# Probabilistic Framework For A Time Series

Siddamsetty Upendra

**Abstract:** This paper proposes probabilistic models of time series data in time series analysis. This accommodates models with a fitted drift and as time trend by defining the stationarity assumptions on time series to discriminate between stationarity and non-stationarity about a deterministic trend also defining the stochastic models for time series.

**Keywords:** Stationary process , non-stationary process , assumptions of stationarity, stochastic models for time series , Review of related Literature, Equations , Basic definitions and notations.

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## Introduction:

By definition, a time series X={X (t); t $\epsilon$ I} is an indexed family of random variables, defined on a given probability space (Ω, Γ, P), I being a countable subset of real line R. It is convenient to take I as the set of all integers. Towards studying the stochastic behaviour of a time series, let R (I) $=\underset{t \epsilon I}{X} \boldsymbol{R_t}$ , $\boldsymbol{R_t}$ being a copy of the real line R, so that R (I) will be the collection of all real sequences. By definition X is mapping from Ω to R (I) and as a notational convenience, the dependence of X on the typical element ω of Ω (and therefore that of X (t, ω) on ω) is suppressed, to write X (ω) =X and with the Borel σ-field β (I), which is the minimal σ-field generated by the class of all cylinder subsets of R (I), so that, [1]. The time series X induces a probability measure $\boldsymbol{P_X}$ on$\boldsymbol{\beta(I)}$, through P, given by [2]. This probability measure $\boldsymbol{P_X}$ is called the probability distribution of the time series X. By inference for time series X, it is meant to be any logical endeavour to draw inference on the probability distribution $\boldsymbol{P_X}$ of X, on the basis of partial realisation ( X(1), ---------, X(N)), say, of X. The probability distribution $\boldsymbol{P_X}$ of X, will lead to the corresponding family of finite dimensional distributions associated with X, a typical member being [3]. It may also be noted that the family of distribution functions in [3] will determine $\boldsymbol{P_X}$ by virtue of Daniel Kolmogorov Theorem, if they satisfy the consistency conditions.

## Stationarity Assumptions On Time Series

### Stationary Process:

In statistics, a stationary process is a stochastic process whose joint probability distribution does not change when shifted time. Consequently, parameters such mean and variance, if they are present, also does not change over time.

### Definition:

Formally, let $\{\boldsymbol{X_t}\}$ be a stochastic process and let $F_X\{x_{t_1+\tau},----,x_{t_k+\tau}\}$ represent the cumulative distribution function of the joint distribution of { $\boldsymbol{X_t}$} at times $t_1+\tau,----,t_k+\tau$ . Then, $\{\boldsymbol{X_t}\}$ is said to be strictly (or strongly) stationary if, for all k, for all $\tau$ , and for all $\boldsymbol{t_1}$, ----- , $\boldsymbol{t_k}$, $F_X(x_{t_1+\tau},----,x_{t_k+\tau})=F_X(x_{t_1},----,x_{t_k})$. Since $\tau$ does not affect $F_X(.)$ , $F_X$ is not a function of time.

### Example:

Let Y be any scalar random variable and define a time series $\{\boldsymbol{X_t}\}$ by $X_t=Y$ ; for all t. Then $\{\boldsymbol{X_t}\}$ is a stationary time series, for which realisation consist of a series of constant values, with a deferent constant value for each realisation. A law of large numbers does not apply on this case as the limiting value of an average form a single realisation takes the random value determined by Y, rather than taking E[Y]. As a further example of stationary process for which any single realisation has an approximately noise-free structure, let Y have a uniform distribution on (0,2π] and define the time series $\{\boldsymbol{X_t}\}$ by $X_t=\cos(t+Y)$; For t $\epsilon$ $\boldsymbol{R}$. Then $\{\boldsymbol{X_t}\}$ is strictly stationary.

## Non-Stationary Process:

A non-stationary time series will have a time-varying mean or a time-varying variance or both. A white noise process: $\in_t$ ~ N (0,$\boldsymbol{\sigma^2}$) such that it is independently and identically distributed (i.i.d) with $\operatorname{cov}(\in_t,\in_{t-s})=0$.

### Example:

A Random Walk Model (RWM) is a non-stationary process. There are two types:
(i) Without a drift and (ii) with a drift.

