Student Reflective Abstraction Of Impulsive And Reflective In Solving Mathematical Problem

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Abstract: The purpose of this study is to describe the reflective abstractions when solving mathematical problems. This research employs a qualitative descriptive method. Two-second semester students of Mathematics Education study program were chosen as research subjects. Results show that students with cognitive impulsive style failed to conduct reversal process, while the interiorization and coordination process satisfactory occurred. Students with cognitive reflective styles were weak in generalization process, however, most of the interiorization, coordination, encapsulation, and reversal were up to standard.

Index Terms: Reflective Abstraction, Mathematical Problem, Impulsive, Reflective, Interiorization

1. INTRODUCTION

The urgency of students' reflective abstraction ability is shown in the form of schemes made for understanding problems, finding solutions, or solving the given problems. Besides, the students' reflective abstraction ability in solving problems is of necessity since the result of one's reflective abstraction is a scheme that is utilized to understand something, to find solutions, or to solve problems. In addition, reflective abstraction is crucial for higher mathematical logical thinking as what happens in children's logical thinking. Therefore, when it comes to developing the idea of reflective abstraction from higher-level mathematical thinking, it is necessary to separate the important features of reflective abstraction, reflect the rules on higher mathematics, recognize and reconstruct them so that theories of mathematical knowledge are similar and instruction. Piaget distinguishes three forms of abstraction, namely empirical abstraction, psycho-empirical abstraction, and reflective abstraction [1], [2]. Empirical abstraction is the process of gaining knowledge about the properties of objects. This process is related to the subject's experience when viewing an object by seeing the visible properties of an object. However, knowledge is internally formed in the subject [1]. The process of pseudo empirical abstraction occurs when the subject is confronted with an object and then discovers the properties of the object through the process of imagining an imposed action on the object. The subject tries to configure the object in space as well as to check for possible connections. The third abstraction is reflective abstraction which is also called by Piaget as general coordination of actions, departing from the subject. The whole process is considered internal. This process directs the subject to a constructive generalization and results in new forms of synthesis among [10], [17], [22] specific rules for gaining new insights. Piaget distinguishes reflective abstraction into four types of construction processes: interiorization, as an internal construction process, namely how to understand perceived phenomena [5]. Coordination or composition of two or more new construction processes. Encapsulation or conversion of a (dynamic) process into an object (static), in the sense, that, "... actions or operations become objects that result from thought or assimilation". Piaget assumed that "... Mathematical entities move from one level to another, operations such as 'entities' will become objects of theory ...". When subjects learn to apply an existing scheme to a broader set of phenomena, the scheme is called Generalization. Generalization can also occur when a process is formulated into an object. The scheme will remain the same unless it has wider application. Piaget defines it as reproduction assimilation or generalization. In other words, it is called extensional generalization. Cognitive style is one of the students' characters which is crucial and influential, especially in their learning achievement. Cognitive style is related to how they learn through their own inherent ways. It is different from one individual to the others [8], [10]. Cognitive style is closely related to how to receive and process all information, especially in learning. Impulsive and reflective cognitive styles are the most common cognitive style owned by students. It is in accordance with a research conducted by Warli that the proportion of children who have impulsive reflective cognitive style is around 73.7% [9] Research from Rozenwajg also shows that children who have 76% impulsive-reflective cognitive style in one class [13], [16].

2. METHOD

This research is a descriptive study by means of a qualitative approach. This study wants to get a picture of students' reflective abstraction in solving mathematical problems from the perspective of children who are impulsive and reflective. The results of this study can be seen from the results of written tests and interviews. Before conducting research, written tests and interviews were initially conducted and then validated. The research subjects comprised 4 first semester students at the Islamic University of Malang. When it comes to analyzing the research subjects, class selection and MFTT (Matching Familiar Image Test) were utilized to classify students' impulsive or reflective cognitive styles. Research subjects also had good communication skills and were able to convey what they wrote verbally and systematically. Data were collected by conducting written tests and interviews [15], [19]. Each subject was given one written test and interview. The written test was in the form of open questions that contained more than one solution.

