Disc-Plate Squeal Investigation Using Finite Element Software: Study and Compare

Ammar A. Yousif Mohammed, Inzarulfaisham Abd Rahim

Abstract:- Brake squeal is an example of noise caused by vibrations induced by contact forces. During brake operation, the contact between the pad and the disc can induce a dynamic instability in the system. The brake squeal which occurs in the frequency range of 1-20 kHz is investigated with the plate on disc as a new model that's presented in this paper to study the instability of the system. The squeal events treated in this paper were identified to be dynamic instabilities of the brake system. The ANSYS product finite element software in 3-D with Matrix27 as a contact element was used to simulate the behavior of the plate-disc system. Coulomb's friction was used at the contact surface with a constant coefficient value. The contact stiffness and the friction coefficient were changed during the simulation in order to analyze the occurrence of the squeal. For this application, temporal simulations showed that separation occurred between surfaces, confirmation of instabilities. It also showed that, the vibrations responsible for the instability were localized in the contact properties. The mode responsible for squealing was obtained by using the modal analysis of the plate–disc system and assuming that the interface was stuck. The result has shown that the maximum degree of instability appeared as a result of changing the contact stiffness effect rather than changing the friction coefficient.

Index Terms:- Contact Stiffness, Contact Element, Matrix27, Diametral Mode, Instability, Modal Analysis, Mode Shape

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1 INTRODUCTION

Brake noise is a problem which is related to the characteristics of geometric design. Squeals and other high frequency noises in various names have been a major concern in the design of disc brake. When the disc brake generates noise, it indicates that the disc vibrates in one of diametral modes, thus the squeal frequency is more dependent on the natural frequency of the disc rotor. However, the dynamic stability of the system studied depends on the diametral mode shape of the disc. The higher squeal frequency appeared at a range of 1000-12000 Hz as [1]. The squeal frequency usually matches well with the modal frequency of the disc brake vibration as [3]. The present analysis proves that the instability in disc brake is an inherent property caused by the geometrical coupling of the vibrating component in the beam-disc system. Nishiwaki [4] found that by using different brake types, the rotor was responsible for the most noise generated during the squeal. Thus the disc diametral mode is the first part that should be considered. McDaniel [2] used a stationary brake system consisting of a pad, calipers, and rotor. In his experimental work, he found that the natural frequencies and mode shape of the operating system are identical to those of the stationary system under the assumption that the systems are linear and the rotation frequency of the rotor is much smaller than the natural frequency of either system. Earlier works had indicated that noise was often found to be close to the natural frequencies of the disc and the pad, and it was suspected that there was a coupling of the natural frequencies of the individual parts when they were close together [6].

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A node is a point along a standing wave (A standing wave, also known as a stationary wave, is a wave that remains in a constant position) which has minimal amplitude. The opposite of a node is an anti-node, which is the farthest point from the node on a wave.

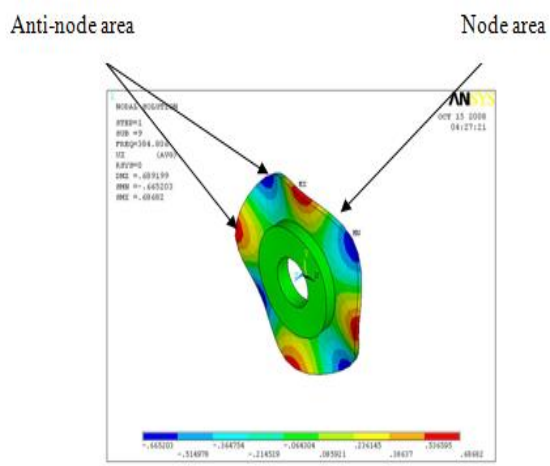


Fig. 1. Illustrates the node and anti-node positions. The maximum amplitudes are referred to as antinodes and the apparent zero position as nodes

According to Fieldhouse [10], the node in Fig. 1 is a stable area. So the researcher tried to make the center of the pad on the node during the squeal. This characteristic is supposed to form a stable vibrating mode when braking is applied and noise is generated. Only even-numbered orders appeared in the diametral mode shape. The reason can be explained as follows: a sold bar has open ends, so the waves propagate without dependence on the vibration phase at both ends. The longitudinal waves of uneven-numbered orders vibrate in the opposite phase at each end. Because a circularly-shaped rotor has no open end in the circumferential direction, the wave in the opposite direction interferes with each another and disappears. The node and anti-node should appear in the response frequency diagram as peak and anti-peak. The response of an annular disc can be conducted by using the equation which was found by McDaniel, J.G. [5].

$$\omega(r,\theta,t) = \mathrm{Re}\{w_n(r)e^{i(n\theta-\omega t)}\}$$

$$w_n(r) = \sum_{n=-\infty}^{n=\infty}[A_nJ_n(K_fr) + B_nY_n(K_fr) + C_nI_n(K_fr) + D_nK_n(K_fr)]$$

$$K_f = \sqrt[1/4]{\rho\omega^2/D}$$

Where ρ is mass per unit area, n the mode shape, K_f the wave number of flexural waves that propagate in an infinite plate, A is the area of the disc, r is the radius while A,B, C and D are constants. Sherif [7] showed the relation between the frequency and the node to be

$$f_n = \frac{\sqrt{(1+n^2)\frac{E_1}{\rho_1}}}{2\pi a} \text{ (Hz), Where n=0, 1, 2, 3, …}$$

The formula above shows that no diametral mode shape will appear when the model number of the shoe (n) is not an integer. From the above equation, the line which represent the relation between the diametral mode and the frequency should not been continues because the value in the vertical direction refers to the diametral mode as mentioned by [6]. The problem of the squeal is not yet solved and so there is a necessity for a new model to predict the occurrence of the squeal at high frequency.

2 Disc Modal Analysis

The disc was plotted inside ANSYS GUI. The disc geometric cross section and the dimensions are shown in Fig. 2. The disc element chosen was solid45. A linear and homogeneous material was used for the disc. The disc material properties were taken from [22].

