Availability Of A Complex System Using MATLAB & Comparison

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Abstact: In this paper, MATLAB has been applied on differential difference equation to obtain availability, MTTF etc. of a complex system. Though availability can also be found by other methods like matrix method [2], Laplace transformation method [7] etc., but with the help of MATLAB it is easy to find availability compared to other methods. Zaidi, Zeenat et. al. [3] applied the method of MATLAB to a simple system. We in this paper applied the MATLAB to a two element standby system having perfect switching and also compared the result with matrix method and found that results obtained match completely.

Index terms- Availability, Differential Difference Equations, Failure Rate, Matrix method, Markov Graph, MATLAB, MTTF, Repair rate

1. INTRODUCTION

To make the reliability analysis technique more computer friendly, in the manuscript MATLAB has been applied on the governing differential difference equations of a complex standby system to get influencing parameters; availability, MTTF etc .Singh, J. and Goyal, Yogesh [2] discussed complex systems using matrix method by developing a computer program and others[8,9] discussed the same using numerical methods and Laplace Transform Method. Zeenat Zaidi et. al. used MATLAB to analyze availability of a simple system. We in this manuscript apply MATLAB on a two element standby system having perfect switching and observe that the result match with that obtained by Matrix method. Though computer application have been applied earlier [7, 8, 9]; in this text we use MATLAB which makes the computation easy. It is the long run or steady state availability in which management is interested. But with passage of time efficacy of machine to do work reduces, so the transient state availability is of more importance. Here, in this paper we calculate both steady state and transient availability. Steady State availability is calculated by solving recursively, the differential difference equations of the system. Calculation of transient state availability is a difficult task. So for ease in calculation we involve computer and solve it by the MATLAB programming and compare the result with matrix method.

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2. LITERATURE SURVEY

Many methods used by researchers [2, 3, and 7] to calculate availability of a complex system are either Laplace transform method [7] or regenerative technique. Use of Laplace transform method yields good results. The numerical methods [8] used for solution of differential equations give approximations. Singh [10] used matrix method for solving differential difference equations to analyze a multiple channel system. The method used by Singh was applied to a simple system by Mahajan [12]. The method requires the computation of Eigen values of the coefficient matrix. Since the calculation of Eigen values for a large order matrix is a difficult task so, the method could not be applied to complex systems. In this paper, we discuss the extension of the matrix method [2] by which it is possible to calculate the probabilities of the various states of a complex system without calculating the eigen values of the matrix. We apply the method to a standby system. We also apply the method of MATLAB to the same set of equations and observe that the result match with that obtained by use of matrix method.

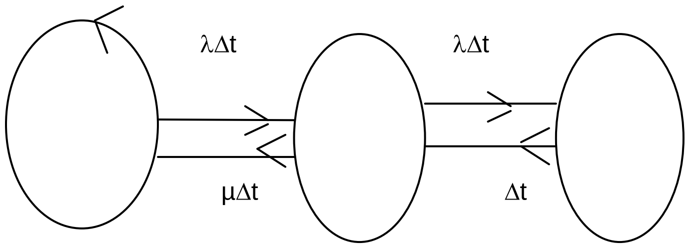
3. THE MODEL

In this paper, we discussed system composed of two elements x_1 and x_2. With passive redundancy x_1 and x_2 are connected in parallel and one of them is standby element. When x_1 fails, x_2 takes over and element x_1 undergoes repair immediately. As soon s the working unit fails, the standby unit is immediately switched on by a perfect switch and sensing device. The failed unit is repaired by a repairman immediately and the repaired unit goes in standby. When the system is in failed state there is no failure in the system. At any time system is in any of three states:-

(a) State S_0- Where the first element is operating the second element is standing by in an operable condition.
(b) State S_1- Where the first element has failed and second element takes over.
(c) State S_2- When both the elements are down.

Here, we are not treating S_2 as an absorbing state transition from S_2 to S_1 is possible.

3.1 The Markov Graph



$s_0 = x_1 x_2$ $\quad s_1 = x_1 \bar{x}_2 + \bar{x}_1 x_2$

$s_2 = \bar{x}_1 \bar{x}_2$

<u>We assume</u>

λ = Failure Rate

μ = Repair Rate

State	Unit I	Unit II
0	Online	Stand by
1	Failed	Online
2	Failed	Failed

3.2 STATE EQUATIONS

$$p_0^{'}(t) = -\lambda p_0(t) + \mu p_1(t)$$
$$p_1^{'}(t) = \lambda p_0(t) - (\lambda+\mu) p_1(t) + \mu p_2(t)$$
$$p_2^{'}(t) = \lambda p_1(t) - \mu p_2(t)$$

Let the initial condition be

$$p_{_0}(0) = 1$$
$$p_1(0) = p_2(0) = 0$$

Where $p_0(t), p_1(t), p_2(t)$ denote the probabilities of state s_0, s_1, s_2 respectively.

