Generation of Strongly Regular Graphs from Normalized Hadamard Matrices

A. A. C. A. Jayathilake, A. A. I. Perera, M. A. P. Chamikara

Abstract: - This paper proposes an algorithm which can be used to construct strongly regular graphs from Hadamard matrices.A graph G is strongly regular if there are integers λ and μ such that every two adjacent vertices have λ common neighbours and every two non adjacent vertices have μ common neighbors. Proposed method is mainly based on basic matrix manipulations. If the order of the normalized Hadamard matrix is n the resulting strongly regular graph will have $n \times n$ number of vertices. Therefore the simplest strongly regular graph generated from this method has 16 vertices since its predecessor normalized Hadamard matrix has the order of 4. This algorithm was implemented using C++ programming language. Then the algorithm was tested for large Hadamard matrices and the results proved that this method is correct and works for any normalized Hadamard matrix of order greater than or equal to 4.

Index Terms: - Adjacency matrix , Hadamard Matrices ,k-regular graphs , Latin Square , Simple Graphs ,Strongly Regular Graphs, Tensor Product .

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1 INTRODUCTION

A hadamard matrix of order n is an $n \times n$ matrix H with entries ± 1, such that $HH^T = nI_n$ where I_n is the identity matrix of order n.[1] The examples for the smallest orders are,

$$H_1 = [1] \;\&\; H_2 = \begin{bmatrix} 1 & 1 \\ 1 & - \end{bmatrix}$$

Figure 1-The smallest order Hadamard matrices

where, we write – for -1.

Also, note that multiplying a row/column by (-1) or permuting a row/column of a Hadamard matrix yields another Hadamard matrix. If H is a Hadamard matrix of order n,then $n = 1, n = 2$or $n \equiv 0(mod 4)$.[2]The most preliminary way of constructing Hadamard matrices is to use the Tensor (Kronecker) product.

Tensor Product: [3] Let $A = \{a_{ij}\}$ be $m \times n$ matrix and B be a matrix.Then the Tensor product of A and B is defined as,

$$A \otimes B = \begin{bmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{bmatrix}$$

Figure 2- The Tensor product of two matrices

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If H is a Hadamard matrix of order n,then the partition matrix $\begin{bmatrix} H & H \\ H & -H \end{bmatrix}$ is a Hadamard matrix of order $2n$.[4]This can be applied repeatedly and results the following sequence of matrices,

$$H_{2^k} = \begin{bmatrix} H_{2^{k-1}} & H_{2^{k-1}} \\ H_{2^{k-1}} & -H_{2^{k-1}} \end{bmatrix} = H_2 \otimes H_{2^{k-1}}$$

Figure 3-Sylvester's method of constructing Normalized Hadamard matrices

where $\otimes$ denotes the kronecker product.This results the normalized Hadamard matrix for which the first row and first column is all ones.[5]Using the Sylvester's method the first few Hadamard matrices that can be obtained are,

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & - \end{bmatrix}$$
$$H_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & - & 1 & - \\ 1 & 1 & - & - \\ 1 & - & - & 1 \end{bmatrix}$$

Figure 4-First two normalized matrices constructed from Sylvester construction

Further, if H_1 and H_2 are Hadamard matrices, then $H_1 \otimes H_2$ is also a Hadamard matrix.[6] These Hadamard matrices which are constructed by the Sylvester's method can be used to construct certain graphs.One can recall the definition of graphs as follows; A ***Graph*** $G = (V, E)$ is an ordered pair of sets, the elements of V are called vertices and the elements of E are called edges.The vertex set of G is denoted by V_G and the set of edges is denoted by E_G.Further,for a graph denote $v_G = |V_G|$ and $e_G = |E_G|$ and the value of v_G is called the order of the graph and the value of e_G is called the size of the graph.[7] For an edge $uv \in G$, vertices u and v are its ends.Further,the vertices u and v are adjacent or neighbours if $uv \in G$ and the edge uv is said to be incident to the vertices u and v.[8]A graph G can be represented in a plane figure by drawing a line between two vertices. Moreover, the graphs can be generalized by allowing loops and multiple edges between vertices.[9] In figure 5 below, the vertex set of the graph G can be written as $V_G = \{u, v, w\}$ and the set of edges can be expressed as $E_G = \{uu, uv, vw, vw\}$.The point u has loop uu and the vertices v and w are connected with multiple edges.