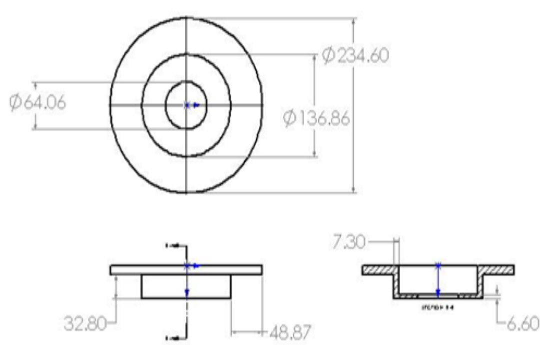
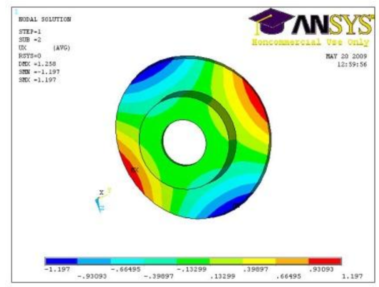
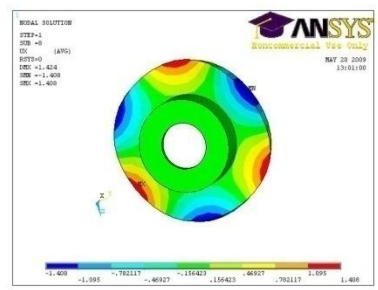


Fig. 2. Illustrates disc brake dimensions (all the dimensions in mm)

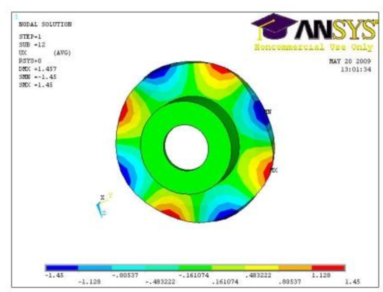
Modal analysis of the disc was conducted by using QR damping method in order to find the disc free-free mode shapes. The analysis was run and the mode shape was conducted in the normal direction of the disc, as illustrated in Fig. 3. The mode shapes are presented in Fig. 3 in the form of normal-displaced shape of the disc. It was observed as well that the disc had repeated roots where two modes had the same modal frequency and same mode shape with rotating angle. Previous studies of squeal in brake system as in [7], [8] they found that the mode shapes involved in the occurrence of drum brake squeal are radial mode shapes. However, the mode shape conducted in this analysis is the diametral mode. When the disc generates noise, the disc vibrates in one of the diametral modes, the shape being rather like a wave washer or carousel. The maximum amplitudes are referred to as antinodes and the aperient zero position as nodes. A 4th diametral mode order would possess 4 diametral nodal which gives 4 positive nodes and 4 negative antinodes (waves). Similarly an 8th diametral mode would also have 8 positive nodes and 8 negative antinodes. The eigenvalue analysis of the disc showed a total of 7 diametral modes within the frequency range of 1-12 kHz. The results are presented in the displacement contour for consideration, to compare with the experimental results which were indicated in the displacement contour in [10].



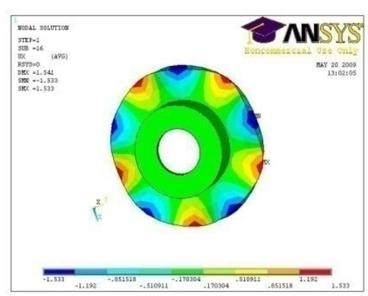
2nd diametral disc mode shape plotted at frequency 1808.2 Hz



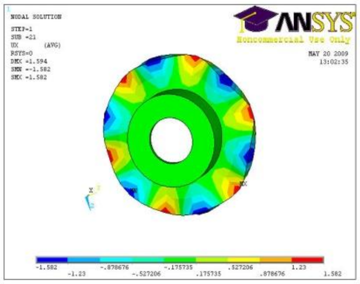
3rd diametral disc mode shape plotted at frequency 2944.6 Hz



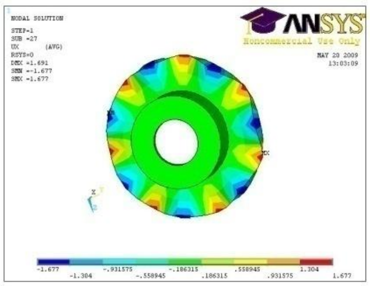
4th diametral disc mode shape plotted at frequency 4312.3 Hz



5th diametral disc mode shape plotted at frequency 6184.6 Hz



6th diametral disc mode shape plotted at frequency 8571.8 Hz



7th diametral disc mode shape plotted at frequency 11474.8 Hz

Fig. 3. Disc free-free mode shape

Each mode order will occur at very specific frequencies depending on the structure of the disc. Each antinode will occupy an equal angular space appropriate to its node order, for example 45 degree for a 4 diameter mode order (8 antinodes) and so on. There will always be an even number of antinodes in a diametral mode order. It is therefore possible to plot a curve of diametral mode orders against the frequency.

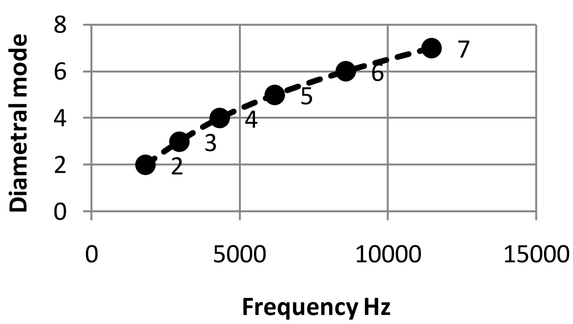


Fig. 4. Vibration modes versus the Frequency

It can be observed that most of the drum mode shapes occurred in pairs. This is because of the typical axially symmetrical geometric shapes of the brake drum. Flint [11] observed as well that the disc brake had repeated roots where two modes had the same modal frequency and similar mode shapes. The axially symmetrical geometry causes the structure to have a repeated root of the eigenvalue. When the damping matrix C is negligible, both complex eigenvalue roots have virtually the same imaginary values while the real values have opposite signs, as [9].

3 PLATE MODEL ANALYSIS

Linear and homogeneous materials were used for the plate. The plate density, Young modulus and Poisson's ratio were 7800 Kg/m^3, 200 GN/m^{2}and 0.27, respectively. The plate was thin in order to achieve a multi-valued frequency. The plate element was shell63. The modal analysis of the plate was conducted in the frequency range of 1000-12000 Hz. The damping was not included in this simulation. As a result, 40 mode-shapes occurred in the specified frequency range. The modes during the squeal were either bending or torsion according to the experiment done by Feildhous [10]. This result showed that the squeal appeared when the pad was at bending or torsion mode shapes. Flint [11] also showed that the frequency (during the squeal) for the brake shims was either bending or torsion. In this analysis, the natural frequency and mode shapes of the particular system (plate) were calculated by using finite element solver which showed that the plate had different mode shapes and frequencies. However, for different mode shapes the frequencies of the paired mode shapes were different because of the open end geometric structure. The plate was modeled with the dimensions as illustrated in Fig.. 5.