4. STEADY STATE AVAILABILITY OF THE SYSTEM

This is also called long run equilibrium availability. For finding steady state availability of the system we take $t \to \infty$

$$\frac{d}{dt}(p_i(t)) = 0$$
$$p_i(t) = p_i$$

On applying this on the given set of equation, we get

$$\lambda p_0 = \mu p_1 \qquad ---(i)$$
$$(\lambda+\mu) p_1 = \lambda p_0 + \mu p_2 \qquad ---(ii)$$
$$\mu p_2 = \lambda p_1 \qquad ---(iii)$$

$$p_1 = \frac{\lambda}{\mu} p_0 \qquad \text{----By Equation } (i)$$
$$p_2 = \frac{\lambda}{\mu} p_1 = \frac{\lambda^2}{\mu^2} p_0 \qquad \text{-----By Equation } (iii)$$

Now, using

$$\mathrm{p}_0 + p_1 + p_2 = 1$$

We get

$$p_0 + \frac{\lambda}{\mu} p_0 + \frac{\lambda^2}{\mu^2} p_0 = 1$$
$$p_0 \left[1 + \frac{\lambda}{\mu} + \frac{\lambda^2}{\mu^2} \right] = 1$$
$$p_0 = \left[\frac{\mu^2}{\mu^2 + \mu\lambda + \lambda^2} \right]$$

And therefore

$$p_1 = \frac{\lambda\mu}{\mu^2 + \mu\lambda + \lambda^2}$$

Here, p_0 and p_1 are working states of the system.

Therefore,

Availability of the system is given by

$$\mathrm{p}_0 + p_1 = 1 - \frac{\lambda^2}{\mu^2 + \mu\lambda + \lambda^2}$$

The above process is Markov and is known as birth-death process.

5. TRANSIENT STATE AVAILABILITY

It is time dependent availability and can be calculated by many methods [2, 3]. Here in this paper we shall apply Matrix method and method of MATLAB programming.

5.1 Matrix Method

Let $p(k,t)$ denote the probability of the system at time t in the state k. If the number of all the possible transition states of a complex system are 'n' then the system of differential difference equations may be written as:

$(\theta I - A)\bar{p}(k,t) = \bar{0}$, where $\theta = \frac{d}{dt}$ is used for differentiation, $\bar{0}$ is the null matrix, matrix A is the matrix of coefficients of $p(i,t), i = 1,2,3..........,n$.

$$\bar{p}(k,t) = (p(1,t), p(2,t)............p(n,t))^T$$

And I_n is a identity matrix of order n.

Let C be a matrix such that $C^{-1}AC = D$, where $D=(d_1, d_2, ..., d_n)$ is the diagonal matrix of the matrix A.

We may write

$C^{-1}(\theta I - A)\bar{p}(k,t) = \bar{0}$

$\Rightarrow C^{-1}(\theta I - A)CC^{-1}\bar{p}(k,t) = \bar{0}$

$\Rightarrow (\theta I - C^{-1}AC)C^{-1}\bar{p}(k,t) = \bar{0}$

$\Rightarrow (\theta I - D)G(k,t) = \bar{0},$

Where

$G(k,t) = C^{-1}\bar{p}(k,t)$

Above equation is a matrix linear differential equation in $G(k,t)$. The solution to the equation is $G(k,t)e^{-Dt} = K$, for some constant K with initial conditions $p(1,0)=1$ and $p(k,0)=0$ otherwise. We get $C^{-1}\bar{p}(k,0) = K$ i.e. $K = C^{-1}\bar{p}(k,0)$ Notice that $\bar{p}(k,0) = (100......0)^T$ $G(k,t) = e^{Dt}K = e^{Dt}C^{-1}\bar{p}(k,0)$ gives $C^{-1}\bar{p}(k,t) = e^{Dt}C^{-1}\bar{p}(k,0)$, so

$$\bar{p}(k,t) = C(1 + Dt + \frac{D^2t^2}{2!} + ...)C^{-1}\bar{p}(k,0)$$

$$= \bar{p}(k,0) + A\bar{p}(k,0)t + \frac{A^2}{2!}t^2\bar{p}(k,0) + ...$$

$$= \bar{p}(k,0) + L_1 t + \frac{L_2}{2!}t^2 + ...(1),$$

where

$$L_N = A^N \bar{p}(k,0)$$

The initial conditions make it clear that $\bar{p}(k,0)$ is the column matrix $(100......0)^T$, $A\bar{p}(k,0)$ is just the 1st column of the matrix A. Let us denote this column matrix by

