential failure time class of models, Weibull and Gamma failure time class of models, infinite failure category models and Bayesian models. These models are based on prior assumptions about the nature of failures and the probability of individual failures occurring. There is no reliability growth model that can be generalized for all possible software projects, although there is evidence of models that are better suited to certain types of software projects.

An important class of SRGM that has been widely studied is the NonHomogeneous Poisson Process (NHPP). It forms one of the main classes of the existing SRGM, due to it is mathematical tractability and wide applicability. NHPP models are useful in describing failure processes, providing trends such as reliability growth and the fault - content. SRGM consider the debugging process as a counting process characterized by the mean value function of a NHPP. Software reliability can be estimated once the mean value function is determined. Model parameters are usually determined using either Maximum Likelihood Estimate (MLE) or least-squares estimation methods. NHPP based SRGM are generally classified into two groups[13].

The first group contains models, which use the execution time (i.e., CPU time) or calendar time. Such models are called continuous time models.

The second group contains models, which use the number of test cases as a unit of the fault detection period. Such models are called discrete time models, since the unit of the software fault detection period is countable. A test case can be a single computer test run executed in an hour, day, week or even month. Therefore, it includes the computer test run and length of time spent to visually inspect the software source code.

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Software reliability growth models with a model are formulated by using a Non-Homogeneous Poisson Process (NHPP). Using the model, the method of data analysis for the software reliability measurement will be developed. SRGM parameters are estimated by using the least square estimation (LSE) or maximum likelihood estimation (MLE) method and using actual software failure data, numerical results are obtained. In the existing system, the software reliability growth model parameters are estimated using least square estimation to obtain the numerical results.

### 3.2 Limitations of Existing System

- It is generally considered to have less desirable optimality properties than maximum likelihood.
- It can be quite sensitive to the choice of starting values.
It can’t generate accurate results for data of sufficiently large samples.

4 PROPOSED WORK & METHODOLOGY

In the proposed system, the reliability growth model parameters are estimated using maximum likelihood estimation to get accurate results, to overcome the problems of LSE, we use MLE.

Maximum likelihood provides a consistent approach to parameter estimation problems. This means that maximum likelihood estimates can be developed for a large variety of estimation situations. For example, they can be applied in reliability analysis to censored data under various censoring models.

MLE has many optimal properties in estimation, i.e.

Sufficiency: complete information about the parameter of interest contained in its MLE estimator.

Consistency: true parameter value that generated the data recovered asymptotically, i.e. for data of sufficiently large samples.

Efficiency: lowest-possible variance of the parameter estimates achieved asymptotically.

4.1 Selected Models for Parameter Estimation and Comparison Criterion

A software reliability growth model characterizes how the reliability of that software varies with execution time. The traditional software reliability models are set of techniques that apply probability theory and statistical analysis to software reliability. A reliability model specifies the general form of the dependence of the failure process on the principal factors that affects it.

The following five models are selected as the candidate and comparison models, which have been widely used by many researchers in the field of software reliability modeling[6].

(1) Goel-Oukumoto model (GO Model) : M1

This model is proposed by the Goel and Okumoto, one of the most popular non-homogeneous poisson process (NHPP) model in the field of software reliability modeling. It assumes failures occur randomly and that all faults contribute an equally to total unreliability. When a failure occurs, it assumes that the fix is perfect, thus the failure rate improves continuously in time.

\[ a(1-\exp(-rt)), a>0,r>0 \]

(2) Musa-Oukumoto model (MO Model) : M2

It is also a nonhomogeneous poisson process with an intensity function that decreases exponentially as failures occur. The exponential rate of decrease reflects the view that the earlier discovered failures have a greater impact on reducing the failure intensity function than those encountered later.

\[ 1/a*\ln(1+art) \]

(3) Delayed S-Shaped Model : M3

Yamada presented a delayed S-shaped SRGM incorporating the time delay between fault detection and fault correction. The Delayed S-Shaped model is a modification of the NHPP to obtain an S-shaped curve for the cumulative number of failures detected such that the failure rate initially increased and later decays.

\[ a(1-\exp(-rt)) \]

(4) Inflected S-Shaped Model : M4

Ohba proposed an inflected S-shaped model to describe the software failure-occurrence phenomenon with mutual dependency of detecting faults.

\[ a(1-\exp(-rt)/(1+c*\exp(-rt))) \]

(5) Generalized GO Model : M5

In GO model, the failure occurrence rate per fault is time independent, however since the expected number of remaining faults decreases with time, the overall software failure intensity decreases with time. In most real-life testing scenarios, the software failure intensity increases initially and then decreases. The generalized GO model was proposed to capture this increasing/decreasing nature of the failure intensity.

\[ a(1-\exp(-rt^s)) \]

In these NHPP models, usually parameter ‘a’ usually represents the mean number of software failures that will eventually be detected, and parameter ‘r’ represents the probability that a failure is detected in a constant period [13]. In this, two real failure data-sets are selected, Ohba and Wood [5], which are popular and frequently used for comparison of SRGM.

