Approximate Method Of Calculation Of Reinforcement Connection With Wood When Pulling Out The Reinforcing Bar

Turdimurod Makhmatkulov

Abstract: Currently, special attention is paid to the connections of elements of wooden structures. The most effective methods of connections are connections on glued rods. The article presents an approximate method for calculating the connections of reinforcement with wood depending on different types of loads.

Keyword: Theoretical research, reference node, glued columns, engineering calculation, analytical dependence, glued connections on the terminals, wooden glued column, basic assumption, stress-strain state, movements, rigidity, sectional.

1 INTRODUCTION.

Theoretical research of support units of glued columns under the action of compressive forces and bending moments is carried out in order to establish analytical dependencies describing the stress-strain state of these units and the creation on their basis of a reliable engineering method of calculation. The article presents the solution of problems using the finite element method, as well as an approximate calculation of the connection of reinforcement with wood used in the support units of columns. As is known, the stress-strain state of the support units of the columns, the clamping of which is provided by means of adhesive joints of reinforcement with wood, is characterized by great complexity. Therefore, it seems appropriate to develop approximate methods of calculation, aimed at identifying simple in form analytical dependencies, which should be sufficiently true to identify the values of the forces pulling or pressing glued into the wood rods, from factors such as the design and size of the adhesive joint, as well as elastic and mechanical characteristics of the materials working together [1,2,3]. When constructing an approximate method of calculation, the author considers it possible to liken the work of a rod glued into a column to its work in a prism with the dimensions of the sides equal to 5-7 diameters of the glued rod, which corresponds to the ratio of the areas of reinforcement and wood μ = 0.14 - 0.20 [4,5]. This assumption agrees quite satisfactorily with the theoretical data obtained by the finite element method. In this regard, we also note that according to the requirements, the strength of the adhesive joints of steel rods with wood is determined on samples-prisms of square section 6d x 6d with a wasp arrangement of the rods [6,7]. In addition to the above assumption, it is also assumed that the joined materials are isotropic and obey Hooke's law: the elastic modulus of materials under compression and tension are assumed to be equal; the thickness of the adhesive seam along the entire length of the seam is constant; the end face of the steel rod is not glued to the wood (there is a gap between them); the normal stress across the cross section of the glued rod and the wooden prism is distributed evenly [1,2,4].

Under the adopted assumptions calculation of bonded joint of a steel rod with wood can be reduced to solving a one-dimensional problem, which allows obtaining a convenient for engineering practice, simple expressions for calculating the bars when pulling and pressing, and connections of the rods with the wood - chipping. The influence of the volume stress state is indirectly taken into account by the proportionality coefficient (Gc) determined experimentally [2]. In the support units, working on the influence of longitudinal compressive forces and bending moments, the most stressed zone is the end section of the column. The operation of this zone depends on the method of fastening the rods to the steel Shoe or directly to the Foundation. If fastening of rods to a Shoe is carried out by means of threaded connection (Fig.2.), perceiving only tensile forces, steel rods, located in the compressed zone of the cross section, are involved in the work of compression only due to the work of the adhesive joint on the shift (Fig. 1.). Consider the solution of the problem for some cases of rods in the support nodes. In any cross-section X within the adhesive joint, the equilibrium between the compressive force N and the forces on the steel rod Nc (x) and the wooden prism Nd (x) is determined by the conditions:

\[ Nc(x) + \frac{N_d}{2} = N \quad (1) \]

\[ N_d = N - Nc(x) \quad (2) \]

The relationship between the shear force T(x) on the adhesive joint and the difference between the displacements of the steel rod Uc (x) and the prism wood Ud (x) is a condition of compatibility of deformation and can be written as:

\[ T(x) = d \cdot \frac{Nc(x)}{dx} = Gc \cdot (U_d(x) - Uc(x)) \quad (3) \]

Where U c (x) - steel rod travel; U d (x) - moving the wooden element; Gc - the proportionality factor or shear modulus of the joint.

This coefficient is determined by shear testing of samples of connections with a short length of the insert (about five diameters of the steel rod).

After differentiation (1,2) take the form:

\[ \frac{\partial^2 Nc(x)}{\partial x^2} = \varepsilon c(x) - \varepsilon d(x) \quad (4) \]

If, as a first approximation, we assume that the normal stresses over the cross-section of the wooden prism are uniformly distributed, then the relative deformations \( \varepsilon c(x), \varepsilon d(x) \) are equal:

\[ \varepsilon c(x) = \frac{Nc(x)}{Ec \cdot Fc} \quad (5) \]
\[ \varepsilon \delta(x) = \frac{N}{E_c} \delta(x) / E_c \cdot F_c \quad (6) \]

Where \( \varepsilon, \varepsilon_c \) - accordingly, the modules of elasticity of steel and wood;

\( F_c, F_d \) – the cross-sectional area of the steel rod and the wooden prism.

**Fig. 1.** Design schemes of connections on the pasted rods at pulling out of a reinforcing bar.

After substituting the values of relative deformations expressed taking into account the boundary conditions, we obtain the following second-order differential equation with constant coefficients:

\[ \frac{\partial^2 N c(x)}{\partial x^2} = G_c \left[ 1 / H_c + 1 / H_d \right] \cdot N c(x) - N / H_c \]

Then the equation takes the form:

\[ \frac{\partial^2 N c(x)}{\partial x^2} = a_2 \frac{N c(x) - n N}{(1+n)} \]

(7)

The General solution of this equation can be represented in the form:

\[ N c(x) = C_1 \cdot \sinh(x) + C_2 \cdot \cosh(x) \quad (8) \]