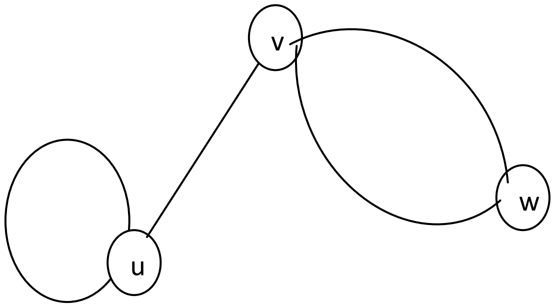

Figure 5-The diagram of a graph G with vertices u, v, w.

Among the extensive combination of graphs the fundamental type of graphs are known as simple graphs.
Simple graph[10]- A graph G is called a simple graph if it has no loops and multiple edges**.** Figure 6 shows a simple graph with three vertices.

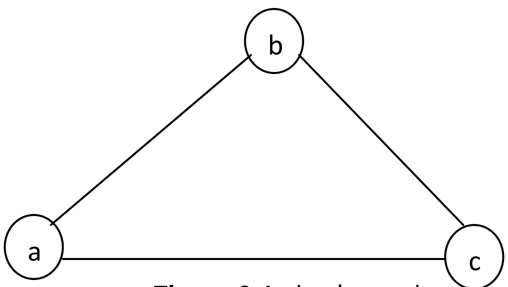

Figure 6-A simple graph

Through these simple graphs, the interesting class of graphs are known to be the k-regular graphs.

***k*-Regular graph[11]-**A simple graph with vertices of equal degree k is known as a k-regular graph.

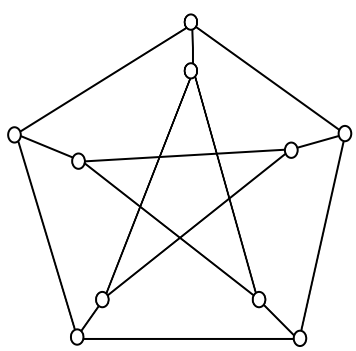

Figure 7- A 3-regular graph (Peterson Graph)

The action of k-regular graphs appear to be the origination of another collection of important graphs which are known to be the strongly regular graphs.

A strongly regular graph,$SRG(v, k, \lambda, \mu)$ is a regular graph with v vertices of degree k such that every two joined vevrtices have exactly λ common neighbours and every two non-joined vertices have exactly μ common neighbours.[11] The parameters(v, k, λ, μ) of a strongly regular graph satisfies the equation $kk(k - \lambda - 1) = (v - k - 1)\mu.\mu$.[12]

These several types of graphs can be represented with their adjacency matrix.[13] The correspondence between the graph and the adjacency matrix is as follows.

The **adjacency matrix A** [14] for a graph with n n vertices is an $nn \times nn$ matrix if its(i, j) entry is,

$$\begin{cases} 1\,; if\ i^{th}\ vertex\ and\ j^{th}\ vertex\ are\ connected \\ 0; \quad otherwise \end{cases}$$

For the graph in figure 5, the adjacency matrix A is shown below.

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Figure 8-the adjacency matrix of the graph G in figure 5

Moreover, a graph G G is a (v, k, λ, μ) strongly regular graph if and only if for the adjacency matrix AA of GG holds,$AJ_v = kJ_v kJ_v$ and $A^2 + (\mu - \lambda)A + (\mu - k)I_v = \mu J_v$, where $I_v I_v$ and $J_v J_v$ are the identity matrix and the matrix with all ones respectively.[15] The Latin squares play an important role in constructing graphs in certain situations. There are several types of Latin squares. For this objective, we may use the symmetric Latin squares. The *Latin square* of order nn is an $nn \times nn$ array such that each cell contains a single element from an $nn - setset$ such that each element occurs in exactly once in each row.[16] For this intention, it will be important to have the set of indices for the rows and columns and the set of symbols be the same set. For some finite index the "obvious" choice for the index sets will be $\{1,2, \dots, n\}$ or $\{0,1,2, \dots, nn - 1\}$, but we could use any finite set. For a specific order n, there will be more than one Latin squares..A Latin square L of order n is symmetric if $L(i, j) = L(j, i)$ for $1 \le i \,\&\, j \le n$ and a Latin square of side n is in standard form if the symbols in the first row and column occur in natural order. [17]

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Figure 9-Latin squares of order 2

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

Figure 10-Symmetric Latin square of order 3

To accomplish the mission of generating a strongly regular graph from a normalized Hadamard matrix, we used the Symmetric Latin Squares where lower triangle separated by the diagonal has a mirror image of the upper triangle. Order 4 symmetric Latin square depicted by Figure 11 and order 8 Latin symmetric Latin square depicted by Figure 12 show two such Latin squares which follows the format of Latin squares used in this research.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 4 & 1 & 2 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