4.2 AdaBoosting based Combinational Model (ACM)

Many SRGMs may result in estimation or prediction bias since their underlying assumptions are not consistent with the characteristics of the application data. To reduce this bias, combining several different SRGMs together in linear or nonlinear manner is a common and applicable method. Many approaches are proposed to determine the weight assignment for the combinational models, such as equal weight, neural-network, genetic programming and etc. In this, an abstract description of how to use AdaBoosting to obtain the dynamic weighted linear combinational model (ACM) of several SRGMs is shown as follows[3]

Input:

1. Let \( M(t)=u(x_1,x_2,...,x_n,t) \) denotes different SRGMs (such as GO model). \( M(t) \) is the cumulated faults detected at time \( t \), \( S \) is the number of the parameters of \( M(t) \), \( x_k \) is the \( k \)-th parameter of \( M(t) \), \( k=1,2,3,......,S \).
2. The failure data-set D0 is denoted by \((t_1, m_1), (t_2, m_2),\ldots, (t_n, m_n)\).

Where \(n\) is the data number of \(D_0\),

\(m_j\) is the cumulated faults detected at \(t_j\) \(j = 1\ldots n\).

**Initialze:**

**Step1:** Selecting \(M\) different SRGMs (denoted by \(M_0(t)\), \(m=1\ldots M\)) as the candidate models for the ACM.

**Step2:** The original weight set of \(D_0\) can be denoted by \(K_0 = \{k_{01}\ldots k_{0n}\}\), where \(k_0\) is initialized by \(1/n\).

**Circulation:**

**Step3:** New training data set \(D_i\) (\(i=1\ldots P\), \(P\) is the training rounds) are generated by repeatedly random sampling from \(D_0\) according to its corresponding weight set \(K_i = \{k_{i1}\ldots k_{in}\}\).

If there are some same data in \(D_i\), only one of them will be reserved in our approach. Hence the data number of \(D_i\) may not be \(n\).

**Step4:** \(D_i\) is used to estimate the parameters of each \(M_0(t)\) in the \(i\)th training round. Then the fitness function (Notation 1) of \(M_0(t)\) can be determined by \(D_i\).

The candidate model whose value of fitness function is the smallest is chosen as the selected model (denoted by \(M(t)\)) in this round, \(i=1\ldots P\).

**Step5:** If \(M_1(t)\) denotes the estimation form of \(M_0(t)\), the loss function \(L_{is}\) (Notation 2) of \(M_0(t)\) can be calculated by the fitting results of \(M_1(t)\) with \(D_i\).

Then \(K_i\) can be updated as \(K_{i+1}\) by \(L_{is}\) (Notation 3). The basic weight \(\beta_{is}\) of \(M_0(t)\) also can be determined by \(L_{is}\) (Notation 4).

**Step6:** Performing Step3-Step5 repeatedly until \(i=P\), and then turning into Step7.

**Output:**

**Step7:** Finally, a combinational linear model is obtained as follows:

\[ M_{ACM}(t) = \sum_{i=1}^{P} W_{is} M_{is}(t) \]

The combination weight \(W_{is}\) of the selected model \(M_{is}(t)\) is \(f(\beta_{is})\) (Notation 5), that is, \(W_{is}\) is a function of the basic weight \(\beta_{is}\).

**Notation 1:**
Fitness function (FF) can be defined according to the estimation method of the parameters of these candidate SRGMs. The general methods to estimate the parameters of SRGMs are least-squares estimation (LSE) and maximum likelihood estimation (MLE).

If Maximum likelihood Estimation is used, FF equation is

\[ FF = 1/ - \log (ML) \]

Where ML is the maximum likelihood function derived in the next section.

