On Comparative Modeling Of Gls And Ols Estimating Techniques

Atanlogun S.K., Edwin O. A. Afolabi Y.O.

ABSTRACT: In this study GLS and OLS estimating techniques were compared. To achieve the goal, GLS and OLS estimating techniques were applied on a simultaneous equation models (that is Per Capital Gross Domestic Product equation model and Foreign Direct Investment equation model). Annual data was collected for Per Capital Gross Domestic Product, Foreign Direct Investment, Lending Rate of Interest and Exchange Rate Index for the period of 1983 to 2008 from the National Bureau of Statistics (NBS) and Central Bank of Nigeria Statistical Bulletin (2009). Results from the analysis showed that GLS and OLS estimating techniques produced the same values of coefficients and standard errors in the two equations. The study however concluded that the two estimators are both efficient alike, which shows that the GLS estimator is an OLS estimator of a transformed isomorphic model. The R-package of statistical software was adopted. The two estimators provide BLUE (Best Linear Unbiased Estimator) under heteroscedasticity/serial correlation.

KEY WORDS: Generalized Least Squares (GLS), Ordinary Least Square (OLS)

1. INTRODUCTION

In statistics, Generalized Least Squares (GLS) is a technique for estimating the unknown parameters in a linear regression model. The GLS is applied when the variances of the observations are unequal (heteroscedasticity), or when there is a certain degree of correlation between the observations. In these cases, Ordinary Least Squares (OLS) can be statistically inefficient, or even give misleading inferences. On the other hand, OLS which emerged in the early years of the nineteenth century was centered on dominant and powerful estimating principle. However, OLS which is the cornerstone of most econometric theory was developed and published by Carl Friedrich Gauss which have been discussed in (Gujarat, 2003; Upton et al, 2002), is an estimation technique used in regression analysis, which is a method that studies the relationship between two or more variables. Supposed that Gauss – Markov assumptions hold for all equations, thus the transformed variables in the equation satisfy the conditions under which OLS is BLUE (Best Linear Unbiased Estimator). The coefficient vector from the OLS regression model is the Generalized Least Squares (GLS) estimator. However, by using the GLS method to estimate the equations jointly, efficiency is obtained. (http://en.wikipedia.org/wiki/generalizedleast-squares-regression).

2. MODEL SPECIFICATION

I. Generalized Least Square

The Generalized Least Square (GLS) model consists of

\[
\begin{align*}
&\{y_{ij}, x_{ij}\; ; i=1\ldots n, j = 1\ldots p\; \text{on n statistical units}. \\
&\text{The study however}.
\end{align*}
\]

and the predictor values are placed in the design matrix

\[
X = [x_{ij}],
\]

whereas the conditional variance of \( Y \) given \( X \) is a known matrix \( \Omega \). This is usually written as

\[
Y_i = X_i\beta_i + \varepsilon_i, \quad i = 1, 2, 3, \ldots, m
\]

Where \( y_i \), \( i \leq (n \times 1) \), \( X_i \), \( (n \times k) \), \( \beta_i \), \( (k \times 1) \)

\[
E[\varepsilon_i / X_i] = 0, \quad \text{var}[\varepsilon_i / X_i] = \Omega.
\]

Here \( \beta \) is a vector of unknown “regression coefficient” that must be estimated from the data. If \( \varepsilon_i \) is the \( t \)-th element of \( \varepsilon_i \), we assumed that \( (\varepsilon_i, \varepsilon_{i2}, \ldots, \varepsilon_{im}) \) is iid with \( E(\varepsilon_{it}) = 0 \).

\[
E(\varepsilon_{it} \varepsilon_{js}, \quad \text{if} \quad t = s \quad \text{(Non zero)}
\]

contemporaneous correlation) If we stack the m equations we have

\[
\begin{align*}
\sigma_{ij} & \quad \text{if} \quad t = s, \\
0 & \quad \text{if} \quad t \neq s.
\end{align*}
\]

Efficiency is one of the properties of a good estimator. The concept refers to the one with smallest variance for any given sample size. (Udom, Akaninyene Udo, 2005). In this study, the GLS and OLS estimating technique will be adopted on a simultaneous equation models (that is Per Capital Gross Domestic Product equation model and Foreign Direct Investment equation model and then confirm which estimator is most efficient).

