Reflective Abstraction: How Can You Find Out In Mathematatcis Learning

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Abstract: This study aims to find out how students' reflective abstraction can be indentified in the process of learning mathematics. Research design uses descriptive qualitative research. The study involves 36 students as research subjects. Data validation uses data triangulation and method triangulation. Research results: reflective abstraction students can more easily appear and be known by using and demonstrating apperception in the learning process; in the learning process in class if the concept has been conveyed directly by the teacher, the reflective abstraction of the student can still be known by giving feedback in the form of asking students about the opposite concept as long as it does not violate the existing concept; by being given a test in the form of questions that represent students' knowledge of the concepts before and continuing about new concepts to be learned by students; by conducting interviews that can be carried out during the learning process or after the learning process is carried out in order to confirm what cannot be observed directly. The two parts of reflective abstraction can be identified in the same way, but only different types of questions are adapted to the parts of each reflective abstraction. The first reflective abstraction part is focused on how students construct new mathematical concepts according to them, while the second reflective abstraction is how students build new concepts in order to solve mathematical problems.

Index Terms: Abstraction, Reflective Abstraction, Mathematics Learning, Mathematical Concept, Mathematical Problem Solving.

1. INTRODUCTION

Mathematics is an important learning because it can develop logical thinking [1],[2]. The importance of learning mathematics is also evident when students consider that learning mathematics can cause students to pass exams with good results [3]. Learning mathematics is inseparable from mathematical concepts. In learning mathematics, the concept of mathematics is an important thing to be mastered by students because the concepts in mathematics learning are connected to each other. By understanding mathematical concepts, students will understand mathematical problems without difficulty and can easily continue the material being studied next [4]. Students are not only expected to understand mathematical concepts, but students are also expected to be able to develop mathematical concepts. Building mathematical concepts is the basis for learning mathematics [5]. Building mathematical concepts is very important because in the process of learning mathematics, mathematical concepts are not immediately transferred to students, but it is done by a process of how students build their own mathematical concepts in order that students remember the important concepts they have built [6]. In developing mathematical concepts, students will certainly find it easier to recall what they have built. It is especially when they meet problems involving these mathematical concepts and when they meet new concepts that they must build and in such processes students will involve abstraction. Building concepts is a step major in abstraction [7]. In the process of learning mathematics, one that is related to the ability of students to build concepts is reflective abstraction. Reflective abstraction in Piaget's view is the highest and basic form of thinking in mathematical thinking [8]. Reflective abstraction is the ability to build and reconstruct new knowledge that is not yet known by students through the mechanism of changing, combining and linking a certain mathematical structure [9], [10]. Another opinion says reflective abstraction is the mapping of mathematical logical frameworks in the process of developing one's dynamics with the aim of constructing new concepts through linking between concepts that are sometimes able to lead to differences in constructional generalizations [11],[12],[13]. Besides giving rise to differences in constructional generalizations for students, teachers can provide, generalize and infer concepts so correctly according to existing mathematical concepts that they did not cause differences in student understanding after the concept development process takes place. The important thing delivered by Piaget is that reflective abstraction did not have the time when the ability of reflective abstraction to develop in students and reflective abstraction will continue until higher mathematical concepts are reached [12], [13]. Students will never stop involving the ability of reflection abstraction before they reach the goal of building concepts that are in accordance with the conditions needed. Considering the importance of reflective abstraction and mathematics learning as explained above, this article discusses "Reflective Abstraction: How to Find Out in Mathematics Learning" with the aim of explaining how students' reflexive abstraction can be known in the mathematics learning process.

2 METHOD

This study used a qualitative research approach with descriptive methods and involved 36 students in class XI. Students were observed during mathematics learning for one month, students were given tests 6 times and as many as 8 students were interviewed to get more information. Data collection using test and non-test techniques (interviews and observations). Data validation uses triangulation techniques namely data triangulation and method triangulation. The research implementation is presented in the following table:.