Figure 11- Latin square of order 4

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 1 & 4 & 3 & 6 & 5 & 8 & 7 \\ 3 & 4 & 1 & 2 & 7 & 8 & 5 & 6 \\ 4 & 3 & 2 & 1 & 8 & 7 & 6 & 5 \\ 5 & 6 & 7 & 8 & 1 & 2 & 3 & 4 \\ 6 & 5 & 8 & 7 & 2 & 1 & 4 & 3 \\ 7 & 8 & 5 & 6 & 3 & 4 & 1 & 2 \\ 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \end{bmatrix}$$

Figure 12- Latin square of order 8

In this paper we have proposed an algorithm which can be used to generate strongly regular graphs from normalized Hadamard matrices of order 2^n for $n > 1$.

2 METHODOLOGY

The proposed method constructs a $SRG(v,k,\lambda,\mu)$ graph from a normalized Hadamard matrix. Since, the finite projective plane is known to be the smallest design ,this method is developed to begin with the normalized Hadamard matrix of order 4.

2.1 Mathematical Background Of The Proposed Method

For any integer $n \geq 1$ the normalized Hadamard matrix of order 2^n can be constructed using the Sylvester's construction. Hence, the Sylvester construction cited in figure 3 can be applied to the smallest order Hadamard matrices defined in figure 1 to obtain the Hadamard matrix of order $4n$.For an example, The normalized Hadamard matrix of order 4 (H_4) can be constructed using H_2 as follows:

$$H_4 = \begin{bmatrix} H_2 & H_2 \\ H_2 & -H_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & - & 1 & - \\ 1 & 1 & - & - \\ 1 & - & - & 1 \end{bmatrix}$$

Figure 13 –The normalized Hadamard matrix of order 4 constructed by Sylvester construction

In the next step label each column vector as c_i for $i = 1,2,\dots,4n$.For the above example, there are four column vectors c_1, c_2, c_3, c_4.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & - & 1 & - \\ 1 & 1 & - & - \\ 1 & - & - & 1 \end{bmatrix}$$

$$= [c_1 \quad c_2 \quad c_3 \quad c_4]$$

Figure 14- The column vectors of the matrix H_4

Then, multiply each column vector by its transpose and label the resulting matrices asC_i, for $i = 1,2,\dots,4n$.For the above example, eachC_i' s can be constructed as follows.

Using the first column vector c_1, the construction of the first block matrix C_1 is as follows:

$$C_1 = c_1 {c_1}^t$$

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} . [1 \quad 1 \quad 1 \quad 1] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = C_1$$

Figure 15-The construction of the first block matrix C_1

Then, applying the above matrix multiplication method to the column vector c_2 gives the next block matrix C_2.

$$C_2 = c_2 {c_2}^t$$

$$\begin{bmatrix} 1 \\ - \\ 1 \\ - \end{bmatrix} . [1 \quad - \quad 1 \quad -] = \begin{bmatrix} 1 & - & 1 & - \\ - & 1 & - & 1 \\ 1 & - & 1 & - \\ - & 1 & - & 1 \end{bmatrix} = C_2$$

Figure 16-The second block matrix C_2

Similarly, the matrix operation on the third column vector c_3 yields the next block C_3.

$$C_3 = c_3 {c_3}^t$$

$$\begin{bmatrix} 1 \\ 1 \\ - \\ - \end{bmatrix} . [1 \quad 1 \quad - \quad -] = \begin{bmatrix} 1 & 1 & - & - \\ 1 & 1 & - & - \\ - & - & 1 & 1 \\ - & - & 1 & 1 \end{bmatrix} = C_3$$

Figure 17-The third block matrix C_3

Since the illustration uses the matrix H_4, there are four scolumn vectors and the last one being c_4. Hence matrix product on c_4 gives the required last block C_4.

$$C_4 = c_4 {c_4}^t$$

$$\begin{bmatrix} 1 \\ - \\ - \\ 1 \end{bmatrix} . [1 \quad - \quad - \quad 1] = \begin{bmatrix} 1 & - & - & 1 \\ - & 1 & 1 & - \\ - & 1 & 1 & - \\ 1 & - & - & 1 \end{bmatrix} = C_4$$

Figure 18-The last block matrix C_4

Then consider the symmetric Latin square with entries $1,2,\dots,4n$ with constant diagonal 1 and replace each i by C_i. For the above example, the symmetric Latin square of four elements is required. The resulting Latin square will be as follows,

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 4 & 1 & 2 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

Figure 19-The Latin square of order 4

Hence, by replacing each element with the corresponding C_i matrix, following large matrix of order 16 can be obtained.