$$A_1 = (a_{11}, a_{12}, ...a_{1n})^T$$

$A^2\bar{p}(k,0) = AA\bar{p}(k,0) = AA_1$ is again a column matrix, let us denote it by

$$A_2 = (b_{11}, b_{12}, ...b_{1n})^T$$

Let $A^{r-1}\bar{p}(k,0) = A_{r-1} = (p_{11}, p_{12}, ...p_{1n})^T$,

By the method of Principle of induction

$A^r\bar{p}(k,0) = AA_{r-1} = (q_{11}, q_{12}, ...q_{1n})^T$, say probability of different stages from equation (1) is:

$$p(1,t) = 1 + a_{11}t + b_{11}\frac{t^2}{2!} + ...$$

$$p(2,t) = a_{21}t + b_{21}\frac{t^2}{2!} + ...$$

....

....

....

$$p(n,t) = a_{n1}t + b_{n1}\frac{t^2}{2!} + ...$$

If $p(1,t), p(2,t)....................p(i,t)$ are the working states of a system then,

$$Av(t) = p(1,t) + p(2,t) + ...p(i,t)$$

$$= 1 + (a_{11} + a_{21} + ... + a_{i1})t + (b_{11} + b_{21} + ... + b_{i1})\frac{t^2}{2!} + ...$$

5.2 Application of matrix method to the system

Notation: We write $p_i(t) = p(i,t)$;i=0,1,2

The coefficient matrix of the given system of equations

$$A = \begin{pmatrix} -\lambda & \mu & 0 \\ \lambda & -(\lambda+\mu) & \mu \\ 0 & \lambda & -\mu \end{pmatrix}$$

$$A\bar{p}(k,0) = \begin{pmatrix} -\lambda & \mu & 0 \\ \lambda & -(\lambda+\mu) & \mu \\ 0 & \lambda & -\mu \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -\lambda \\ \lambda \\ 0 \end{pmatrix}$$

$$A^2\bar{p}(k,0) = A.A\bar{p}(k,0) = \begin{pmatrix} -\lambda & \mu & 0 \\ \lambda & -(\lambda+\mu) & \mu \\ 0 & \lambda & -\mu \end{pmatrix}\begin{pmatrix} -\lambda \\ \lambda \\ 0 \end{pmatrix} = \begin{pmatrix} \lambda(\lambda+\mu) \\ -\lambda(2\lambda+\mu) \\ \lambda^2 \end{pmatrix}$$

$$A^3\bar{p}(k,0) = A.A^2\bar{p}(k,0) = \begin{pmatrix} -\lambda & \mu & 0 \\ \lambda & -(\lambda+\mu) & \mu \\ 0 & \lambda & -\mu \end{pmatrix}\begin{pmatrix} \lambda(\lambda+\mu) \\ -\lambda(2\lambda+\mu) \\ \lambda^2 \end{pmatrix} = \begin{pmatrix} -\lambda(\lambda^2+3\mu\lambda+\mu^2) \\ \lambda(3\lambda^2+5\mu\lambda+\mu^2) \\ -2\lambda^2(\lambda+\mu) \end{pmatrix}$$

$$p(0,t) = 1 + (-\lambda)t + \lambda(\lambda+\mu)\frac{t^2}{2!}$$

$$+(-\lambda)(\lambda^2 + 3\mu\lambda + \mu^2)\frac{t^3}{3!} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$p(1,t) = \lambda t + (-\lambda)(2\lambda+\mu)\frac{t^2}{2!} + \lambda(3\lambda^2 + 5\mu\lambda + \mu^2)\frac{t^3}{3!} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Now,

$$Availabillity = p(0,t) + p(1,t)$$

$$= 1 - \frac{\lambda^2 t^2}{2!} + 2\lambda^2(\lambda+\mu)\frac{t^3}{3!} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

……………………………….(a)

5.3 MATLAB (R 2010a)

MATLAB is the defacto language of technical computing community, the world over. Most of the modern day communication systems simulation is done with help of powerful high level language.MATLAB provides an interactive programming environment with hundereds of built-in functions for technical computations, graphics, and animation. The built-in functions provide excellent tools for linear algebra, data analysis, signal processing, optimization, numerical solution of ordinary differential equations(ODEs), quadrature and many other types of scientific computations.

5.4 Application of MATLAB on given system of equations

Taking p(0,t)=x; p(1,t)=y; p(2,t)=z; λ =x1; μ =y1.