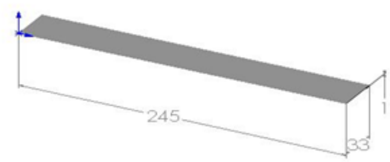
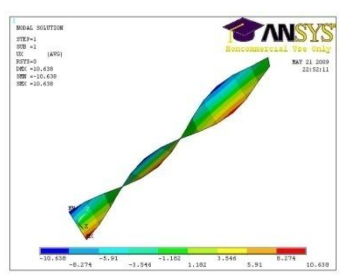
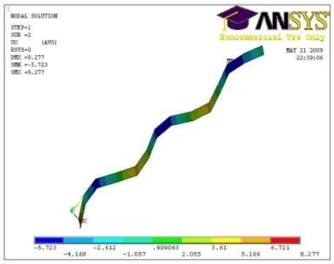


Fig. 5. Plate geometric dimensions, the entire dimension in millimeter

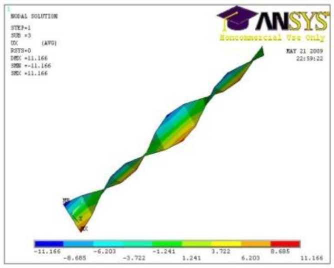
The experimental works done by Fieldhouse and Tworzydlo [12], [13] showed that the squeal is due to the coupling between the disc and the pad natural frequencies. That coupling can happen when the brake system natural frequencies come close to the other, which is the reason to find the plate natural frequency. Different parameters are responsible to make the system frequencies come close to the other. The mode shapes were which, when coupled with the disc generated squeals are illustrated in Fig.. 6.



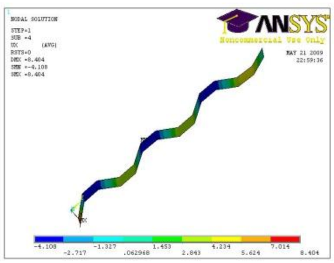
1st mode shape (torsion), f=1056.8 Hz



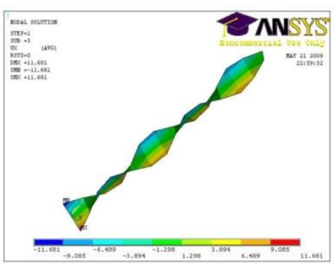
2nd mode shape (bending), f=1194.2 Hz



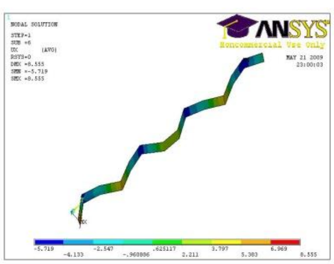
3rd mode shape (torsion), f=1528.4 Hz



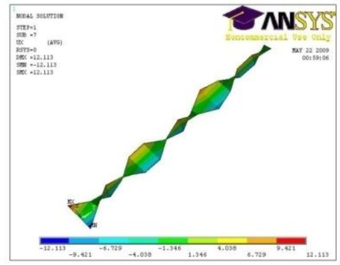
4th mode shape (bending), f=1675.1 Hz



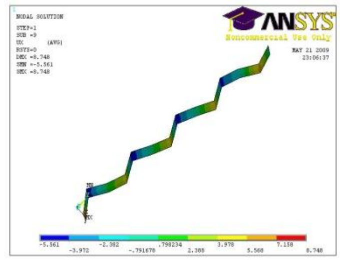
5th mode shape (torsion), f=2042.3 Hz



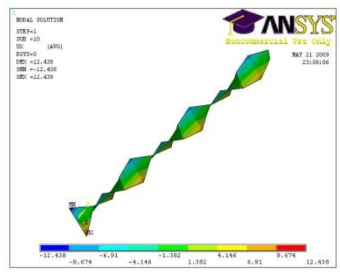
6th mode shape (bending), f=2236.7 Hz



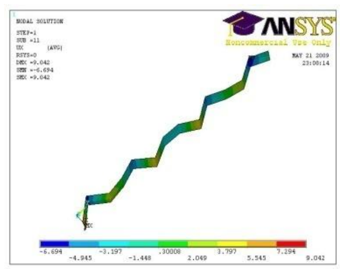
7th mode shape (torsion), f=2603.8 Hz



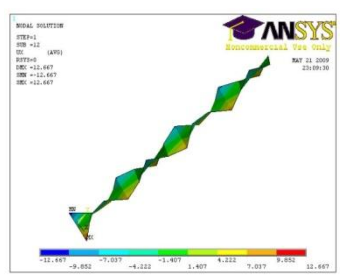
9th mode shape (bending), f=2878.4 Hz



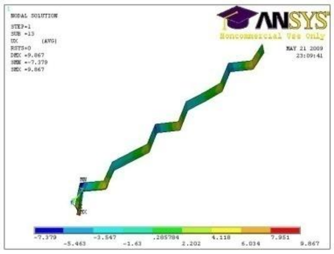
10th mode shape (torsion), f=3214.6 Hz



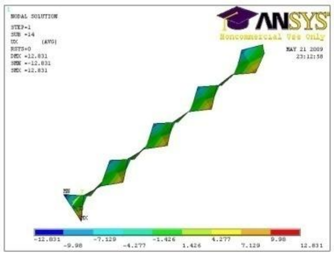
11th mode shape (bending), f=3597.9 Hz



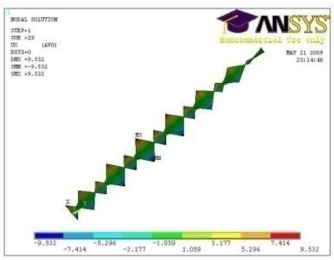
12th mode shape, (torsion) f=3874.2 Hz



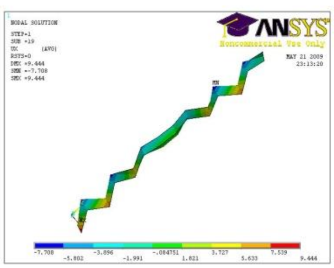
13th mode shape (bending), f=4384.8 Hz



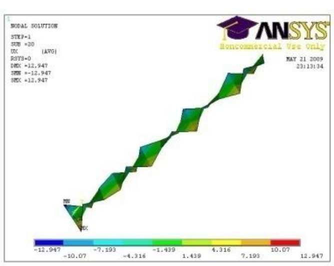
14th mode shape (torsion), f=4578.3 Hz



29th mode shape (torsion), f=7247.5 Hz
Fig.. 6 plate mode shapes



19th mode shape (bending), f=5235.3 Hz



20th mode shape (torsion), f=5316.5 Hz

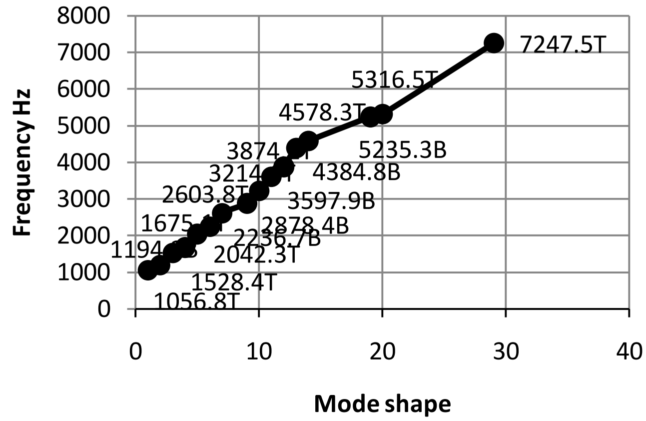


Fig. 7. Mode shape versus plate frequency, the B and T symbols refer to Torsion and Bending mode shape