**(i) Without a drift:**

$Y_t=Y_{t-1}+\in_t$

Where value Y at time t is equal to its previous value and random stock. Efficient market hypothesis states that stock prices in efficient markets follow a random walk process without a drift such that there is no scope for profitable speculation in the stock market, the change in the in the stock price from one period to the next essentially random and unpredictable. We can write: $Y_1=Y_0+\in_1, Y_2=Y_1+\in_2, Y_3=Y_2+\in_3$ and replacing $Y_3=Y_0+\in_1+\in_2+\in_3$ such that $Y_t=Y_0+\sum\in_t$ . Therefore, $E[Y_t]=Y_0$ since errors have zero expectation, i.e., $E[\in_t]=0$. Similarly, $\operatorname{var}(Y_t)=t\sigma^2$ , i.e., it is dependent on time, not time invariant. Hence, RWM without drift is a non-stationary process. Although, it's mean is constant over time, its variance increase over time. In this model, shocks persist as the current value is equal to the initial value plus a series of random shocks over time. A random walk has a infinite memory.

**(ii) With a drift:**

$Y_t = \alpha + Y_{t-1} + \in_t$

We write $Y_1 = \alpha + Y_0 + \in_1, Y_2 = \alpha + Y_1 + \in_2, Y_3 = \alpha + Y_2 + \in_3$ and replacing $Y_3 = \alpha + \alpha + \alpha + Y_0 + \in_1 + \in_2 + \in_3$ such that $Y_t = \sum \alpha + Y_0 + \sum \in_t$. Therefore, $E[Y_t] = \alpha t + Y_0$ since errors have zero expectation, i.e., $E[\in_t] = 0$. Similarly, $\mathrm{var}(Y_t) = t\sigma^2$. Hence, RWM with a drift is non-stationary process. Both its mean and its variance increase over time such that it is again a non-stationary process. A time series X={X (t); t $\epsilon$ I}, (I being the set of integers) is said to be strictly stationary if for any k ≥ 1, $t_{1,\_\_\_\_\_\_\_}, t_k$ distinct members of I and for any t∈I. [4]. A time series X={X (t); t $\epsilon$ I} is said to be stationary in the wide sense if [5], Where Q (.) Is an everywhere finite real valued function and Q (.) is also known as the auto covariance function associated with the wide sense time series X. Stationarity is statistically convenient assumption. Although many time series do not possess this property, the assumption of stationarity is a common practice in the statistical analysis of time series. If one were to classify a time series as non-stationary whenever it is not stationary (in either sense), a usual approach is to identify and isolate the non-stationary part, so that the residual behaves as a stationary time series. From this context, the assumption of stationarity helps indirectly to solve some problems relating to time series which is non-stationary in some sense. Further the fact that strict stationarity is preserved when taking in probability or in mean square of a sequence of strictly stationarity time series, the asymptotic theories can be developed without much difficulty. Towards covering same non-stationary time series which behave as stationary time series, when't' is sufficiently large, one can introduce the concept of asymptotic stationarity. A time series X={X (t); t $\varepsilon$ I} is said to be asymptotically stationarity in the wide sense if [6], Where Q (.) is the auto covariance function of wide sense stationary time series. An immediate advantage of this assumption over wide sense stationarity is that one can restrict I to be the set of positive integers, (on defining, if necessary, X (t) to be arbitrary constant whenever t≤ 0). Further the assumption of wide sense stationarity provides a new dimension to time series analysis in terms of Spectral theories. The spectral representation theorem asserts the existence of a unique non-negative, non-decreasing and right continuous function F (λ) on [-π, π] such that [7], $\mu_F$ being the Borel Measure induced by F on the Borel class associated with R. F(**λ**) is called the spectral distribution function of the time series X, and, if F(**λ**) is such that [8], Then F (**λ**) is called the spectral density function of X and is given by [9], Whenever $\sum_{r \in I} Q(r)$ is absolutely convergent. The problems associated with the estimation of F (**λ**) have been widely discussed in the literature (Anderson (1971), Bartlett (1966), Fuller (1976), Hannan (1960)). One can define the asymptotic spectral distribution function and the asymptotic spectral density function for an asymptotically wide sense stationary time series, using Q (.) introduced in (6). Venkataramana (1972) has highlighted the estimation methods relating to such spectral density function.

