A Method To Find The Area Of Sector Without The Usage Of Angle Made By The Chord

Syed Abdul Lateef

ABSTRACT: This is the simplest method to find the area of the segment of the circle without the usage of the angle made by the chord and thus the area of sector could be found without the usage of angle made by chord. This is done by relating the area of segment to the area of sector.

INTRODUCTION-

As we know that to find the area of sector the angle made by the chord (that is chord which divides the circle) is required. But in the below method we find the ratio of the segments of the circle. Thus by relating the area of segment to the area of sector the area of sector could be found. The ratio of area of segments is related to tangents that are drawn through diameter on either side.

THEOREM-

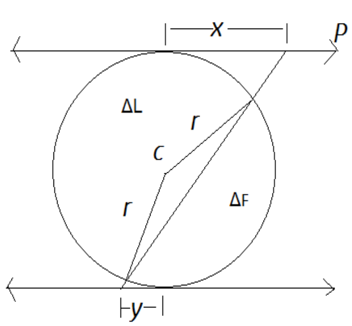
The two points considered at the alternate sides, of the tangents through the diameter of the circle, and then the line joining these points divides the circle in the ratio of the distances between the considered points to the points of contacts.

DIAGRAM AND CONSIDERATIONS-

Let the two points be D and E taken on the tangents P and U respectively to the circle of radius 'r' with centre C.

Let AB be the diameter and the distances between the considered points (D and E) and point of contacts (A and B) be x and y respectively.

The line DE divides the circle into two segments. Let the areas of the two segments be ΔL and ΔF respectively

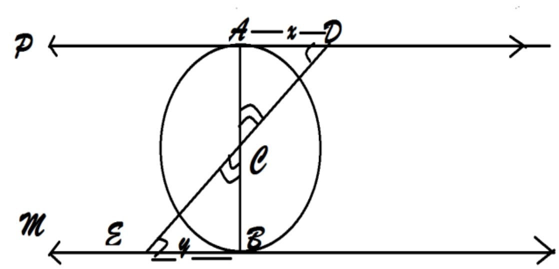


- *Syed Abdul Lateef, Address- D.No.5-114, R.T.C 2nd colony, Hindupur-515201, Andhra Pradesh, India.*
- *Institution: Christ University, Bangalore. B.Sc 1st Year, Email Id- abdullateefmath@gmail.com Contact- +918095968473/+919502686665.*

Now the area of segment is given by:
$\Delta L = \pi r 2x/(x+y)$ and $\Delta F = \pi r 2y/(x+y)$.
Thus the area of sector can be calculated by:
(1/2) × (length of chord× perpendicular distance from center to chord) + ΔF
Figure 1:

CASE 1: IF X=Y

Which implies $x/y = 1$.
Let the two points be D and E taken on the tangents P and M respectively to the circle with centre C.
Let AB be the diameter and the distances between the considered points (D and E) and point of contacts (A and B) be x and y respectively.
Let Q be the point of contact of lines AB and ED.
The line DE divides the circle into two segments. Let the areas of the two segments be ΔL and ΔF respectively.



Proof:
Consider the triangles ADQ and BEQ.
Given AD = BE. The angles AQD and BQE are equal (since opposite angles are equal).
The angles ADQ and BEQ are equal (since alternate angles are equal).
Therefore from A.S.A congruency, the triangles ADQ and BEQ are congruent.

So $AD/BE = AQ/BQ$
$x/y = AQ/BQ$ (since $x/y = 1$)
Therefore AQ = QB

Thus the point Q is equidistant from A and B (midpoint). But AB is the diameter. Hence the point Q coincides with the centre C and therefore the segments become semi-circles.
$\Delta L = \Delta F$ $\Delta L/\Delta F = 1$ and also $x/y = 1$
$\Delta L/\Delta F = x/y$ Thus the proof
Figure 2:

CASE 2: IF X IS NOT EQUAL TO Y (PROVIDED THE OPPOSITE ANGLES ARE 45^0)

Then x/y is not equal to 1.

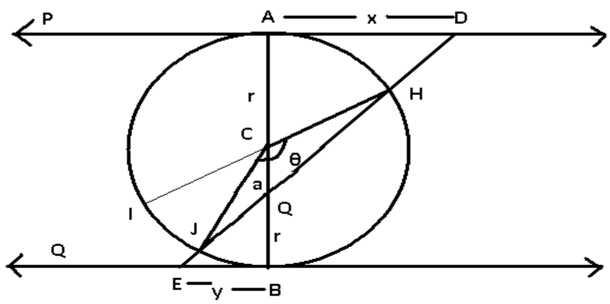
Say x/y = k (constant)

Let the two points be D and E taken on the tangents P and U respectively to the circle of radius 'r' with centre C.

Let AB be the diameter and the distances between the considered points (D and E) and point of contacts (A and B) be x and y respectively.

The line DE divides the circle into two segments. Let the areas of the two segments be ΔL and ΔF respectively.

The distance CQ is considered as 'a'.

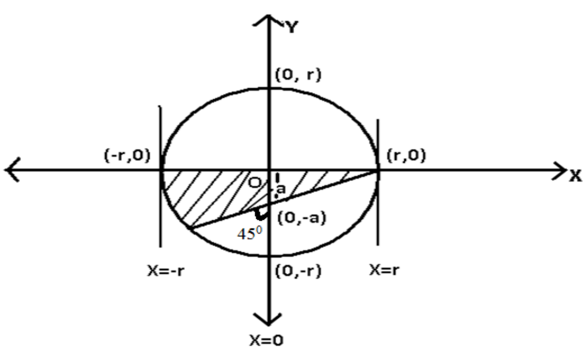


Proof: *Consider the triangles ADQ and BEQ.*

The angles AQD and BQE are equal (since opposite angles are equal).
The angles ADQ and BEQ are equal (since alternate angles are equal).
Therefore from A.A similarity, the triangles ADQ and BEQ are similar. So

AD/BE = AQ/BQ
x/y = AQ/BQ » x/y=(r+a)/(r-a)

Thus if we prove that ΔL/ΔF =(r+a)/(r-a) then the theorem is proved. Now let us consider a circle with centre at origin and radius 'r'. Thus the equation of circle is $x^2+y^2=r^2$.The equation of the chord at 'a' distance from center is ax-ry-ar=0 or Y= a/r(x-r).



Thus by the integration method of finding areas we get required area= (1/2) πra
So ΔL/ΔF= (area of major segment)/(area of minor segment = [(πr²)/2+ (πra)/2] ÷ [(πr²)/2-(πra)/2] =r+a/r-a =k. But k=x/y

So ΔL/ΔF = x/y Thus the proof

CONCLUSION-

Thus the area of sector can be found by relating it to area of segment where the area of segment is found without the usage of angle made by the chord.

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