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\[
\begin{pmatrix}
 y_1 \\
 y_2 \\
 \vdots \\
 y_m \\
\end{pmatrix} = \begin{pmatrix}
 x_1 & 0 & \ldots & 0 \\
 0 & x_2 & \ldots & 0 \\
 \vdots & \vdots & \ddots & \vdots \\
 0 & 0 & \ldots & x_m \\
\end{pmatrix} \begin{pmatrix}
 \beta_1 \\
 \beta_2 \\
 \vdots \\
 \varepsilon_m \\
\end{pmatrix} + \begin{pmatrix}
 \varepsilon_1 \\
 \varepsilon_2 \\
 \vdots \\
 \varepsilon_m \\
\end{pmatrix}
\]

that is

\[Y = X\beta + \varepsilon\]

\[E(\varepsilon) = 0\]

\[E(\varepsilon') = v = \sum \otimes I_n = \begin{bmatrix}
 \sigma_{11} I_1 & \sigma_{12} I_1 & \ldots & \sigma_{1m} I_n \\
 \sigma_{21} I_1 & \sigma_{22} I_1 & \ldots & \sigma_{2m} I_n \\
 \vdots & \vdots & \ddots & \vdots \\
 \sigma_{m1} I_1 & \sigma_{m2} I_1 & \ldots & \sigma_{mm} I_n
\end{bmatrix}\]

The GLS estimator of \(\beta\) is:

\[\hat{\beta}_{GLS} = (X'v^{-1}X)^{-1} X'v^{-1} y = \left[X' \left(\sum_{i=1}^{n} \otimes I_n \right) X\right]^{-1} X' \left(\sum_{i=1}^{n} \otimes I_n \right) y\]

Since from Kronecker product \((A \otimes B)^{-1} = A^{-1} \otimes B^{-1}\)

\[V(\hat{\beta}_{GLS}) = (X'v^{-1}X)^{-1} = \left[X' \left(\sum_{i=1}^{n} \otimes I_n \right) X\right]^{-1}\]

Which have been discussed in Greene, 1998; Zellers, 1962; Griffiths et al 1993; Ferdinand et al, 2008; GLS (http://200.32.4.58/econometrical/ssem.pdf); GLS (http://psweb.sbs.ohiostate.edu).

II. Ordinary Least Squares

The method of least squares is a standard approach to the approximate solution of over determined systems i.e. sets of equations in which there are more equations than unknown. The least squares method is usually credited to Carl Friedrich Gauss (1794), but it was first published by Adrien – Marie Legendre. “Least Squares” means that the overall solution minimizes the sum of the squares of the errors made in the result of every single equation. (http://en.wikipedia.org/wiki/leastsquares) Multiple regression model is a type of model in which a response variable \(Y\) is determine by two or more predictor/explanatory variables \(X_1, X_2, \ldots, X_k\) (A. I. Arua et al, 1999). The model is of the form \(Y = X\beta + \varepsilon\) and the desire formula is \(\hat{\beta} = (X'X)^{-1} X'Y\). The variance is given by \(V(\beta) = \sigma^2(X'X)^{-1}\), we determine the scalar \(\sigma^2\), being discussed in (Pindyck and Rubinfeld, 1981). The coefficient of determination (denoted by \(R^2\)) measures the proportionate reduction of total variation in \(Y\) associates with the use of the set of \(X\) variable \(X_1, \ldots, X_p\). (Peter et al, 2003). It is given by \(R^2 = \frac{SS_{total} - SS\varepsilon}{SS_{total}}\). have values of \(R^2\) close to 0 indicate a poor fit.

III. Simultaneous Equation Models

An economic model may contain multiple equations, which are explanatory of each other on the surface. They are not estimating the same response variable; they have different explanatory variable e.t.c. (http://en.wikipedia.org/wiki/seemingly-unrelated-regression).

IV. The Model

The models of interest are

\[\text{PGDP} = \alpha_0 + \alpha_1 \text{ERI} + \alpha_2 \text{LDR} + \alpha_3 \text{FDI} + \varepsilon_1\]

\[\text{FDI} = \beta_0 + \beta_1 \text{ERI} + \beta_3 \text{LDR} + \varepsilon_2\]

Here we have two equations with two endogenous variables (PGDP and FDI) and two exogenous variables (ERI and LDR).

V. Definition of Terms

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>PDGP</td>
<td>Per Capital Gross Domestic Product</td>
</tr>
<tr>
<td>FDI</td>
<td>Foreign Direct Investment</td>
</tr>
<tr>
<td>LDR</td>
<td>Lending Rate of Interest</td>
</tr>
<tr>
<td>ERI</td>
<td>Exchange Rate Index</td>
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</tbody>
</table>

3. MATERIAL

The data used for this study was obtained from the National Bureau of Statistics; Annual Abstract of Statistics 2009 Federal Republic of Nigeria, December, 2009.

