Analytical Thinking Process Of Student In Proving Mathematical Argument

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Abstract: In mathematics there are two concepts of thinking, analytical thinking and procedural thinking. Analytical thinking is needed because the objects studied in mathematics are abstract. Theorems, Definitions, and Lemmas are objects in mathematics. the arguments in this research are arguments in the form of definitions, theorems, and lemmas. The research subjects were 5th -semester students of the Mathematics Education Department. The research instrument is a test question in the form of a description. The problem consists of one problem that asks students to prove a mathematical argument that exists in set material. The results of the study show that there are two subjects use the pre-analytical thinking process which is characterized by the process of simplifying the theorem, determining the proof plan regardless of the conclusions of the theorem. One subject uses an analytical thinking process characterized by a process of simplifying theorems, determining the proof plan by considering the conclusions of the theorem.

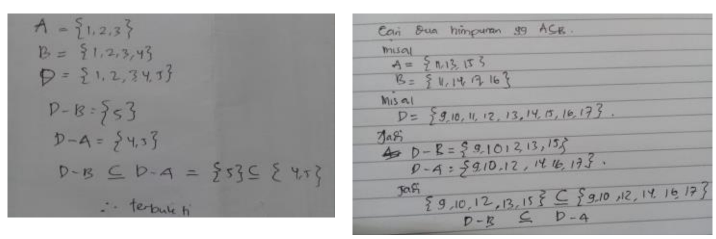
Index Terms: analytical thinking, proof, mathematical argument

1. INTRODUCTION

ONE of the significant effects of mathematics learning is the occurrence of logical and systematic thinking processes in testing the truth of an argument [1]. At the college level, the arguments in this research are arguments in the form of definitions, theorems, and lemmas [2]. Real analysis is a course that contains abstract concepts such as definitions, theorems, lemmas and corolarry which are connected with each other [3]. One way for students to understand this connection is to practice proving theorem, lemma, and corollary [4][5]. Therefore, the achievements in learning real analysis courses emphasize the ability of students to think mathematically in doing proof [6][7]. Learning activities are always related to the thinking process because the processes that occur in learning activities cannot be separated from the mental processes that occur in the brain. The process of thinking is a procedure or way that students do to respond and present information [8]. Analytical thinking is one model of mathematical thinking [8] [9]. The ability of analytical thinking is needed by students because the objects studied in mathematics are abstract [10]. [11] states that students need to be trained to be skilled in problem-solving, reasoning, and analytical thinking. In mathematics learning, there is always a process of restructuring the scheme that is owned by students. Restructuring is the process of identifying the suitability of new knowledge structures and knowledge that students already have. The ability to think analytically plays a role in the restructuring process. [12] view analytical thinking in terms of its usefulness. He says that analytical thinking is used to process the information obtained so that information can be received. In terms of mathematical thinking skills, analytical thinking is one of mathematical thinking skills that is equivalent to problem solving and mathematical evidence [13]. [14] distinguish between holistic thinking and analytical thinking. According to Nisbett analytical thinking is limited to three aspects there are can separate objects from their context, there is a tendency to focus on an attribute in categorizing an object, and using certain rules categories to explain and predict the behavior of observed objects. [10] states that analytical thinking is equivalent to critical and practical thinking where aspects of critical thinking include activities in analyzing, comparing, and evaluating. [15] states that analytical thinking is a sub-part of creative thinking. Based on the description of the experts above, it can be concluded that analytical thinking cannot be categorized as a specific thinking model. However, in the thinking process, the analytical thinking model has certain characteristics and learning experiences for students. So it needs to be studied more deeply about this analytical thinking process. The ability to prove a theorem, lemma or corollary is needed to understand mathematics. This happens because the concepts contained in mathematics are abstract. The need for the ability of analytical thinking to understand and the link between these concepts. Not only memorizing theorems but also understanding the truth of the theorem through the process of proof is important to note in understanding mathematical concepts [16]. Facts in the field show that in proving a theorem, lemma, or corollary students are indicated using procedural thinking processes. This can be seen from the process of proof being carried out. The following are excerpts of the proving process carried out by students in proving one of the theorems on the set material.

$A, B,$ and D are sets, proof that if $A \subseteq B$ then $D \backslash B \subseteq D \backslash A$

The following are some excerpts of student work,



(a) (b)

Fig. 1. *Work results of students using procedural techniques*

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Based on Figure 1 part (a) the student performs a procedural process of proving that the student takes one case example to test the truth of the theorem given. This also happens to other students as shown in figure part (b). Even if you prove by taking one case, the other cases must also be proven. Therefore such verification is invalid [17] because the results do not apply to all cases. Based on these findings, the researcher wants to examine more deeply the student's analytical thinking process in proving a theorem.