Fig. 7 represents the plate frequency as a function of the mode shape. By referring to Figs. 7 and 4, squeal can be expected when the disc and the plate have close frequency. When the frequency is below 4500Hz, the plate shows that the mode frequencies are close to one another but when the frequency is above 4500Hz, the plate shows that the mode frequencies are far from one another. This indicates that at high frequency, the plate damping increases and as mention by Flint [11], the thin plate shows high damping at high frequency.

4 PLATE-DISC SYSTEM MODAL ANALYSIS

A three-dimension finite element model of the disc-plate system was simulated by using ANSYS software. The system was coupled by Matrix27 as illustrated in Fig. 8.

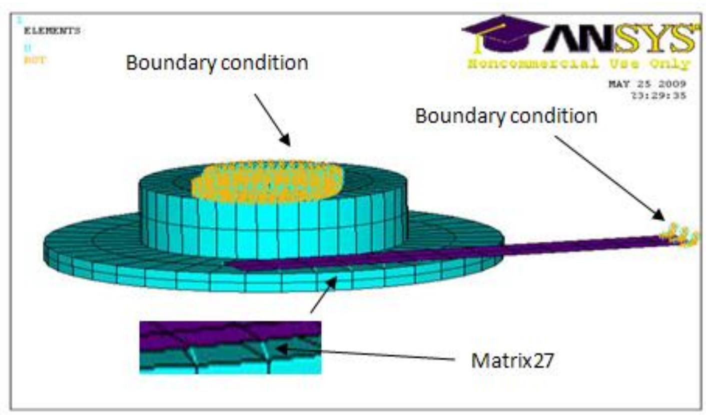


Fig. 8. Plate-Disc system coupled with matrix 27 as a contact element (Finite element model of Disc-plate system geometry)

The analysis can predict the occurrence of instability in the coupled disc-plate system. A modal analysis is the process of describing a structure in terms of its natural characteristics or dynamic properties. The natural characteristics of a structure are represented by the natural frequencies and mode shapes. These dynamic properties are also called modal parameters [14]. The positive real value indicates the occurrence of self-excited vibration, or squeal. The disc-plate system boundary conditions, element type and materials are the same as mention in sections 2 and 3.

5 STUDY THE EFFECT OF THE CONTACT STIFFNESS ON THE INSTABILITY

In order to provide the model with various operation conditions, a range of contact stiffness values K, 200, 250, 300, 350, and 400 MN/m was applied on the system. The friction coefficient value was assumed to be constant 0.4 which was distributed uniformly among the contact nodes.

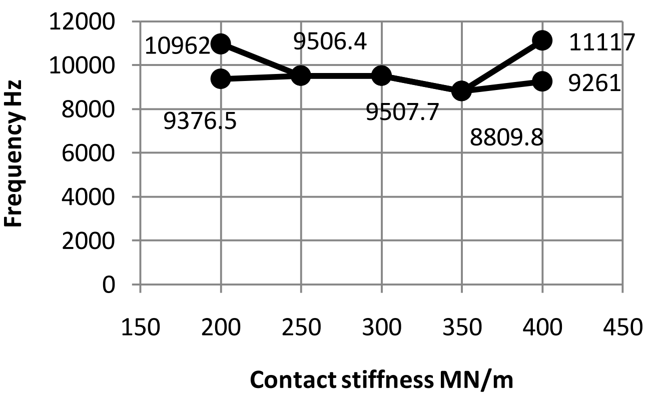


Fig. 9. Effect of contact stiffness on the system mode frequencies at μ=0.4 for the system sixth mode shape

The result showed that at contact stiffness 200 MN/m, the system frequencies (10962Hz, 9376.5Hz) diametral mode began to converge until they merged at contact stiffness equal to 250 MN/m with frequency 9506.4Hz. The modes continued to merge until contact stiffness 350 MN/m. The two modes started to split and separate into two modes with different frequencies (1117Hz, 9261Hz) at contact stiffness 400 MN/m. The symmetrical annular disc had two vibrating modes with the same frequency. When the disc and the plate at a contacted pressure, a violation of the symmetrical conFig.uration will split the frequency into two modes with different frequencies (10962Hz, 9376.5Hz).