CBN Statistical Bulletin, December, 2009

PERIOD: - 1983 to 2008

NOTE:- GDP per capital was generated by dividing National Income by population i.e. \(\frac{\text{National Income}}{\text{Population}}\)

4. DISCUSSION OF RESULTS

Two different estimation techniques used to obtain the estimated parameters are GLS and OLS. The \(R\) – package of statistical software was adopted to obtain the results and necessary discussions were made.

(i) Per Capital Gross Domestic Product model for GLS and OLS estimators are displayed below:

Estimator: Generalized Least Squares
Dependent Variable: Per Capital Gross Domestic Product.

From the GLS output in table 1(b) below, the model becomes:

\[
\text{PGDP} = -2071.6403 + 155.8892\text{ERI} + 399.1797\text{LDR} + 0.1218\text{FDI}
\]

Table 1a: OLS OUTPUT

|       | Estimate  | Std. Error | t value | Pr(>|t|) |
|-------|-----------|------------|---------|----------|
| Intercept | -2071.6403 | 4090.2818 | -0.5064 | 0.6175590 |
| ERI    | 155.8891   | 59.98038  | 2.599   | 0.0163810 |
| LDR    | 399.1796   | 216.6851  | 1.842   | 0.0789646 |
| FDI    | 0.121842   | 0.03212   | 3.792   | 0.0009972 |

Table 1b: GLS OUTPUT

|       | Estimate  | Std. Error | t value | Pr(>|t|) |
|-------|-----------|------------|---------|----------|
| Intercept | 8874.8206 | 26485.955 | 0.3350 | 0.74060 |
| ERI    | 1700.887  | 160.631    | 10.588  | 0.0009792 |
| LDI    | -166.971  | 1406.099   | -0.1187 | 0.90651  |

Mean of dependent variable = 20428.18
Standard deviation of dependent variable = 20244.61
Sum of squared residuals = 727703018.385395
Standard error of residuals = 5751.296369
R – Squared = 0.928978

Interpretation: The two estimators produced the same value of coefficients and standard errors. This result revealed that both of them are efficient alike, which shows that the GLS estimator is an OLS estimator of a transformed isomorphic model.

(ii) Foreign Direct Investment model for GLS and OLS estimation are displayed below:

Estimator: Generalized Least Squares
Dependent variable: Foreign Direct Investment
From the GLS output in 2(b) below, the model becomes:

\[
\text{FDI} = 8874.8206 + 1700.8870\text{ERI} - 166.9712\text{LDR}
\]

Table 2a: OLS OUTPUT

|       | Estimate  | Std. Error | t value | Pr(>|t|) |
|-------|-----------|------------|---------|----------|
| Intercept | -2071.6402 | 4090.2818 | -0.5064 | 0.6175590 |
| ERI    | 155.8891   | 59.98038  | 2.5990  | 0.0163810 |
| LDI    | -166.971   | 1406.099   | -0.1187 | 0.90651  |
| FDI    | 0.121842   | 0.03212   | 3.7929  | 0.0009972 |

Mean of dependent variable = 71840.95
Standard deviation of dependent variable = 91823.35
Sum of squared residuals = 32055238567.287
Standard error of residuals = 37332.372342
R – Squared = 0.847927

**Estimator: Ordinary Least Squares**


Dependent variable: Foreign Direct Investment

From the OLS output in the table 2(a) above, the model becomes:

\[ FDI = 8874.8206 + 1700.8870 \text{ERI} – 166.9712 \text{LDR} \]

Mean of dependent variable = 71840.95

Standard deviation of dependent variable = 91823.35

Sum of squared residuals = 32055238567.287

Standard error of residuals = 37332.372342

R – Squared = 0.847927

**Interpretation:** The two estimators produced the same value of coefficients and standard errors. This result revealed that both of them are efficient alike, which shows that the GLS estimator is an OLS estimator of a transformed isomorphic model.

5. **CONCLUSION**

In this study, the two estimators – GLS and OLS were compared. Result from the analysis showed that GLS and OLS estimators produced the same values of coefficients and standard errors in the simultaneous equations. This study therefore concluded that the two estimators are efficient alike, which shows that this GLS estimator is an OLS estimator of a transformed isomorphic model.

6. **REFERENCES**