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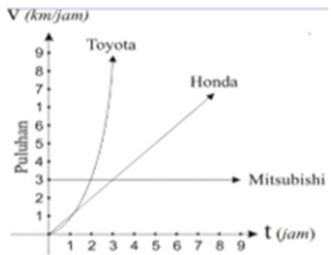
There are 3 cars, namely Toyota, Honda and Mitsubishi that travel on the same track from city X to city Y which is 720 km at the speed of Vtoyota, Vhonda, Vmitsubishi. The velocity graph V for time t of the three cars is illustrated as shown below



a. What is the distance traveled by each car when t = 3
b. Sort the arrival of cars in the city of Y starting from the earliest

***Fig. 1**. Open Ended Problem.*

The students' reflective abstraction data in solving mathematical problems comprised interiorization, coordination, encapsulation, and generalization. Interiorization is an internal construction process, which is a way to understand perceived phenomena, coordination is a combination of two or more new construction processes carried out by the subject consciously, encapsulation is the change of a process (dynamic) into an object (static), in the sense that the subject is able to make mathematics in his mind, generalization is the application of schemes to a wider set of phenomena [20], [8], [10]

3 RESULT AND DISCUSSION

This study aims to describe the process of students' reflective abstraction in terms of impulsive and reflective cognitive styles in solving mathematical problems. For more details can be seen in Table 1.

3.1. Abstraction reflective of a reflective student

Departing from the results of this study, in the stage of reflective style interiorization, students could already understand the given problem, this is in accordance with the notion [27] which says that reflective-style children are easier to understand the provided problem. Moreover, at the stage of interiorization, they could already understand Toyota car charts in the form of curves so that the equation becomes squared. They could also find the distance of Toyota car that

Component Abstraction Reflective	Cognitive	Style Student
	Reflective	Impulsive
Interiorization	1. Read the given math problem 2. Understand the graphs of Toyota cars in the form of curves so that the equation becomes squared 3. Find the distance of Toyota cars that are sought using the integral 4. Enter the upper limit 3 and the lower limit 0	1. Finding the distance traveled using straight-motion changes irregularly 2. Look for acceleration using vt = v0 + at 3. Calculate the distance traveled when t = 3 using s = v0t + 1/2at^2
Coordination	1. Coordinate the equation y = 10x^2 by using integrals 2. Coordinate $\int_0^3 10x^2$ dx to find the distance 3. Coordinate graphs in the form of straight lines with integrals	1. Coordinate acceleration with the distance sought 2. Coordinate speed relationships to find the acceleration
Reversal	1. Look for an acceleration of a known speed 2. Find the time taken by finding the upper limit of the integral	Acceleration achieved by using the speed of a known graph
Encapsulation	1. Use L = $\int_0^3 10x^2$ dx to find the travel time 2. Find the upper limit of the integrals used from city X to city Y	1. Determine the time taken by making a table 2. Use the triangle area formula to find the time taken
Generalization	1. Use a differential to determine the travel time of each car 2. Sort the arrival of the car in the city of Y tang the earliest	1. Not yet able to determine the arrival time of each car

***Table 1.** A Summary of student reflective abstraction of impulsive and*

reflective in solving a mathematical problem was sought by utilizing integrals through entering their lower boundary 0 and upper limit 3. At the coordination stage, the students utilized coordinate L = dx to find the mileage. However, they could also coordinate the graph in the form of a straight line by integrals. At the reversal stage, students could already find the acceleration of the known speed and the taken time by finding the upper limit on the integral. In the encapsulation stage, students employed L = dx to find travel time. They could find the upper limit of integrals from city X to city Y. At the generalization stage, reflective force students had employed differentials to determine the time travel of each car as well as sorting out the arrival of cars in the city Yearly.

3.2 Abstraction reflective of an impulsive student

The results of this study for the reflective abstraction of impulsive students at the interiorization stage were that they had been able to find the mileage by utilizing irregularly changing straight movements (GLBB) as well as to look for acceleration by employing $vt = v_0 + at$ and calculate the mileage when t = 3 by employing $s = vt + 1 / 2at^2$. At the coordination stage, students could already coordinate the acceleration with the distance sought and coordinate the speed relationship to look for acceleration. At the reversal stage of reflective abstraction, impulsive force students could already find the obtained acceleration by utilizing the velocity of a known graph. In the encapsulation stage, students could determine the time needed by creating a table and utilizing the triangle area formula to find the needed time. At the generalization stage, students could not determine the arrival time of each car.

CONCLUSION

The process of reflective abstraction in students with reflective and impulsive cognitive styles had distinctions in solving mathematical problems. In the interiorization stage of students who had a reflective style, they were careful and meticulous in understanding the problems that were given. When there were mistakes, students immediately realized them and tried to fix them. Meanwhile, impulsive students were inclined to be less careful in understanding this problem because they were somewhat rushed in understanding the problem, but they also tended to be less aware when there were mistakes in solving the given problem. In the coordination stage, reflective style students were able to build two or more new processes, whereas impulsive style students tended to be imperfect in coordination. For the stage of encapsulation and generalization, reflective-style students were able to carry it out, whereas impulsive style students had not done it perfectly. Therefore, it is suggested that teachers should pay more attention to students who have different cognitive styles so that the learning process can be well understood by the students.

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