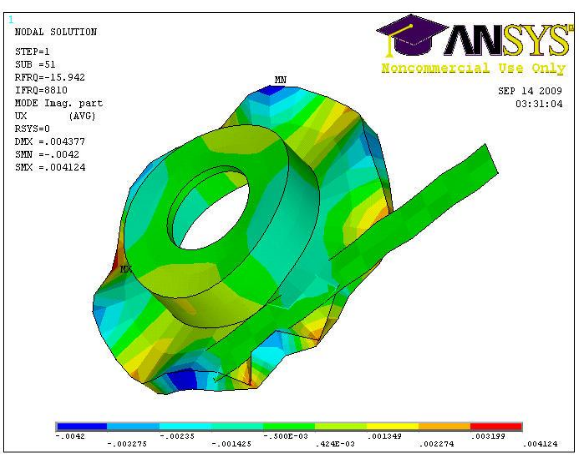


Fig. A. Mode shape at frequency 10692 Hz with contact stiffness 200 MN/m

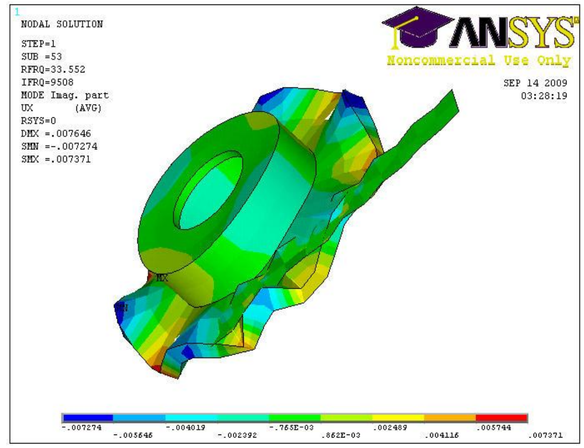


Fig. B. Mode shape at frequency 9507.7 Hz with contact stiffness 300 MN/m

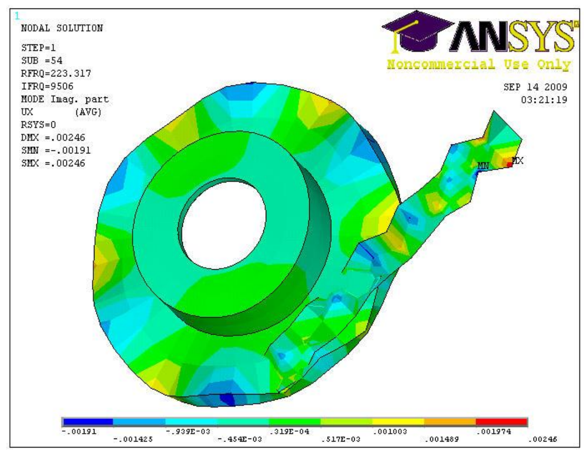


Fig. C. Mode shape at frequency 8809.8Hz with contact stiffness 350 MN/m

Fig. 10. Disc-Plate mode shapes at the unstable frequency

The brake disc is a major source of emitting squeal noise due to its larger area compared to the pad, as mentioned by Flint [11]. For this reason, the diametral mode shape of the disc is important and should be considered as a major source in generating the squeal. A thick layered plate gives high damping at low frequencies, while a thin layered gives plate higher damping at high frequencies [11]. This is the reason why the plate showed very small deformation (motion) during the squeal. It can be observed from Fig.s 10 A and B, that the mode during the instability did not show any angular rotation even though the instability decreased. The mode shape before splitting at contact stiffness 350MN/m was conducted, as illustrated in Fig. 10C. The mode showed angular rotation before the split. The real part of the eigenvalue as a function of the contact stiffness was conducted with contact stiffness ranging between 200 and 400 MN/m, as in Fig. 11 which shows that the real part of eigenvalue started to increase when the contact stiffness was 200MN/m.

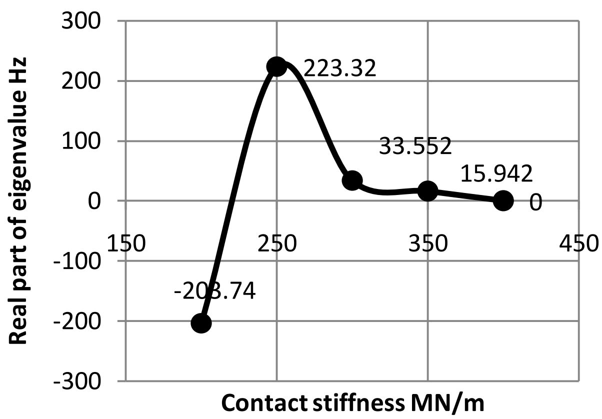


Fig. 11. Effect of contact stiffness on the system instability at μ=0.4 for the system fifth mode shape

The system instability began to increase until the contact stiffness reached 250 MN/m. The system showed that the maximum degree of instability 223.2Hz at contact stiffness value 250MN/m. The real part of eigenvalue showed decrease in the degree of the instability when the contact stiffness was 300MN/m, indicating that the system tended to be stable. The instability of the system decreased until it reached to 15.94Hz at contact stiffness 350MN/m. It can be observed that the frequency decreased during the instability, from 9506.4Hz at contact stiffness 250MN/m to 8809.5Hz at contact stiffness 350MN/m. It can also be said that the system with decreased the instability showed decrease in the unstable frequency, even though the contact stiffness increased. The result showed that instability is the reason to increase or decrease the frequency. The values of the positive real part refer to the instability which depends on the number of nodes and antinodes which are in contact. The mode shape under the pad will be compressed if two antinodes (even numbers of antinode) are held. This causes the free antinodes to expand resulting in a lower frequency being generated, compared to one antinode (odd numbers of antinode) being held below the pad [15]. The cause for high squeals is mostly due to the involvement of tangential mode, above 5 KHz [23]. However, it can be observed from the mode shape at contact stiffness 250MN/m that the disc had three antinodes under the plate and a frequency above 5 kHz. According to Fieldhous [15], these two points made the instability rise to a higher value at contact stiffness 250MN/m than at contact stiffness 350 MN/m. By providing the model with a range of contact stiffness of 200 MN/m to 500MN/m, another unstable mode in the system was observed, as illustrated in Fig. 12. The value of the friction coefficient was assumed to be constant and distributed uniformly among the contact nodes. The system showed two frequencies (2402.7Hz and 1862.9 Hz) at contact stiffness 200MN/m, indicating a second mode shape. These two modes converged with increasing contact stiffness until they merged at contact stiffness 300MN/m. This merging continued until the contact stiffness was 350 MN/m. The system instability was conducted at contact stiffness 300 MN/m to 350MN/m, as in Fig. 12. The modes started to split and separate with increased the contact stiffness. The split appeared clearly at contact stiffness 400 MN/m and showed two frequencies 2613.2Hz and 1912.8Hz. Return to Fig.s 7 and 4, and observe that the frequency which is close to the merged frequency 2093.4Hz is 2042.3 Hz (plate torsion shape) and 1808.2Hz (disc diametral mode). This converging of plate-disc frequencies in free-free mode made the generation of unstable mode possible.

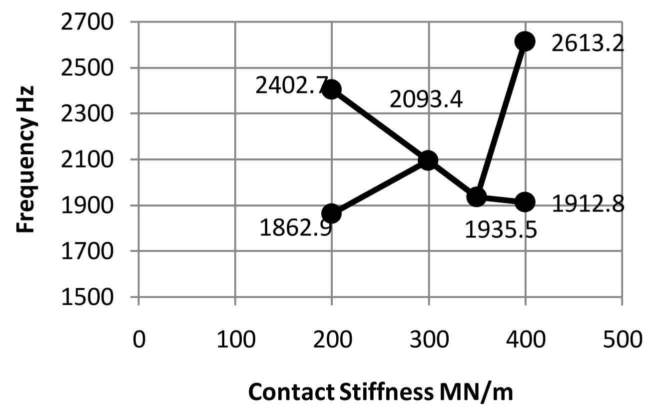


Fig. 12. Effect of contact stiffness on the system mode frequencies at μ=0.4 for the system second mode shape.