The syntax dsolve is used to solve the differential equations using initial conditions in MATLAB.

```
>>syms x1 y1

>>[x,y,z]=dsolve('Dx+x1*x=y1*y','Dy+(x1+y1)*y=x1*x+y1*z','Dz+y1*z=x1*y','x(0)=1','y(0)=0','z(0)=0')
```

Result

Result of the above equations in MATLAB

```
x=((x1^2*y1^2)/(x1^2+x1*y1+y1^2)+(x1*exp(t*(x1*y1)^(1/2)-t*y1-t*x1)*(x1*y1)^(1/2)*(x1^2*y1+x1^3+x1^2*(x1*y1)^(1/2)))/(2*x1^2*(x1*y1)^(1/2)+2*y1^2*(x1*y1)^(1/2)+2*x1*y1*(x1*y1)^(1/2))+(x1*(x1*y1)^(1/2)*(x1^2*y1+x1^3-x1^2*(x1*y1)^(1/2)))/(exp(t*x1+t*y1+t*(x1*y1)^(1/2))*(2*x1^2*(x1*y1)^(1/2)+2*y1^2*(x1*y1)^(1/2)+2*x1*y1*(x1*y1)^(1/2))))/x1^2

y=-((x1*(x1^2*y1+x1^3-x1^2*(x1*y1)^(1/2)))/(exp(t*x1+t*y1+t*(x1*y1)^(1/2))*(2*x1^2*(x1*y1)^(1/2)+2*y1^2*(x1*y1)^(1/2)+2*x1*y1*(x1*y1)^(1/2)))-(x1*exp(t*(x1*y1)^(1/2)-t*y1-t*x1)*(x1^2*y1+x1^3+X1^2*(x1*y1)^(1/2)))/(2*x1^2*(x1*y1)^(1/2)+2*y1^2*(x1*y1)^(1/2)+2*x1*y1*(x1*y1)^(1/2))-(x1^2*y1)/(x1^2+x1*y1+y1^2)+(exp(t*(x1*y1)^(1/2)-t*y1-t*x1)*(x1*y1)^(1/2)*(x1^2*y1+x1^3+x1^2*(x1*y1)^(1/2)))/((2*x1^2)*(x1*y1)^(1/2)+2*y1^2*(x1*y1)^(1/2)+2*x1*y1*(x1*y1)^(1/2))+((x1*y1)^(1/2)*(x1^2*y1+x1^3-x1^2*(x1*y1)^(1/2)))/(exp(t*x1+t*y1+t*(x1*y1)^(1/2))*(2*x1^2*(x1*y1)^(1/2)+2*y1^2*(x1*y1)^(1/2)+2*x1*y1*(x1*y1)^(1/2))))/x1

z= x1^2/(x1^2+x1*y1+y1^2)-(exp(t*(x1*y1)^(1/2)-t*y1-t*x1)*(x1^2*y1+x1^3+x1^2*(x1*y1)^(1/2)))/(2*x1^2*(x1*y1)^(1/2)+2*y1^2*(x1*y1)^(1/2)+2*x1*y1*(x1*y1)^(1/2))+(x1^2*y1+x1^3-x1^2*(x1*y1)^(1/2))/(exp(t*x1+t*y1+t*(x1*y1)^(1/2))+(2*x1^2*(x1*y1)^(1/2)+2*y1^2*(x1*y1)^(1/2)+2*x1*y1*(x1*y1)^(1/2)))
```

On simplification

x=

$$\frac{y_1^2}{x_1^2 + x_1 y_1 + y_1^2} + \frac{x_1\left(x_1 + y_1 + \sqrt{x_1 y_1}\right)}{2(x_1^2 + x_1 y_1 + y_1^2)} e^{(\sqrt{x_1 y_1} - y_1 - x_1)t} + \frac{x_1\left(x_1 + y_1 - \sqrt{x_1 y_1}\right)}{2(x_1^2 + x_1 y_1 + y_1^2)} e^{-(\sqrt{x_1 y_1} + y_1 + x_1)t}$$

y=

$$-\frac{x_1^2\left(x_1 + y_1 - \sqrt{x_1 y_1}\right)}{2\sqrt{x_1 y_1}(x_1^2 + x_1 y_1 + y_1^2)} e^{-(\sqrt{x_1 y_1} + y_1 + x_1)t} + \frac{x_1^2\left(x_1 + y_1 + \sqrt{x_1 y_1}\right)}{2\sqrt{x_1 y_1}(x_1^2 + x_1 y_1 + y_1^2)} e^{(\sqrt{x_1 y_1} - y_1 - x_1)t} + \frac{x_1 y_1}{(x_1^2 + x_1 y_1 + y_1^2)}$$

Availability=x+y

=

$$1 - \frac{x_1^2 t^2}{2!} + \frac{2x_1^2(x_1 + y_1)t^3}{3!} - \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$= 1 - \frac{\lambda^2 t^2}{2!} + 2\lambda^2(\lambda+\mu)\frac{t^3}{3!} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

……………….(b)

If we take $t \to \infty$ in x and y then we will get steady state availability otherwise transient state availability. It is clear from (a) and (b) that results match completely.