**Stochastic Models for Time Series**

What distinguishes a time series from other discrete parameter stochastic processes is the prevalence of dependence between successive components of the time series, with the nature of dependence varying as one move along the time axis. Stochastic models are constructed with the primary objective to highlight mathematically and form a probabilistic angle, such a dependence that exists in a time series. The dependence among the components may be either self induced (endogenous) or due to extraneous forces (exogenous). A general stochastic model for exposing such dependence may include lagged variables to explain the endogenous dependence, and, regressors to explain the exogenous dependence, in addition to error terms to introduce the stochastic nature of the time series. Linear versions of such models have been widely discussed in the literature, a typical model having specification [10] Where (i)$f_i$ (t) are known real valued functions of t satisfying certain regularity conditions and (ii) Z (t); t≥ $\mathbf{1}$ is an error process with a specified probability distribution, but for a finite number of constants occurring in the distribution. The dynamic stability of the time series that flows from [10] depends on the location of the roots of its characteristic polynomial [11]. with reference to the unit circle. If all the roots lie within the unit circle, the time series is said to be autoregressive in nature. Such time series, with regression component have been discussed in the literature, under varying assumptions on Z(t) [ (Anderson (1971), Fuller (1976), Hannan and Nicholls (1972), Fuller, Hasza and Goebel (1981)] , and Venkataramana and Viswanathan (1981-(a))]. Fuller etal (1981), and Venkataramana and Viswanathan (1981)-(a), (b)) have also studied the explosive situations when some or all the roots of $\bar{P}$(Z) = 0 lie outside the unit circle. Stochastic models for non-stationary time series are constructed so as to identify and isolate the non-stationary part of the time series, so that their elimination will result in a process which is stationary in some sense. Estimation techniques associated with models, have been developed, solely resting on such possibility. In many applications, X (t) will turn out to be vector valued, resulting in X being a vector (multiple) time series. Stochastic models for vector time series will then have to be a vector version of the models of the type (10). In this context, simultaneous linear models, with as many equations as there are components in X (t), which include lagged variables, find their applications in quantitative econometric research. The simultaneous linear model, generating a pair $\boldsymbol{X_1}$={$\boldsymbol{X_1(t)}$, $\boldsymbol{t\epsilon I}$}, $\boldsymbol{X_2}$={$\boldsymbol{X_2(t)}$, $\boldsymbol{t\epsilon I}$} of related time series, considered in this , has the specification.[12], Where f (t) and g (t) are linear functions of known real valued functions of t, and{$\boldsymbol{\epsilon_1(t)}$, $\boldsymbol{t\epsilon I}$}, and {$\boldsymbol{\epsilon_2(t)}$, $\boldsymbol{t\epsilon I}$} are error processes governed by some probabilistic assumptions. This simple simultaneous linear model of first order in two variables, is considered in this only to expose the problems (and methods of solving them), encountered in the process of statistical estimation of the unknown parameters in (12).One can easily generalise the techniques and arguments put forth, without any loss of rigour in such generalisation. However the underlying algebra gets aggravated when either the number of variables is increased or the order of the lag is increased.

## Review of Related Literature

Anderson (1971) and Fuller (1976) have studied the problems of estimation relating to the model (10) generating a scalar time series. Hannan and Nicholls (1972) have also considered the model (10) when Z (t) is generated by a moving average process. Recently Venkataramana and Viswanathan (1981)-(a), (b)) have considered the least squares estimation of the coefficients in (10), obtained by missing the sum of squares [13] with respect to these unknown coefficients. They have imposed Anderson-Hannan type regularity conditions (vide Anderson (1971) p.572) on $f_i$(t). To be specified, let

(i) $F_i(N) = F_i = \sum_{t=1}^{N} f_i^2(t) \to \infty$ as N→ ∞

(ii) Among ( $F_i$ , i=0,1,........,p) there exists a F(N) = F such that as N→ ∞ $F^{-1/2}F_i^{1/2} \to \Theta_i$ (Real) $F^{-1/2}N^{1/2} \to \partial_0$ (Real)

(iii) As N→ ∞,

$$F_i^{-1/2}F_j^{-1/2} \sum_{t=1}^{N-Max(m,n)} f_i(t+m) f_j X(t+n) \to T_{ij}(m,n)$$

(Real)

(iv) The matrix T = $(T_{ij}(0,0), i,j = 0,1,\ldots\ldots,p)$ is non-singular

(v) Letting $c_i(N) = c_i = max|f_i(t)|; t = 1,2,\ldots..,N$ $N^{\partial}F_i^{-1/2}c_i(N) \to 0$ as N→ ∞,With $\partial = \frac{1-\alpha}{2}$ and $\frac{1+\alpha}{4}$ for some $\alpha \varepsilon (0,1)$, for i=0,1,...,p.