The asymmetrical plate-disc system illustrated why the system tended to be stable within a short range of contact stiffness, when compared to [22] results. It will be noted that the disc vibration is of a diametral mode suggesting that an asymmetry shape needs to be built into the disc in order to disrupt the symmetrical modes of vibration, [10]. This indicates that the introduction of asymmetry into the rotor may be a solution to inhibit the formation of symmetrical modes of vibration and the tendency to generate noise. It was found that the pad vibration can make a significant contribution to the noise propensity of the brake system. A non-uniform cross section was performed to find a simple and effective method of reducing brake squeal by partially changing the shape of the shoes, [16]. The result of changing the cross-section has simple and effective method to reduce the squeal as in [16]. The instability as a function of contact stiffness was conducted as in the Fig. 13, which shows that the degree of instability started to increase when the two modes merged.

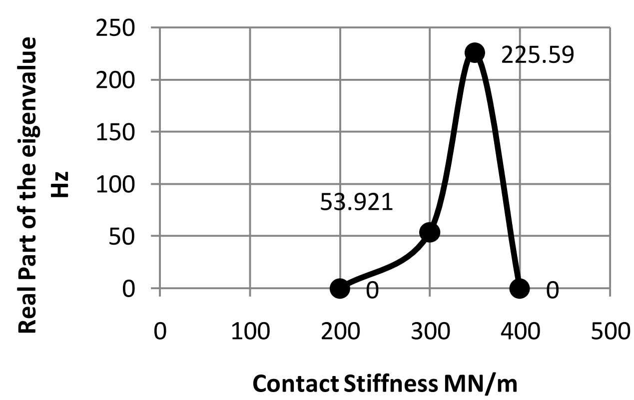


Fig. 13. Effect of contact stiffness on the system instability at μ=0.4 for the system second mode shape.

This merging increased the instability from zero at contact stiffness 200Mn/m to 53.921Hz at contact stiffness 300MN/m. The increase was continued until the real part of eigenvalue reached to 225.59Hz at contact stiffness 350 MN/m. The system tendency to generate noise began to decrease, referring to the system tendency to split the coupled modes until they start to separate at contact stiffness 350MN/m. The system returned to stability at contact stiffness 400MN/m. It can be observed that the instability started with the merging of the two modes but did not end with instantaneous splitting. By providing the model with a range of contact stiffness of 400 MN/m to 550MN/m, another unstable mode in the system was observed, as illustrated in Fig. 14. The value of the friction coefficient was assumed to be constant and distributed uniformly among the contact nodes.

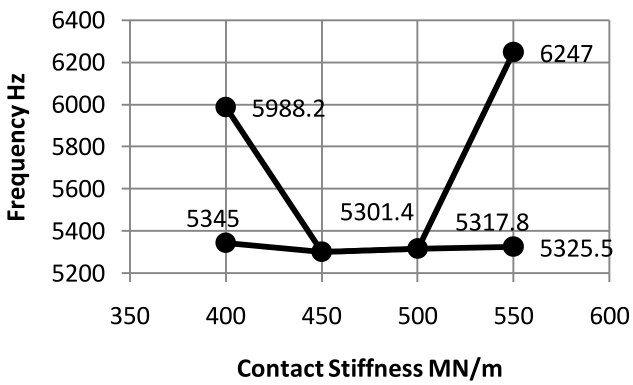


Fig. 14. Effect of contact stiffness on the system mode frequencies at μ=0.4 for the system fourth mode shape

The system showed two frequencies (5988.2Hz and 5345Hz) at contact stiffness 400MN/m, indicating a fourth mode shape. These two modes converged with increasing contact stiffness until they merged at contact stiffness 450MN/m. This merging continued beyond the value of that contact stiffness 450 MN/m until the split was generated at contact stiffness 500MN/m. The system instability was conducted at contact stiffness of 450 MN/m to 500 MN/m, as in Fig. 14. The modes started to split and separate with increased contact stiffness beyond 500MN/m and become two stable modes at contact stiffness 550MN/m (6247Hz and 5325.5Hz). The instability as a function of contact stiffness was conducted as in the Fig. 15 which shows that the degree of instability started to increase when the two modes merged.

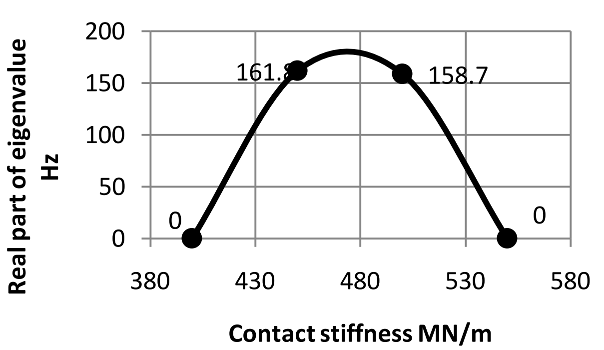


Fig. 15. Effect of contact stiffness on the system instability at μ=0.4 for the system fourth mode shape

This merging increased the instability from zero at contact stiffness 400Mn/m to 161.8Hz at contact stiffness 450MN/m. The system at contact stiffness 500MN/m showed the same instability at contact stiffness 450MN/m. Thus the system tendency to generate the squeal was approximately equal. This explains why the frequencies stayed approximately constant between 450 MN/m to 500MN/m, as in Fig. 14. The system instability began to decrease beyond contact stiffness 500MN/m indicating the system tendency to be stable again. The real part of eigenvalue showed that the system was completely stable when the contact stiffness reached 550MN/m.

6 STUDY THE EFFECT OF FRICTION COEFFICITENT ON THE SYSTEM STABILITY

The purpose of conducting this parameter is to recognize the effect of friction coefficient in the occurrence of instability in the disc-plate system. It is expected that the result of the analysis could provide a guideline for proposing some methods to suppress the occurrence of disc brake squeal. The friction force will not be uniformly distributed over the entire contact area between the pad and the disc surface [24]. The system stability is affected by increasing or decreasing the friction force value. This proves that the role of friction is to couple the different vibrating modes of sliding components [7]. The effect of friction coefficient on the squeal was analyzed by applying a range of friction coefficient values from 0.0 to 0.4 as in Fig. 16 with an increment of 0.1. The analysis was conducted with contact stiffness value of 200 MN/m which was assumed to be constant and distributed uniformly among the contact nodes. The system showed two frequencies (4441.9Hz and 3268.9Hz) at friction coefficient 0.1 indicating that the mode is the third diametral mode. These two modes began to converge until they merged (2919.8Hz) at friction coefficient 0.2. The merging continued beyond the friction coefficient 0.2 until discontinuity was achieved with friction coefficient 0.3.