$$\begin{bmatrix} C_1 & C_2 & C_3 & C_4 \\ C_2 & C_1 & C_4 & C_3 \\ C_3 & C_4 & C_1 & C_2 \\ C_4 & C_3 & C_2 & C_1 \end{bmatrix}$$

Figure 20-The constructed matrix which is replaced by the block matrices

$$\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & - & 1 & - & 1 & 1 & - & - & 1 & - & - & 1 \\
1 & 1 & 1 & 1 & - & 1 & - & 1 & 1 & 1 & - & - & - & 1 & 1 & - \\
1 & 1 & 1 & 1 & 1 & - & 1 & - & - & - & 1 & 1 & - & 1 & 1 & - \\
1 & 1 & 1 & 1 & - & 1 & - & 1 & - & - & 1 & 1 & 1 & - & - & 1 \\
1 & - & 1 & - & 1 & 1 & 1 & 1 & 1 & - & - & 1 & 1 & 1 & - & - \\
- & 1 & - & 1 & 1 & 1 & 1 & 1 & - & 1 & 1 & - & 1 & 1 & - & - \\
1 & - & 1 & - & 1 & 1 & 1 & 1 & - & 1 & 1 & - & - & - & 1 & 1 \\
- & 1 & - & 1 & 1 & 1 & 1 & 1 & 1 & - & - & 1 & - & - & 1 & 1 \\
1 & 1 & - & - & 1 & - & - & 1 & 1 & 1 & 1 & 1 & 1 & - & 1 & - \\
1 & 1 & - & - & - & 1 & 1 & - & 1 & 1 & 1 & 1 & - & 1 & - & 1 \\
- & - & 1 & 1 & - & 1 & 1 & - & 1 & 1 & 1 & 1 & 1 & - & 1 & - \\
- & - & 1 & 1 & 1 & - & - & 1 & 1 & 1 & 1 & 1 & - & 1 & - & 1 \\
1 & - & - & 1 & 1 & 1 & - & - & 1 & - & 1 & - & 1 & 1 & 1 & 1 \\
- & 1 & 1 & - & 1 & 1 & - & - & - & 1 & - & 1 & 1 & 1 & 1 & 1 \\
- & 1 & 1 & - & - & - & 1 & 1 & 1 & - & 1 & - & 1 & 1 & 1 & 1 \\
1 & - & - & 1 & - & - & 1 & 1 & - & 1 & - & 1 & 1 & 1 & 1 & 1
\end{bmatrix}$$

Figure 21-The matrix obtained by substituting the block matrices

Then, replacing 1 to 0 and -1 to 1 yields to the adjacency matrix of the strongly regular graph obtained from the normalized Hadamard matrix of order $4n$.For the Hadamard matrix of order 4 we have the following adjacency matrix.

$$\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

Figure 22-The adjacency matrix of the strongly regular graph obtained from H_4.

The same method is applicable for the other normalized Hadamard matrices of order 2^n for $n > 1$.

2.2 Implementation Of The Proposed Method

This algorithm was implemented using C++ programming language. The first task of this computer program is to generate a normalized Hadamard matrix with a specified order. This task is achieved by implementing the kronecker product of matrices starting from the normalized Hadamard matrix of order two mentioned above. User can input the value of the required Hadamard matrix of order 2^n for $n \geq 0$.Depending on the user input, the program generates the relevant normalized Hadamard matrix and stores it in an external file (File 1) to reduce the burden of holding large matrices in the computer RAM (Random Access Memory). In the same way, the program creates the relevant symmetric Latin Square and stores it in another external file (File 2).

Next the program reads column vectors (simple c's ex: $(c_1, c_2$,....) from the normalized Hadamard matrix stored in that external file (File 1) and execute the product of its transpose to get the resulting matrices (capital C's ex C_1, C_2, ..). In the latter part of the computer program the generated matrices C_1, C_2..etc will be substituted to the relevant locations of the above generated Latin square. The resulting matrix will then be saved in another external file (File 3) Finally, it will automatically replace $+1$ with *0* and -1 with *1* and results the required adjacency matrix of the strongly regular graph will be generated and this resulting adjacency matrix will be saved as another external file (File 4). This File 4 will finally be used to draw the strongly regular graph on the computer display by drawing lines between all the adjacent vertices.