TABLE 1
RESEARCH IMPLEMENTATION

	Activity		
	1	2	3
First Meeting	Tes	Observation	Test
Second Meeting	Interview		
Third Meeting	Observation	Test	
Fourth Meeting	Observation	Test	
Fifth Meeting	Interview		
Sixth Meeting	Test	Observation	
Seventh Meeting	Test		
Eighth Meeting	Interview		

3 RESULT AND DISCUSSION

Result

Observation Result

Observations were made during mathematics learning. Researchers only observed mathematical learning activities, without regulating what learning methods should be used by the teacher so that the teacher can teach what they were as usual.The material taught by the teacher during the process of conducting research was the Material Linear Inequality of Two Variables. Some of the concepts were discussed in the Material Variable Linear Inequality System (SPtLDV), including the area of the settlement of SPtLDV and the use of mathematical models. Before teaching the concept of SPtLDV, the teacher made an apperception by asking students to mention the linear equations they had learned as many as 3 students tried to answer while recalling what they knew about the two variable linear equations. After that, the teacher asked students to give examples of two variable linear equations as many as 7 students tried to give examples of two variable linear equations. The next activity carried out by the teacher asked students about the symbol of inequality. there were 4 students who tried to answer the symbol of inequality. The teacher asked the students to conclude about the inequality of two variables and SPtLDV, then two students tried to deduce what was the linear inequality of two variables and SPtLDV. The first student concluded that it was an inequality while the second student concluded that an inequality had two variables. What the teacher does next is to conclude completely about the linear inequality of the two variables and SPtLDV. The teacher asked students to repeat what had been concluded previously and the teacher asked students to give an example of SPtLDV about 4 students who answered examples of two variables linear inequality. In the next activity, the teacher explained how to determine the set completion area (SCA) of the SPtLDV. The teacher explained with complete and sequential steps how to look for the SCA, then students were given examples of questions, and the teacher asked them to solve them individually. As students worked on sample problems in looking for it, researchers walked around the class to ask several students what they understood about it that the teacher had explained. Students can explain again what they knew about SAC exactly as explained by the teacher. Researchers tried to give questions that are different from those explained by the teacher (but do not violate mathematical concepts), such as "can we make SAC into shaded areas?" Because what the teacher explained was that SAC was an unshaded area, as students make in their notes:

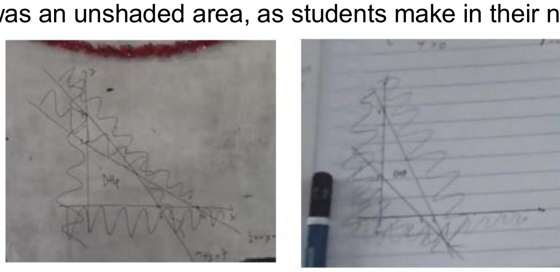
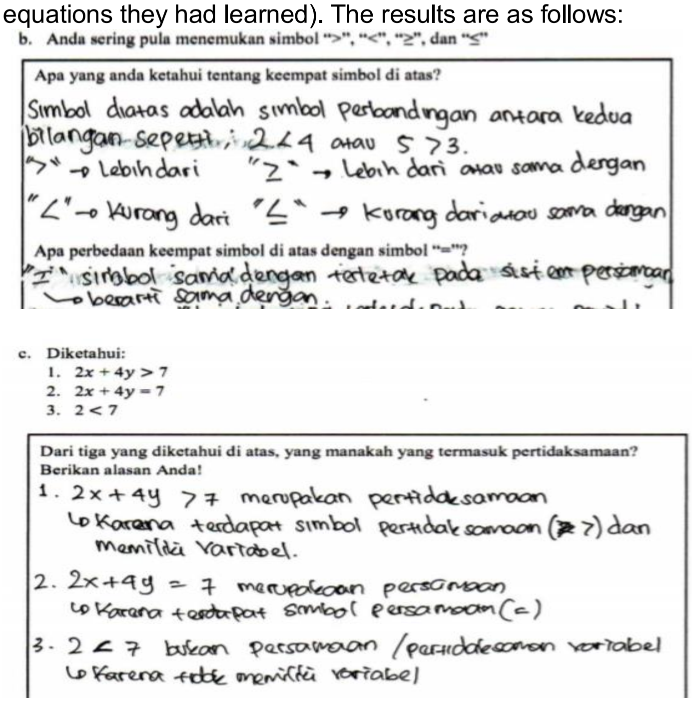


Fig 1. SAC explained by the teacher (left), SAC made by students (right)

The researcher asks 6 students about the question. As many as 2 students state that "SAC can also be shaded areas as long as the rules are the same as explained by the teacher only reversed", while 2 more people answere "do not know whether SAC can be used as shaded area" and two others answere "no Yes, the shaded area is a non-SAC area, and the SAC area is a clean area".