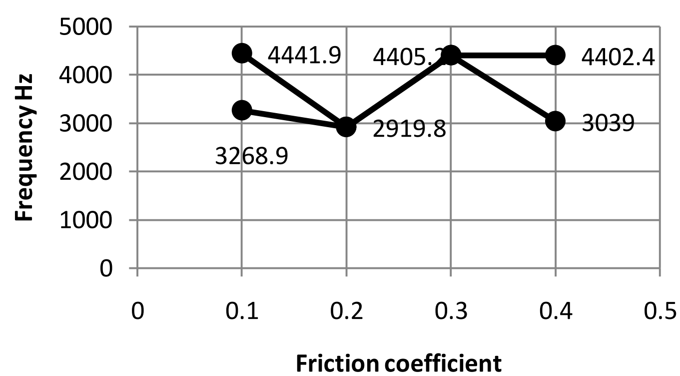


Fig. 16. Effects of friction coefficient on the system mode frequencies at K=200 MN/m for the system third mode shape

A non-conservative effect can be achieved as a friction forces tend to couple the two modes. This effect enable the system to exchange energy in a way that makes the unstable behavior of the brake system continues. The effects of friction coefficient enforce the system to generate the squeal at lower frequency and not those of separate stable modes. The instability as a function of friction coefficient was conducted for the third mode shape, as in Fig. 17. The Fig. shows that instability increased from zero at friction coefficient 0.1 to 258.07 Hz at friction coefficient 0.2. Beyond friction coefficient

0.2 the system tended to be stable again. When the friction coefficient was 0.3, the system showed a small degree of instability 3.899Hz. At friction coefficient 0.4, the system returned stability.

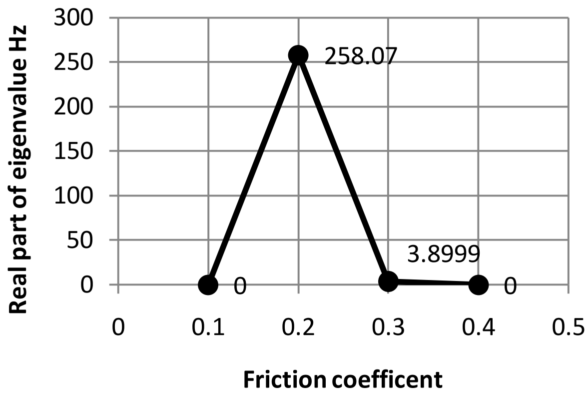


Fig. 17. The effect of friction coefficient on the system instability at K=200 MN/m for the system third diametral mode shape

Even when the contact stiffness is constant, the separation can happen with the increase in the friction coefficient because increased friction coefficient will increase the stress between the contact surfaces. Another way to decrease the instability is to position the nodes at the center of the pad, [10]. The contact stiffness was increased from 200MN/m to 370MN/m in order to study the effects of different friction coefficient with other contact stiffness, as in Fig. 18. By providing the model with a range of friction coefficient from 0.1 to 0.4 with an increment of 0.1, the unstable mode was conducted. The value of the contact stiffness was assumed to be constant and distributed uniformly among the contact nodes. The system generated unstable mode until a friction coefficient 0.3. The unstable frequency at friction coefficient 0.1 was 4893.8Hz, indicating the generation of a third unstable mode. By increasing the friction coefficient to 0.4, the system showed a stable frequency mode at 4348.8Hz and 3487.1Hz.

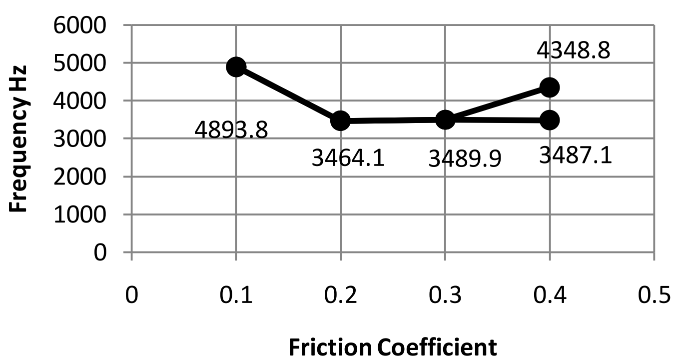


Fig. 18. Effect of friction coefficient on the system mode frequencies at K=370 MN/m for the system third mode shape

It is thought that when the brake has a high friction coefficient, it is noisier but this is a misconception. A noisy brake is not always the result of a high friction coefficient. The result in Fig. 18 agrees with Fieldhouse 1999 results, showing that the high friction coefficient can lead to a stable system. It is clear that with increase in friction coefficient, the negative damping decreases. The evidence of Matrix27 (contact matrix) does not mean that there is no separation between the contact surfaces [17]. Ouyang [17] found that to prevent the surface separation, high contact stiffness should be applied between the surfaces during the use of Matrix 27. That could be the reason why the system showed a long period of instability with increased contact stiffness Fig. 18 than with lower contact stiffness Fig. 16. The instability of the system at contact stiffness 370MN/m which was distributed uniformly among the contact nodes was conducted as shown in the Fig. 19. The results showed that the instability decreased with increase in the friction coefficient.

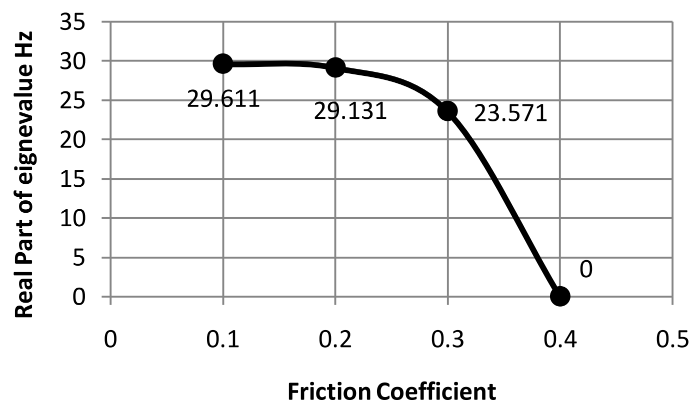


Fig. 19. Effect of friction coefficient on the system instability at K=370 MN/m for the system third mode shape

By increasing the friction coefficient the system became stable and the two modes separated when the friction coefficient reached 0.4. By comparing Figs. 19 and 17, it can be observed that at low contact stiffness of 200MN/m, the system showed a higher degree of instability than at contact stiffness 370MN/m but with a smaller period of instability. By increasing the contact stiffness from 200 MN/m to 370Mn/m, another unstable mode was generated, as illustrated in Fig. 20. The system stable frequency at friction coefficient 0.5 was 2682.3Hz and 1884.8Hz, indicating the second mode shape. The system was considered as unstable system when the friction coefficient between 0.05 and 0.3, a coupling between the two modes generated the squeal.