(vi) For any root ∅, say, of $\bar{P}$(Z) lying outside the unit circle, the following requirements hold: $\emptyset^{-N}F^{1/2}$ $\emptyset^{-N}F_i^{1/2} \to 0$ as N→ ∞

(a) $\sum_{t=1}^{\infty} \phi^{-t} f_i(t)$ is absolutely convergent.

(b) $\sum_{t=1}^{\infty} \phi^{-t}(\in(t) + \sum_{i=0}^{p} \alpha_i f_i(t))$ (defined in the mean square sense) is continuous at zero.

Venkataramana and Viswanathan (1981-(a),(b)) have studied the least squares estimation under the following three possibilities:

**CASE-A:** All roots of $\bar{P}(Z)$ lie within the unit circle (Autoregressive time series)

**CASE-B:** Some roots of $\bar{P}(Z)$ lie outside the unit circle and others lie within the unit circle. (Partially explosive time series).

**CASE-C:** All roots of $\bar{P}(Z)$ lie outside the unit circle. (Purely explosive time series)

Further they assume that the roots whenever they lie outside the unit circle are real and distinct. Under Case-A, setting Z(t) to be independent errors with mean zero, common variance and finite fourth order moments, it has been show that if M is the coefficient matrix of normal equations determining the least square estimators $(\hat{\beta}, \hat{\alpha})$, [14]. The following result has been established.

**PROPOSITION (I):** Under Case-A, let $d_1 \neq 0$, and then $(F^{-1/2}(\hat{\beta}-\beta), F_i^{-1/2}(\hat{\alpha}_i - \alpha_i), i = 0,1,----,p)$ converges in law, as N→ ∞, to a normal vector with mean zero and a non-singular covariance matrix. Under Case-B, let the k-roots of $\bar{P}(Z)$ have the placement:

$$|\phi_1| > |\phi_2| > \ldots. > |\phi_m| > 1 > |\rho_i| \quad i=1,2,------, q=k-m$$

With m≥1, q≥1, so that k≥ $\mathbf{2}$. Then under the same choice of Z(t) under Case-A, it is shown that

$$F^{-q} \underset{i=1}{\overset{m}{\pi}} \phi_i^{-N} \underset{\mu=0}{\overset{p}{\pi}} F_\mu |M| \xrightarrow{P} d_2 \text{ (say) as N} \to \infty .$$

Under such circumstances the following result holds.

**PROPOSITION (II):** Under Case-B, let $d_2 \neq 0$, Then $(F^{1/2}(\hat{\beta}-\beta), F_i^{1/2}(\hat{\alpha}_i - \alpha_i), i = 0,1,----,p)$ converges in law, as N→ ∞, to a normal vector with mean zero and a singular covariance matrix of order p+q+1. The singularity of the limiting distribution is due to m linear relations between the components of the limiting vector. Under Case-C, the roots of $\bar{P}(Z)$ have the placement [15]. Under this case Z (t) can be a linear process which will include independent errors, moving averages and auto regressively generated errors as particular cases. It is shown that [16]. This is nonzero almost surely. Then the following result holds.