MTTF

$$t - \frac{\lambda^2 t^3}{3!} + 2\lambda^2(\lambda+\mu)\frac{t^4}{4!} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

choose

$$\lambda = 0.002$$

$$\mu = 0.01$$

$$Av(t) = 1 - 0.0002t^2 + 0.0000016t^3 \ldots\ldots\ldots\ldots\ldots\ldots$$

$$MTTF = t - 0.0004\frac{t^3}{3!} + 0.0000096\frac{t^4}{4!} \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Table of Availability

Time	1	2	3	4	5	6	7	8	9
Availability	0.999896	0.9992128	0.9982432	0.9969024	0.9952	0.9931456	0.9977488	0.9880192	0.9849664
Time	10	11	12	13	14	15	16	17	18
Availability	0.9816	0.9779296	0.9739648	0.9697152	0.9651904	0.9604	0.9553536	0.9500608	0.9445312
Time	19	20	21	22	23	24	25	26	27
Availability	0.9387744	0.92128	0.9266176	0.9202368	0.9136672	0.9069184	0.9	0.8929216	0.8856928

Table of MTTF

The MTTF for constant failure and repair rates taken in transient state for different time intervals

Time	10	20	30	40	50	60
MTTF	10.003984	23.005334	28.524	36.757334	44.166667	50.784

Conclusion

It is observed that application of above methods on a standby system gives same results. We observed that MATLAB makes the computations easy. It is better than traditional programming like C, C++, FORTRAN. Matrix method also holds good for complex systems, but it has applied on many complex systems [7, 8 &9]. We hope that the application of MATLAB will also make ease in calculation of availability of more complex systems. With the help of MATLAB both Steady state and Transient State Availability can be calculated simultaneously

References

[1] Bellman, Richard (1974), Introduction to matrix analysis, 2nd edition, New Delhi, Tata Mc Graw Hill.

[2] Singh, J. and Goyal, Yogesh Kumar (2006), Availability and Behavior Analysis of Bread Manufacturing plant, JMASS, Vol 2, pp 35-45.

[3] Zaidi, Zeenat and Goyal, Yogesh Kumar (2011), Evaluation of Availability- Comparison of important techniques, JMASS, Vol 7(2), pp 23-30.

[4] Dhillon, B.S and Singh, C. , Engineering reliability New techniques and applications, John Willey and sons, 1981

[5] Garg, Deepika, Kumar, Kuldeep, Singh, J, Availability Analysis of cattle feed plant using Matrix method, International Journal of engineering, Vol. 3, issue 2. Pp201-219.

[6] Giri, Sanjeev, Goyal, Yogesh Kumar , Singh, J; Analysis of Agriculture Based Industry (Liquid Milk Plant)- A shortcut approach JMASS, Vol 5, Dec-2009,pp 95-109

[7] Goel, Pradeep and Singh, J. (1995) Availability analysis of butter manufacturing system in a dairy plant, Opsearch, pp 109-116.

[8] Gupta Pawan et al. (2005), Numerical analysis of reliability and availability of serial process in butter - oil processing plant, International Journal of quality and reliability management, pp 303-306.

[9] Kumar, D. and Singh, J. (1989), Availability analysis of washing system in a paper industry, Microelectronics and Rel.29 pp 775-778.

[10] Singh, J. (1975), on multiple parallel channels with a finite source, Jnanabha Sect. A, Vol. 5, pp 1-7.

[11] Singh, J, Reliability Technology, Theory and Applications, I.K. international publishing house.

[12] Singh, J and Mahajan, P. , Reliability of utensils manufacturing plant- A case study, Opsearch, Vol. 36, No. 3, pp 260-269,1999.

[13] Goyal, Yogesh Kumar (2006), Mathematical analysis of some industrial systems, PhD thesis, Kurukshetra University, Kurukshetra.

[14] Zaidi, Zeenat and Goyal, Yogesh Kumar, Availability Analysis of Box Manufacturing Plant, Mathematical View, Vision International Journal, IMT Faridabad.