For the case where the glued rods are working on pulling out (Fig.1) substituting the boundary conditions in equation (8), we obtain a system of two equations:

\[ N c(0) = C_1 \cdot \sinh(x) = 0 \quad \text{by} \quad x = 0, \]

\[ N c(l) = C_1 \cdot \sinh(l) + C_2 \cdot \cosh(l) = n \quad \text{by} \quad x = l. \]

From which we find the value of the constant coefficients \( C_1 \) and \( C_2 \):

\[ C_1 = \frac{n}{\sinh(l)} \quad \text{and} \quad C_2 = \frac{-n \cdot \cosh(l)}{\sinh(l)} \]

(9)

Substitute the found values \( C_1 \) and \( C_2 \) in the solution of equation (8), we obtain:

\[ N c(x) = \frac{n}{\sinh(l)} \left[ \sinh(x) + \frac{1}{\cosh(l)} \cdot \cosh(l) \cdot \sinh(x) - \sinh(x) \right] \]

\[ = \frac{n}{\sinh(l)} \cdot \sinh(x) \quad (10) \]

Transforming the expression (9) taking into account the above boundary conditions, we find the value of the force in any section \( X \) for a steel rod:

\[ N c(x) = \frac{n}{\sinh(l)} \cdot \sinh(x) \quad (10) \]

The value of the shear force on the adhesive joint after differentiation of the expression will be equal to

\[ T(x) = \alpha \cdot N \left[ \cosh(x) \left[ \cosh(x) - \cosh(l) \right] \right] / (1+n) \sinh(l) \]

(11)

The displacements of the connection according to scheme b are determined from the sum of the deformations of the wooden element \( U_d(x) \) and the steel rod \( U_c(x) \):

\[ U(x) = U_c(x) - U_d(x) \quad (12) \]

The force in the steel rod after conversion (10) can be found by the formula:

\[ \mathcal{N}_c(x) = \frac{n}{1+n} \left[ \sinh(x) - \sinh(l) \right] \]

(13)

and the force that falls on the wooden element is determined from the expression

\[ \mathcal{N}_d(x) = \frac{n}{1+n} \left[ \sinh(x) - \sinh(l) \right] \]

(14)

Movements in steel and wood elements

\[ U_{c,0} = U_{c,0} + \int_0^l \mathcal{N}_c(x) \, dx \]

\[ U_{d,0} = U_{d,0} + \int_0^l \mathcal{N}_d(x) \, dx \]

Substituting the values \( U_{c,0} \) and \( U_{d,0} \) in the expression (12), we get:

\[ U(x) = U_{c,0} - U_{d,0} = \frac{T(x)}{G_c} \]

(15)

\[ \alpha(x) = \alpha(1+n) \cdot \sinh(x) \]

Therefore, when \( \mathcal{N}_d(0) = 0 \)

\[ U_{d,0} = \frac{n}{(1+n) \sinh(x)} \left[ \cosh(x) - \cosh(l) \right] \]

After substituting the value \( U_{d,0} \) into the expression (15), the displacements will be equal to:

\[ U_{c,0} = \frac{n}{(1+n) \sinh(x)} \left[ \cosh(x) - \cosh(l) \right] \]

(16)

Thus, analytical dependences are obtained to determine the stress-strain state of the rebar connection with wood depending on the loading scheme. In addition, expressions are obtained to determine the displacement for the considered cases of connections. Let us proceed to the comparison of the results of the theoretical study with the data obtained by numerical methods, that is, by machine calculation using the finite element method [1,6]. It should be noted that the analytical dependences obtained above (1-8) make it possible to study the peculiarities of changes in normal and tangential stresses along the length of the glued rod. The theoretical values of deformations in rebar \( E_r \) and in wood \( E_w \) are obtained from expressions (5) by dividing the forces \( N_c(x) \) and \( N_d(x) \) obtained by the formula (13) by the deformation modules of rebar \( E_c \) and wood \( E_d \cdot F_d \). On tab. 1 and 2 are the values and in Fig.2 a, b shows graphs of the distribution of relative strains and shear forces along the length of the rods obtained by the above formulas, as well as by machine calculation using the finite element method, at a load of \( N = 50 \) kN [6].

The change in relative deformation along the length of the paste-in-rods.

**Table 1.**

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<th>The test setup</th>
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**Table 2.**

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b)

**Fig. 2.** Distribution of relative deformations (a) and shear forces (b) along the length of the glued rod during pulling.

In conclusion, we note that the above formulas can be easily converted into a convenient form for practical calculations. For example, the calculated bearing capacity of the rod-wood joint at shear can be determined from the expression 17 at \( X = 0 \) and \( n < 1 \).

\[
\tau = \frac{T}{S} = \frac{\alpha n}{S_{(1+\eta)\text{shal}}} [n + \text{chal}] \leq R_{\text{ck}} \quad (17)
\]

Hence the calculated bearing capacity of the rod when pulling is equal to

\[
N = R_{\text{ck}} \frac{S_{(1+\eta)\text{shal}}}{a[1+(\text{chal})]} = R_{\text{ck}} \cdot K_{c,\delta} \cdot S \cdot I_{ck} \quad (18)
\]

The coefficient \( K_{c,\delta} \), which takes into account the uneven distribution of shear stresses along the rod, can be represented in tabular form depending on the ratio \( I/\delta \). The proposed method of calculation of support nodes is based on the finite element method. It allows you to accurately characterize the work of these nodes under the action of appropriate loads. The proposed solution of the problems allows us to determine the stress-strain state of the connection on the value of the rods at any combination of loads, the results of which are in good agreement with the results of previous studies. The developed theory allows us to identify the calculated length of the column, taking into account the flexibility of the nodes. According to the results of the method of determining the deformability of the support node with metal stepchildren, expressions are obtained that accurately describe the deformed state of these nodes.

**REFERENCES**