**PROPOSITION (III):** Under Case-C, $(\phi_k^N(\hat{\beta}-\beta))$ converges in law, as N→ ∞ to random vector with covariance matrix of rank unit. Further $(F_i^{1/2}(\hat{\alpha}_i - \alpha_i); i = 0,1,----,p)$ converges in law, as N→ ∞ to a normal vector with mean zero and a non-singular covariance matrix. It is shown that $d_1$ and $d_2$ is zero for the choice $f_i(t) = t^i$ (polynomial regression) although for this choice of $f_i(t)$, PROPOSITION (II) is satisfied. However, under purely explosive situation, the choice $f_i(t) = t^i$ is covered under PROPOSITION (III). It is also concluded that $\partial = 0$ is a sufficient condition for $d_1$ and $d_2$ is to be zero, although this is not necessary condition. This left the problem of least squares estimation open under polynomial regression under Case-A. It is relevant to note her that Fuller et al (1981) have also considered the problem of estimation of the coefficients [10] under both auto regressive and explosive placements of roots of $\overline{P}(Z)$ they have provided consistent and asymptotically normal estimators, even under polynomial regression after effecting Gram-Schmidt reparametrization techniques. Comparatively, the studies of vector versions of [10] are rare in literature, especially under explosive situation. Venkataraman (1974) made a detailed attempt to study the least squares estimation relating to the model [10], when f(t) and g(t) are constants. It can be check that components $X_1(t)$ and $X_2(t)$ satisfy the stochastic difference equations of second order with the same characteristic polynomial, given by [17]. One can view this common characteristic polynomial, as the characteristic polynomial associated with the model [12] (although it is not mentioned in the literature specifically). The dynamic stability of the two processes is therefore, linked to the placement of roots $\rho_1\rho_2$, say, of P (Z) with reference to the unit circle. Setting $f(t)=\alpha_{13}$, $g(t)=\alpha_{23}$ and the asymptotic properties of the least squares estimator $(\hat{\alpha}_1,\hat{\alpha}_2)=(\hat{\alpha}_{11},\hat{\alpha}_{12},\hat{\alpha}_{13},\hat{\alpha}_{21},\hat{\alpha}_{22},\hat{\alpha}_{23})$ obtained by minimising the sum of squares [18], With respect to $(\alpha_{11},\alpha_{12},\alpha_{13},\alpha_{21},\alpha_{22},\alpha_{23})$, under the following situations: [19]. The major findings of the investigations are revealing that Under [19]-(i) and when $\in_1(t)$ and $\in_2(t)$ are independent errors with mean zero and with common variances $\sigma_1^2$ and $\sigma_2^2$ respectively, $(N^{1/2}(\hat{\alpha}_1-\alpha_1),N^{1/2}(\hat{\alpha}_2-\alpha_2))$ converges in law as N→ ∞ to a normal vector with mean zero and a non-singular covariance matrix. Under [19]-(ii), $\in_1(t)$ and $\in_2(t)$ are in (a) above, and under certain conditions normally imposed on explosive models $(N^{1/2}(\hat{\alpha}_1-\alpha_1),N^{1/2}(\hat{\alpha}_2-\alpha_2))$ converges in law as N→ ∞ to a normal vector with mean zero and a non-singular covariance matrix of rank 4. Under [19]-(iii) and even when $\{\in_1(t),t\geq 1\}$ and $\{\in_2(t),t\geq 1\}$ are linear processes $(\rho_2^N(\hat{\alpha}_{11}-\alpha_{11}),\rho_2^N(\hat{\alpha}_{12}-\alpha_{12}),\rho_2^N(\hat{\alpha}_{21}-\alpha_{21}),\rho_2^N(\hat{\alpha}_{22}-\alpha_{22}))$, Converges in law, as N→ ∞ to a normal vector with mean zero, and a non-singular covariance matrix of rank 2. However $(N^{1/2}(\hat{\alpha}_{13}-\alpha_{13}),N^{1/2}(\hat{\alpha}_{23}-\alpha_{23}))$ has limiting bi-variant normal distribution. Under [19]-(iv) and even when $\{\in_1(t),t\geq 1\}$ and $\{\in_2(t),t\geq 1\}$ are linear processes $(\rho_0^N N^{-1}(\hat{\alpha}_{11}-\alpha_{11}),\rho_0^N N^{-1}(\hat{\alpha}_{12}-\alpha_{12}),\rho_0^N N^{-1}(\hat{\alpha}_{21}-\alpha_{21}),\rho_0^N N^{-1}(\hat{\alpha}_{22}-\alpha_{22}))$ has limiting non-normal distribution with covariance matrix of rank 2. Further, $(N^{1/2}(\hat{\alpha}_{13}-\alpha_{13}),N^{1/2}(\hat{\alpha}_{23}-\alpha_{23}))$ has limiting bi-variant normal distribution.