Test Result

Reflective abstraction are divided into two parts, namely, organizing or reconstructing existing knowledge to form new structures and projecting existing knowledge into higher forms of thought. Based on this, the researchers conducted two types of tests, namely tests before entering into learning about the mathematical concepts of SPtLDV, determining SAC, and the use of mathematical models, and tests after students learned the mathematical concepts.It is test before students learned a mathematical concept in the form of a test with questions in the form of a related mathematical concept. after that, only direct questions was about the concept. For example, for tests on linear inequality of two variables, what they can conclude about linear inequality of two variables, researchers asked questions of their knowledge about variables and asked them to model what variables were, equality symbols, then inequality symbols then to the core questions about inequality (concepts linear inequality was a concept that they had not learned, while the concept of linear equations they had learned). The results are as follows:



b. Anda sering pula menemukan simbol ">", "<", "≥", dan "≤"

Apa yang anda ketahui tentang keempat simbol di atas?

Simbol diatas adalah simbol perbandingan antara kedua bilangan seperti: 2 < 4 atau 5 > 3.
">" → Lebih dari "≥" → Lebih dari atau sama dengan
"<" → Kurang dari "≤" → Kurang dari atau sama dengan

Apa perbedaan keempat simbol di atas dengan simbol "="?

"=" simbol sama dengan terletak pada sisi persamaan
→ berarti sama dengan

c. Diketahui:

1. $2x + 4y > 7$
2. $2x + 4y = 7$
3. $2 < 7$

Dari tiga yang diketahui di atas, yang manakah yang termasuk pertidaksamaan? Berikan alasan Anda!

1. 2x + 4y > 7 merupakan pertidaksamaan
→ Karena terdapat simbol pertidaksamaan (>) dan memiliki variabel.
2. 2x + 4y = 7 merupakan persamaan
→ Karena terdapat simbol persamaan (=)
3. 2 < 7 bukan persamaan /pertidaksamaan variabel
→ Karena tidak memiliki variabel

***Fig 2.** The answer to one of the research subjects from the test results about the concept of linear inequality*

It is test after students learn mathematical concepts about SPtLDV, DHP SPtLDVThe picture above is the answer of one of the research subjects. The research subject can explain what he knows about the symbols of inequality and equality so that when answering questions about grouping which belongs to linear inequality and those that are not subject can answer. In his opinion, only No.1 is known to be inequality because there are symbols of inequality and variables, whereas known no.2 is not a linear inequality because it has an equation symbol then known no.2 is a linear equation, whereas no.3 is known not to include inequality linear because there are no variables, and the use of mathematical models. The results of this test are as follows:

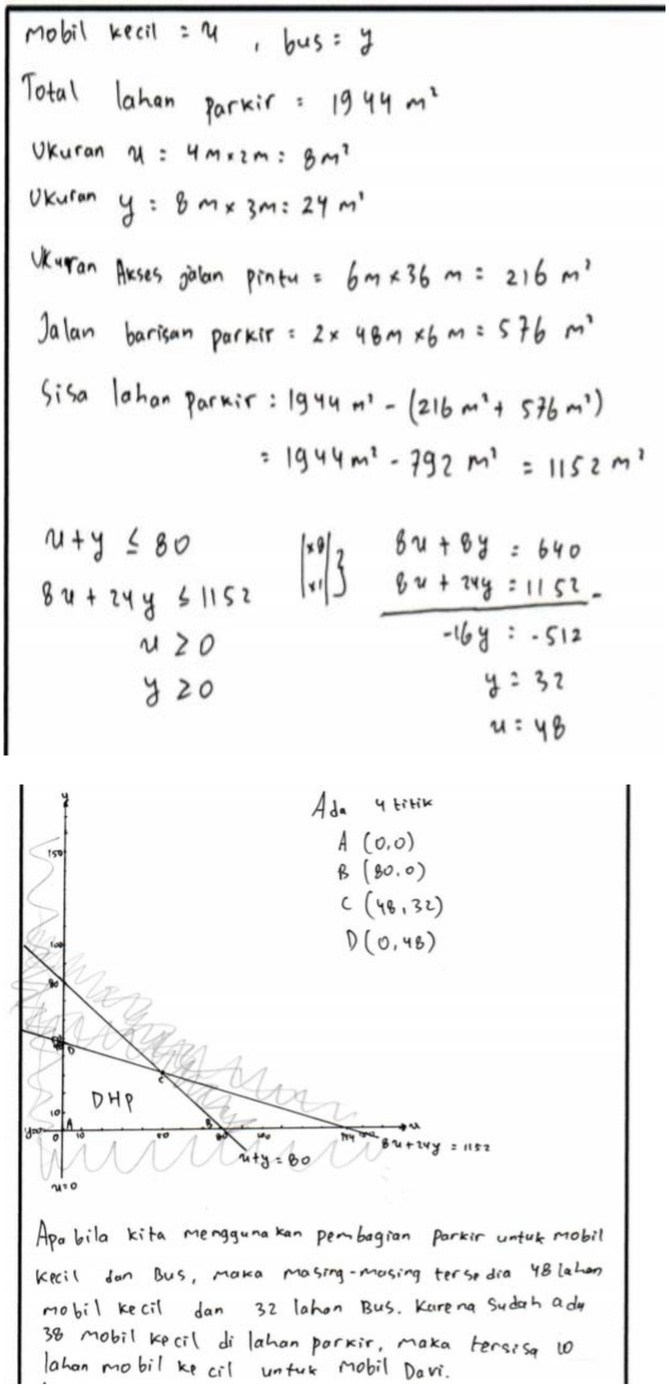


***Fig 3.** Answer one of the research subjects from the test results after studying the SPtLDV concept.*

From the picture above, students consider small cars to be variables and buses to be variables y. Subjects look for parking spaces for small cars and buses by reducing the total area of parking lots by the total area of the road on the parking lot, so that the results obtained 1152. Mathematical models made subjects 8x + 32y = 1152 is the right mathematical model, because it involves 1152 which is a small car and bus parking areas without roads while 1944 is the total area of parking lots involving road area. By involving models, graphs, and some steps made by the subject, then produced below there are 10 parking lots available to park small cars for new visitors who will park their vehicles in the form of small cars.

Interview Result

Interviews were conducted to confirm the subject's answers on the written test results. Here is the interview conversation to confirm the subject's answers as follows:

P : Well, Jihan answers the inequality, why is this answer the inequality while the two are not?

S1 : Because the one symbol is the symbol of inequality that I told you earlier, then if these three are not equations or inequalities because they do not have variables only 2 <7

P : Oww, so if there is a linear inequality there must be a variable?

S1 : Linear equations are also variable

From the interview snippet above, the researcher can confirm what was written by the subject on the test answers. The subject said that which included linear inequality are those that have the symbol inequality and have variables, whereas if there are only signs of inequality without having a variable, not including linear inequality. But it is not only inequality that has variables, even linear equations have variables that differ only in symbols.