**Notation 2:**

\[ L_{is} = \sum_{j=1}^{P} k_{ij} * L_{is} \]

\[ L_{is} = \frac{\text{AE}_{is}}{\text{Dent}} \]

Where \(\text{AE}_{is} = m_{is} - M_{is}(t_i)\), \(\text{Dent} = \max \{\text{AE}_{is}\}\)

**Notation 3:**

\[ K_{i+1} = \{ K_{i+1,1} \} \]

\[ K_{i+1, j} = k_{ij} * L_{is} / (\sum_{j=1}^{P} k_{ij} * L_{is}) \]

**Notation 4:**

\[ W_{is} = f(\beta_{is}) = \log (1/ \beta_{is}) / \sum_{j=1}^{P} \log 1 / \beta_{is} \]

**Notation 5:**

\[ W_{is} = f(\beta_{is}) = \log (1/ \beta_{is}) / \sum_{j=1}^{P} \log 1 / \beta_{is} \]

### 4.3 Parameter Estimation

Once the analytic expression for the mean value function is derived, it is required to estimate the parameters in the mean value function, which is usually carried out by using Maximum likelihood Estimation technique[11].

**Maximum Likelihood Estimation:**

Once a model is specified with its parameters, and data have been collected, one is in a position to evaluate its usefulness. Goodness of fit is assessed by finding parameter values of a model that best fits the data, a procedure called parameter estimation.

The general methods to estimate the parameters are least-squares estimation (LSE) and maximum likelihood estimation (MLE).

Fitting a proposed model to actual fault data involves estimating the model parameters from the real test data sets. Here we employ the method of MLE to estimate the parameters ‘a’ and ‘r’. All parameters of different Reliability models can be estimated by the method of MLE. For example, suppose that the limits are determined for the observed data pairs:

\( (t_0, m_0), (t_1, m_1), (t_2, m_2), \ldots, (t_n, m_n) \)

Then the likelihood function for the parameters \(a\) and \(r\) in the models with \(m(t)\) is given by

\[ ML = \prod_{i=1}^{n} \exp[-(m(t) - m(t)))] \]

Where, \(m(t)\) is the cumulative number of faults detected by \(t_i\) test cases \((j=1,2,3,\ldots, n)\),

\(t_i\) is accumulated number of test run executed to detect \(m_j\) faults,

\(m(t)\) is the expected mean number of faults detected by the \(n\)th test case.

Taking the natural logarithm of the above equation, we get

\[ \ln ML = \sum_{j=1}^{n} (m_j-m(t)) \ln[m(t)-m(t_i)] - [(m(t_i)-m(t_i)) - \ln n] \]
\[ \sum_{n=1}^{N} \ln[(m_n-m_{n-1})!] \]

The values of these parameters that maximize the sample likelihood are known as the Maximum Likelihood Estimator.

4.4 Analysis of Data

**Data Set #1:**
The first data set employed was from the paper by Ohba[5] for a PL/1 database application software system consisting of approximately 1 317 000 LOC. Over the course of 19 weeks, 47.65 CPU hours were consumed, and 328 software faults were removed. Although this is an old data-set, we feel it is instructive to use it because it allows direct comparison with the work of others who have used it.

**Data Set #2:**
The second data set presented by Wood[5] from a subset of products for four separate software releases at Tandem Computers Company. Wood reported that the specific products & releases are not identified, and the test data sets have been suitably transformed in order to avoid confidentiality issues. Here we only use Release 1 for illustrations. Over the course of 20 weeks, 10 000 CPU hours were consumed, and 100 software faults were removed.

5 Experimental Results

In order to implement the fitting and prediction performance of the five selected models and ACM by using maximum likelihood estimation to estimate the parameters of models can be shown as follows.

5.1 Fitness and Prediction Performance of five Models and ACM with failure datasets

The Figures (Fig 5.1 & Fig 5.2) represents the Fitness and Prediction graphs of various SRGMs and ACM respectively. Here the failure data-set Ohba [5] is taken for the comparison purpose.

6 Conclusion and Future Scope

In this paper, the Fitness and Prediction of various Software Reliability Growth Models (SRGMs) can be compared with AdaBoosting based Combinational Model (ACM) with the help of Maximum likelihood estimation to estimate the model parameters. From the results, the fitting and prediction performance of ACM is better compare with individual reliability growth models with real failure data-sets.

The further enhancements those are possible for this, Examining the statistical significance of the estimation and prediction results of the ACM and Comparing with the other combination approaches such as Genetic-based Combinational Model (GCM)[9], Dynamic Weighted Combinational Model (DWCM)[7] etc.

References


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