**Equations**

$\boldsymbol{X^{-1}(B)\in\Gamma,\ \forall B\in\beta(I)}$ ------- [1]

$\boldsymbol{P_X(B)=P(X^{-1}(B))}$ , $\forall \boldsymbol{B\in\beta(I)}$ ---------- [2]

$\boldsymbol{F_{t_1\text{-------}t_k}(X(t_1),-----,X(t_k))=P\{\ \omega\ /X(t_1\omega)\leq X(t_1),-----,X(t_k\omega)\leq X(t_k)\}}$ ------ [3]

Where (i) k is a positive integer,
(ii) $\boldsymbol{t_1}$,_______, $\boldsymbol{t_k}$ are distinct members of I, and
(iii) $\boldsymbol{X(t_1)},-----,\boldsymbol{X(t_k)}$ are real numbers.
$(\boldsymbol{X(t_1)},-----,\boldsymbol{X(t_k)})$ And
$(\boldsymbol{X(t_1+t)},-----,\boldsymbol{X(t_k+t)})$ --------- [4]

are identically distributed.

$$\left.\begin{array}{l}\mathrm{E}\,(\mathrm{X(t)})=\mu\\ \mathrm{E(X(t)}-\mu)\,(\mathrm{X(s)}-\mu)=\mathrm{Q}(|\,\mathrm{t-s}\,|)\end{array}\right\}\quad \text{-----------------} [5]$$

$$\left.\begin{array}{ll}\mathrm{E(X))}\to\mu & \text{as } t\to\infty\\ \mathrm{Cov}\,(\mathrm{X\,(t),X(\,t+s)})\to\mathrm{Q(s)} & \text{as } t\to\infty\end{array}\right\}\quad \text{--------} [6]$$

$$\mathrm{Q(s)}=\int_{-\pi}^{\pi}\exp(\lambda)\,d\mu_F \quad \text{--------------------------------} [7]$$

$$\mathrm{F}\,(\boldsymbol{\lambda})=\int_{-\pi}^{\lambda} f(\mu)\,d_{\mu} \quad \text{-----------------------------------} [8]$$

$$\mathrm{F}\,(\boldsymbol{\lambda})=(\boldsymbol{2\pi})^{-1}\sum_{r=-\infty}^{\infty}Q(r)\ \mathbf{cos\,\lambda_r}\ ;\ -\boldsymbol{\pi}\leq\boldsymbol{\lambda}\leq\boldsymbol{\pi}\text{---------} [9]$$

$$X(t+k)-\boldsymbol{\beta_1 X(t+k-1)}-\ \ldots\ldots\ldots.-\boldsymbol{\beta_k X(t)}-\sum_{i=0}^{p}\alpha_i f_i(t+k)=z(t+k)\ ;\ \ t\geq \mathbf{1}\ \text{----} [10]$$

$$\boldsymbol{\overline{P}}(z)=\boldsymbol{Z^k-\beta_1 Z^{k-1}}-\ \ldots\ldots\ldots-\boldsymbol{\beta_k}\quad \text{------------------} [11]$$

$$\begin{array}{l}\boldsymbol{X_1(t+1)-\alpha_{11}X_1(t)-\alpha_{12}X_2(t)-f(t+1)=\epsilon_1(t+1)}\\ \boldsymbol{X_2(t+1)-\alpha_{21}X_1(t)-\alpha_{22}X_2(t)-f(t+1)=\epsilon_2(t+1)}\end{array}\quad \text{----} [12]$$

$$\sum_{t=1}^{N-k}(X(t+k)-\beta_1X(t+k-1)-\ldots\ldots-\beta_kX(t)-\sum_{i=0}^{p}\alpha_i f_i(t+k))^2 \quad \text{-------} [13]$$

$$F^{-k}\mathop{\pi}_{i=0}^{p}F^{-1}|M|\xrightarrow{P}d_1 \quad \text{(say) as N}\to\infty \quad \text{--------} [14]$$

$$|\phi_1|>|\phi_2|>\ldots.>|\phi_k|>1 \quad \text{-----} [15]$$

$$\mathop{\pi}_{i=1}^{k}\phi_i^{-N}\mathop{\pi}_{\mu=0}^{p}F_{\mu}|M|\xrightarrow{P}d_3 \quad \text{(say)} \quad \text{------} (16)$$

$$P(Z)=Z^2-(\alpha_{11}+\alpha_{22})Z+(\alpha_{11}\alpha_{22}-\alpha_{12}\alpha_{21}) \quad \text{-----} [17]$$

$$\sum_{t=1}^{N-1}(X_1(t+1)-\alpha_{11}X_1(t)-\alpha_{12}X_2(t)-\alpha_{13})^2+ \sum_{t=1}^{N-1}(X_2(t+1)-\alpha_{21}X_1(t)-\alpha_{22}X_2(t)-\alpha_{23})^2$$ ------ [18]