Discussion

In the process of learning mathematics, before entering into learning about the concept of material linear inequality of two variables, students have learned the concept of material linear equations of two variables and symbols about inequality. In the process of learning mathematics in the class under study, the teacher gives apperception in the form of asking students to know what they know about the linear equations of two variables and asking students to give examples, and asking students to mention symbols of inequality. After that the teacher asks students for their opinions about the linear inequality of the two variables based on the concepts they know and have mentioned and explained. By linking, combining, organizing similarities and differences about related concepts, students conclude about the linear inequalities of two variables, some students even try to express their opinions in front of the class. The opinions of some students were incorrectly re-concluded by the teacher so that there was no misconception about the linear inequality of the two variables. The learning process of the material determines SAC, the concept of determining SAC is directly conveyed and explained by the teacher without involving apperception and without acting as scaffolding as in the previous learning process, related concepts and examples of SAC are directly explained by the teacher. Students when asked by researchers about "does SAC have to be a non-narrated area, can it be otherwise"? some students answered there were those who answered did not know, could (with the correct argument), and could not (they assumed must be in accordance with what was explained by the teacher). The learning process in class using apperception which is used as a reminder of concepts known to students related to the concept of the material being studied is very useful for students to be able to build new concepts. Students' reflective abstraction can be known and seen because the teacher uses the apperception. But it turns out that not only by using apperception can reflective abstraction of students be known, and without using apersepipun, it can be known even though the concept is directly explained by the teacher such as the SAC concept. We can know the reflective abstraction of students as researchers do when the teacher has finished explaining the concept of SAC, the researcher asks things that are different from what the teacher says but does not violate the concept (as long as it includes a new concept for students). From the results of students conclude the concept of linear inequality of the two variables and from the results of students answering the researchers' questions about SAC. it can be seen that the reflective abstraction of the student is at what level (if viewed from the level of reflective abstraction) or in what category of reflective abstraction (if reviewed on the high, medium or low category). Using apperception in class really helps students' reflective abstraction to be easily known because the new mathematical concepts are really the result of organizing, compiling, looking for similarities and differences, and gathering between related concepts. Mathematical concepts that are new to students if all delivered by the teacher without involving students' abilities about previous concepts will cause reflective student abstraction to be difficult to know if there is no feedback to them about concepts that are new to them as researchers asked students when learning SAC. After being asked a question, then the student's abstraction can be known even though that way it is not easy to know the student's reflective abstraction because someone answers the question with the answer "don't know".Reflective abstraction is divided into two parts based on the previous explanation above. In the process of learning mathematics, so it is not difficult to know the students 'abstraction (if students' reflective abstraction is difficult to know just by observation) tests can be done. The reflective asbtraction section "organizes or reconstructs existing knowledge to form new structures" can be known by giving tests before entering learning. This test is not in the form of a pretest, but only to find out the reflective abstraction of students by giving questions in the form of their knowledge about previous concepts related to what they will learn after that is still on the same test, then given questions about the material they will learn (questions about new material concepts). By administering the test, each student's reflective abstraction can be identified. As in the above results, the student's abstract abstraction that answers as in Figure 2 is included in level 4 reflective abstraction, namely Structural awareness. In the reflective abstraction section "projecting existing knowledge into higher forms of thinking" can be seen in the learning process in class when students build new concepts in order to solve mathematical problems by giving students a mathematical problem, the teacher can discuss directly so that each student's abstraction visible. However, if you want to know the results of students' reflective abstraction very clearly, then you can test the students after they learn the material. Just like the previous explanation, that each student's reflective abstraction can be known by looking at the results of the test given to them. Interviews can be carried out when the process of learning mathematics takes place as conducted by researchers when the teacher has explained the concept of SAC, researchers' questions in the form of concepts are contrary to what is taught by the teacher to students who answer these questions can be known for their reflective abstraction. Interviews can also be conducted after the mathematics learning process is complete. However, because reflective abstraction is the ability of students to build new concepts based on the results of organizing and projecting the similarities and differences of new concepts and old concepts they already know it is better for interviews to be done very carefully in order that what they convey is the result of constructing concepts that are it was not their knowledge after the new concept was conveyed. Interviews conducted by researchers after the learning process in class is finished in the form of interviews to confirm the results of students' answers to the test. As in the interview results presented above, based on the student's explanation it can be seen that he is at level 4 reflective abstraction, namely structrural awareness, thus written test results and confirmation results through interviews get the same results about the student's reflective abstraction.

4 CONCLUSION

In the process of learning mathematics, students' reflective abstraction can more easily appear and be known by using and expressing apperception in learning. The teacher acts as Scaffolding in order that students can build new concepts for them from the results of linking and organizing differences and similarity of concepts related to new concepts, combining. However, if teacher-centered learning (the teacher does not provide apperception and does not act as scaffolding) does not mean that students' reflective abstractions cannot be present and known. Teacher-centered learning with mathematical concepts explained by the teacher directly, we can regulate the learning process by providing feedback to students in the form of questions about concepts that are different from those taught by the teacher (but not violating the concept), by answering these questions the students' reflective abstraction will appear and be known. Before starting the learning process in class, tests can also be given to students in the form of questions relating to the material they have learned that relate to the core material to be studied and in the form of core questions about the mathematical concepts of new material. Knowing students' reflective abstraction can also be done by interviews, direct interviews in the classroom as researchers do when the teacher does not act as scaffolding or interviews confirm the results of the learning process and test results. Reflective abstraction consists of two parts, each part can be seen in the process of learning mathematics takes place. In the first part reflective abstraction "organizing / reconstructing existing knowledge to form new structures" and the second part "projecting existing knowledge into higher forms of thinking" both of which can be known by students' reflective abstraction through the learning process in class, the difference is if The first is seen from the results of students building mathematical concepts that already exist but are new to them, if the second part looks at how students build concepts in order to solve mathematical problems.

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