3 Results and Discussion

It is easy to observe that, each block C_i that are constructed by the operation $c_i c_i^t$ for $i = 1,2, \ldots, 4n$ are symmetric. That is $C_i = C_i^t$ for $i = 1,2,3, \ldots, 4n$.Since the normalized Hadamard matrices have been used, in every construction the first column vector will be a column matrix with all entries being one. Hence the first block C_1 in each construction is a matrix of order $4n$ whose entries are equal to one. That is,$C_1 = J_{4n}$,where J_{4n} is a matrix of order $4n$ with all entries equal to one. Further, it can be seen that $C_i J_{4n} = J_{4n} C_i = 0$ for $i = 2,3,4, \ldots, 4n$. The implementation of the above method was tested for several normalized Hadamard matrices of order 2^n for $n > 1$. For the program, the following Hardware and software were used.

Hardware:
System Model: Inspiron N5110
Computer: Intel(R) Core(TM) i7-2670QM CPU @ 2.20GHz (8 CPUs), ~2.2 GHz
Memory: 8192 MB RAM

Software:
Operating System: Windows 7 Home Basic 64-bit (6.1, Build 7601)

IDE: Turbo C++; Version 3.0; Copyright(c) 1990, 1992 by Borland International, Inc;
DosBox 0.74

Here, in the program, the user is free to enter the order of the Hadamard matrix, coordinates of the graph (x, y) radius of the circle which is used to draw the graph and the colour of the graph.

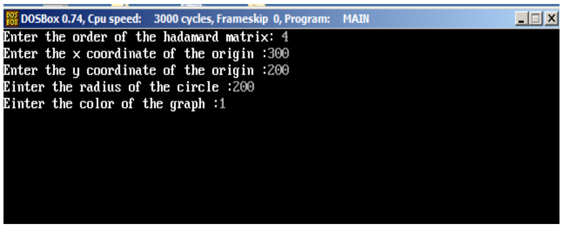

Figure 23-The user input interface

Using the Hadamard matrix of order 4,we can obtain SRG (16,6,2,2), that is the graph consists of 16 points for which each vertex has the degree 6 and each pair of adjacency vertices appear exactly twice and each non-adjacency pair of vertices have 2 common neighbours.Hence the required Regular graph is drawn as follows.

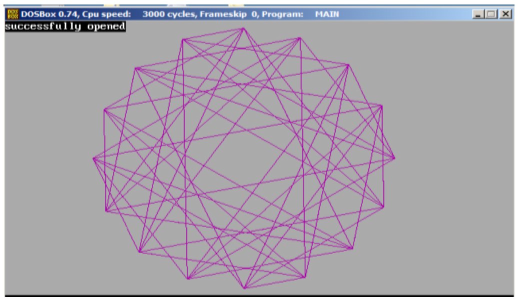

Figure 24-The strongly regular graph obtained from H_4

The corresponding adjacency matrix of the above graph is shown below.

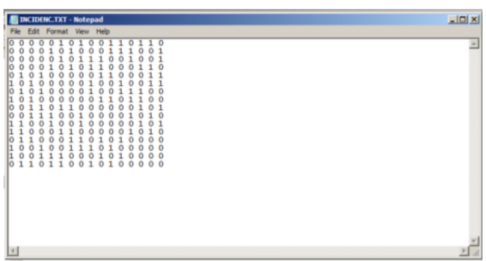

Figure 25-The adjacency matrix of the graph obtained from

H_4

Moreover, the regular graph (64,28,12,12) which can be obtained from the Hadamard matrix of order 8 is shown below.

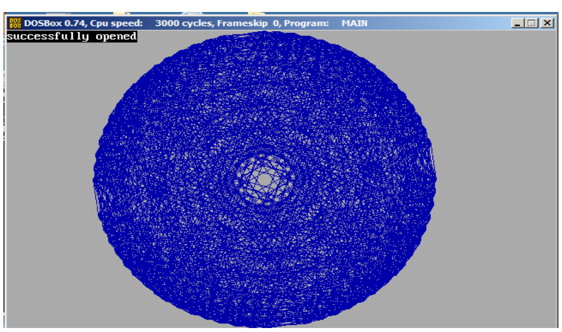

Figure 26- The regular graph obtained from H_8

Thus, from this method, one can obtain the strongly regular graph of order $16n^2$ from the normalized Hadamard matrix of order $4n$.At the same time, one can get the adjacency matrix of the relavant graph.

4 Conclusion

The adjacency matrix A of each graph is symmetric and the diagonal elements are all zeros and satisfies the properties $AJ = (2n^2 - n)J$ and $A^2 = n^2 I + (n^2 - n)J$,where I denotes the identity matrix of order v and J denotes the matrix with all ones. Hence, all the graphs generated from this method are strongly regular graphs with $v = 4n^2$,$k = 2n^2 - n$,$\lambda = \mu = n^2 - n$.

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