(i) $|\rho_i|<1$ i =1,2 (autoregressive)

(ii) $|\rho_1|>1>|\rho_2|$ (partially explosive)

(iii) $|\rho_1|>|\rho_2|>1$ (purely explosive)

(iv) $\rho_1=\rho_2=\rho_0;\ |\rho_0|>1$ (purely explosive)------ [19]

## Basic Definitions And Notations

A sequence $\{(X_{1n},\dots\dots,X_{kn})\}$ of random vectors is said to converge in law $\left(\xrightarrow{\mathcal{L}}\right)$ or in distribution, as n→ ∞, to a random vector $(X_1,\dots\dots,X_k)$ say if $P(X_{1n}\le x_1,\dots\dots,X_{kn}\le x_k)\to P(X_1\le x_1,\dots\dots,X_k\le x_k)$

$=F_{X_1,\dots\dots,X_k}(x_1,\dots\dots,x_k)$ (Say) at every continuity point of F.

A sequence $(X_{1n},\dots\dots,X_{kn})$ of random vectors, is said to converge in probability ($\xrightarrow{P}$) as n→ ∞, to a random vector $(X_1,\dots\dots,X_k)$ say, for every $\in>0$ P $(|X_{in}-X_i|\ge\in)\to 0$ as n→ ∞, for i=1, 2, ……., k. A sequence $X_n$ of random variable is said to be bounded in probability if given$\in>0$, there exists a positive constant M (∈) such that $\lim_{n\to\infty}\sup P\,(|X_n|>M(\in))<\in$. The boundedness in probability of the sequence $X_n$ is also implied by the stronger requirement that $\lim_{n\to\infty}\sup E|X_n|\le A_0A_0$ being a positive memorizing constant. The internal implications of convergence types relating to a sequence of random variables, namely Convergence in probability

$\Longrightarrow$ Convergence in law

$\Longrightarrow$ Boundedness in probability

The proofs of lemmas and theorems based on this discussion are concluding on an appeal to standard convergence theorems. The following proposition summarises these convergence theorems, for an easy reference.

**PROPOSITION:** Let $((X_n,Y_n),n\ge 1)$ be a sequence of bivariate random variables on a given probability space. Then the following statements hold.

(a) If $X_n\xrightarrow{\mathcal{L}}X$ and $Y_n\xrightarrow{P}c$ (constant) then

(i) $X_n+Y_n\xrightarrow{\mathcal{L}}X+c$

(ii) $X_n.Y_n\xrightarrow{\mathcal{L}}X.c$

(iii) $X_n/Y_n\xrightarrow{\mathcal{L}}X/c$ (if c$\neq 0$)

(b) If $(X_n-Y_n)\xrightarrow{P}0$, $X_n\xrightarrow{\mathcal{L}}X$ then $Y_n\xrightarrow{\mathcal{L}}X$

(c) If $X_n\xrightarrow{P}X$ and $Y_n\xrightarrow{P}Y$ imply

(i) a$X_n\xrightarrow{P}aX$ (a is real)

(ii) $X_n+Y_n\xrightarrow{P}X+Y$

(iii) $X_n.Y_n\xrightarrow{P}X.Y$

(iv) $X_n/Y_n\xrightarrow{P}X/Y$
If P $(Y_n=0)=P(Y=0)=0$.

(d) If f(X) is continuous and $X_n\xrightarrow{P}X$, then f $(X_n)\xrightarrow{P}f(X)$

(e) If $(X_{1n},\dots\dots,X_{kn})\xrightarrow{\mathcal{L}}(X_1,\dots\dots,X_k)$ then $(l_1X_{1n}+\cdots\dots+l_kX_{kn})\xrightarrow{\mathcal{L}}(l_1X_1+\cdots\dots+l_kX_k)$ Whenever $l=(l_1,\dots\dots,l_k)\neq 0$.

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