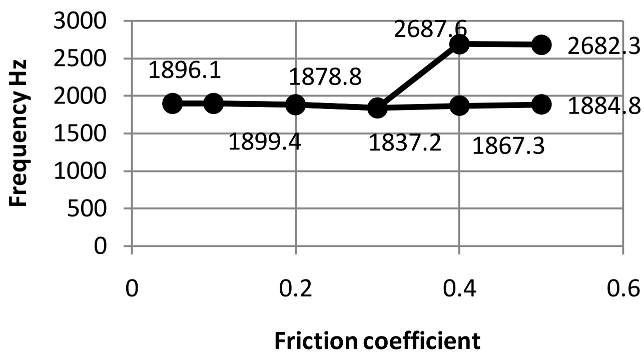


Fig. 20. Effect of friction coefficient on the system mode frequencies at K=370 MN/m for the system second mode shape

The system started as a stable system when the friction coefficient was 0.3. The system remained stable when the friction coefficient reached 0.4 showing two split modes at 2682.3 Hz and 1884.8 Hz. It can be said that even with increased contact stiffness, it is still possible to achieve the stability of the system. McDaniel [5] said, "The increase of contact stiffness between the two surfaces will achieve the separation between the two modes". The real part for Fig. 21 was conducted in order to understand the behavior of the system during the instability. The complex eigenvalue had a positive real part at friction coefficient less than 0.4, indicating the instability of the system.

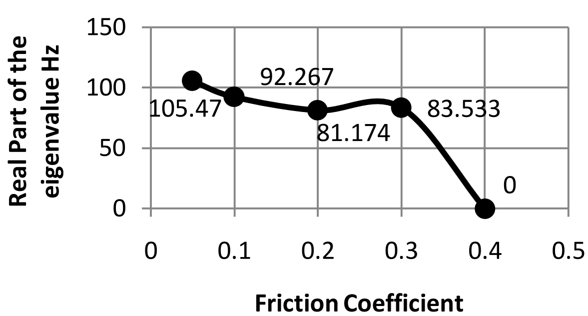


Fig. 21. The effect of friction coefficient on the system instability at K=370 MN/m for the system second mode shape

The system demonstrated negative damping characteristics and therefore, the corresponding mode was unstable and squeal was likely to occur [18]. Fig. 21 shows that at low friction coefficient 0.05, higher instability was achieved. However, by increasing the friction coefficient, the system tendency to be stable increased except at 0.3, when the system showed a little rise in the instability. At friction coefficient 0.4, the second mode shape became a stable mode.

7 MODE SHAPE AND THE FREQUENCY RANGE

Fig. 9 shows unstable frequencies of 9504.6Hz-8809.8Hz. This frequency range lies between the 6th diametral mode and the 7th diametral mode of the disc model under free-free conditions. Since the frequency must increase as a result of contact (the restrained stiffness increases) it is expected that the disc diametral mode order in Fig. 9 should be sixth. Fig. 12 shows unstable frequency range of 2093-1935.5 Hz. This frequency range lies between the 2nd diametral mode (1808.2 Hz) and the 3rd diametral mode (2944.6 Hz) of the disc under free-free conditions. Since the frequency must increase as a result of contact, it is expected that the disc diametral mode order in Fig. 12 should be the second. Fig. 14 shows unstable frequency range of 5301.4-5317.8 Hz. This frequency range lies between the 4th diametral mode (4312.3 Hz) and the 5th diametral mode (6184.6 Hz) of the disc model under free-free conditions. Since the frequency must increase as a result of contact, it is expected that the disc diametral mode order in Fig. 14 should be the fourth. The method in this section is built on Joe [19] result in which he showed that the free-free frequency of the disc is less than the brake system frequency. Besides that, Ripin [22] used this method to evaluate his results. The number of the contact element (Matrix27) should be considered, as the distance between the plate or the pad and the disc. Changing this distance or the Matrix27 element length is going to change the simulation result. The number of the contact element used between the plate and the disc in this paper is eleven. The mesh shape should be similar to the element used for the geometry, otherwise a reduced integration result will be achieved. An axial symmetric mesh should be applied for the disc because the disc element chosen was solid45, so the element shape will not be changed. As a non-axial symmetric mesh can cause the double modes of the disc to split even in the free-free condition, the number of the contact element can also cause the split. If the number of the contact element (Matrix27) is less than the minimum number it can result in a split for the unstable mode (mode has a positive real part). A split in the two stable diametral modes of the system (disc-plate) to three stable modes can also result. Therefore, the entire mode should be observed in order to notice if the number of the element is above the minimum number or not. Besides that, the frequency value should match the condition that was explained in the beginning of this section. However, this may be a theoretical validation for the simulation and can also explain why there is a difference in the element number used by Massi and Ripin [21], [22] to achieve accurate result close to the experimental result.

8 CONCLUSION

When the damping matrix is negligible, both roots of the complex pair have virtually the same imaginary values and the real values have opposite signs. Using matrix27 has shown us how the contact stiffness and friction coefficient can affect the mode shape and natural frequency. The squeal can be reduced by increasing the friction coefficient and decreasing the contact stiffness of the disc-plate system. Changing the K and μ will change the surface roughness and contact stiffness and as a result the stability. The contact stiffness which induced the squeal can be estimated by changing the K and μ values until the unstable mode shape appears. It should be noted that not all unstable modes will squeal. An unstable mode is just one cause of squeal [25]. However, a variation of the contact stiffness values or the friction coefficient values could confirm whether the unstable mode will squeal or not. The result supports the conclusion of Murakami [20] that the noise occurs as a result of the coupling of the natural frequencies of the individual parts when they are close together. According to Nishiwaki [4], brake squeal is strongly influenced by the natural frequency and the mode of the disc component. It is well known that any variation of the boundary conditions affect mode shapes and especially natural frequencies. Thus we cannot deduce directly the brake system behavior from the components modal characteristics of the free-free component. That is why it is necessary to take into account the contact between the different components and modelise it precisely so as to be able to predict the dynamic behavior of the system. However, it has been shown that even with a constant friction coefficient, the system may be unstable [7]. The friction stiffness acts as spring causing the stiffness matrix to be asymmetric. Eigenvalue with positive real parts are identified as unstable modes, which always appear in complex coupled pairs due to the presence of non-conservative frictional work produces a non-symmetric stiffness matrix. It can be said that the squeal is a complex phenomenon caused by the dynamic behavior of the brake parts and friction mechanism. The plate can represent the brake-pad system better than the beam-disc system due to the large contact area between the plate and the disc. This can be noticed clearly by comparing the result of this paper with Ripin's thesis [22]. The effect of changing the contact stiffness with constant friction coefficient showed three unstable modes. The effect of changing the friction coefficient with constant contact stiffness showed one unstable mode at contact stiffness 200 MN/m and two unstable modes at contact stiffness 370 MN/m. This result indicates that low contact stiffness value will generate fewer numbers of unstable modes but the degree of instability is higher than high contact stiffness value and vice verses.

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