2. METHOD

This research is a qualitative study with the subject of the research are 45 5th semester students of the Mathematics Education Department at the University of Muhammadiyah Malang, Indonesia of who take real analysis courses. The research instrument is a test question in the form of a description. The problem consists of one problem that asks students to prove a mathematical argument that exists in set material. This instrument is used to extract data about students' analytical thinking processes in proving a mathematical argument. Data collection was carried out for 15 minutes before the lecture began. The research subject resolves the problems given individually. After that, some students were asked to present the results of their work classically. Then the researcher discussed in detail the solution to the problem given. The discussion was done by discussing classically and the researcher gave scaffolding. The data obtained are data from student work in the form of writing and then analyzed using descriptive techniques.

3. RESULT AND DISSCUSSION

This study aims to determine the analytical thinking process of students in proving a systematic argument. Previously researchers have examined literature which will be a research variable, namely the process of analytical thinking and mathematical argumentation. In this study, the mathematical argument in question is the theorem on set material, namely

$A, B,$ and D are sets,
proof that if $A \subseteq B$, then $D \setminus B \subseteq D \setminus A$

The test was conducted for 15 minutes at the beginning of the lecture. The test problem consists of one description question that asks students to prove one of the theorems that exist in the set material. During the test, no student is asking or asking for clarity from the question. This is an indication that the question the researcher is making is non-ambiguous. The following is presented the work results of Subject 1, Subject 2, and Subject 3.

1. Subject 1

Figure 2 shows the results of Subject 1 in solving the problem given,

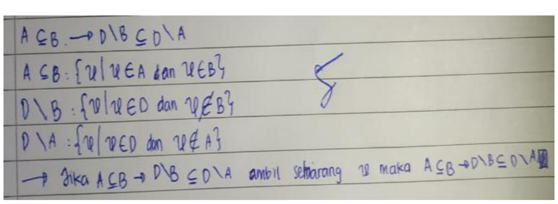

Fig. 2. *Work results of Subject 1*

Based on Figure 2, the first step taken by Subject 1 is to simplify the form of the problem given by rewriting the problem in the form of a mathematical logic symbol. This can be seen in row one, namely subject 1 writes

$$A \subseteq B \rightarrow D \setminus B \subseteq D \setminus A$$

Next is to write the premise of the theorem that will be proven, namely $A \subseteq B$ and write down the definition of the premise. The conclusion of the theorem is also written along with the definition. At the end of the proof of subject 1 write back the theorem given. Next subject 1 tries to process the premise. This can be seen from Subject 1 writing "take everything x". But again subject 1 writes the theorem that was asked to prove.Based on the evidentiary process performed, subject 1 indicated a pre-analytical thinking process. According to [18] the pre-analytical thinking process can be seen from the process of solving the problem given. If the settlement process used tends to use standard procedures even though the procedure cannot be used. Besides, the pre-analytical thinking process is marked when students only describe the surface properties of the problem given. In other words, students only interpret the problems given without regard to symbols that can be manipulated.

This can be seen in the process of proof carried out by Subject 1. Because the theorem which is proven is in the form of implications, namely

$$a \rightarrow b$$

The standard procedure performed to prove the theorem in the form of an implication is to process the premise so that it produces a conclusion that matches the theorem given [19]. In mathematics, this procedure of proof is often called direct evidence. But before doing so we must also look at the form of conclusions given. In this case, the conclusions given are shaped

$$D \setminus B \subseteq D \setminus A$$

This conclusion contains a subset ($\subseteq$). The definition of a subset according to [3] the definition of $A \subseteq B$ is if x element of A then x element of B. So the conclusion to the theorem in question is in the form of implications. So to prove the theorem given students can process the premise from the conclusion of the theorem to prove the conclusions from the conclusions of the theorem.

Subject 1 does not do this. Subject 1 performs the verification process in a procedural way, namely by determining the premise and conclusion without regard to the form of conclusions from the theorem. This can be seen from the process of proving that is done, namely subject 1 defines the premise and conclusion gradually. This makes subject 1 confused the next step. this can be seen from the answer in the last line on Figure 2 that is subject 1 rewrite the theorem that will be proven after that write "take any x". And the proof is considered complete. Based on the process of evidence conducted by subject 1, it can be concluded that subject 1 performs the process of pre-analytical thinking.

2. Subject 2

The process of proof of subject 2 is presented in Figure 3 below.

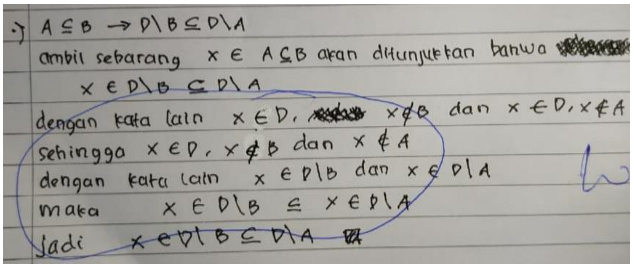

Fig. 3*. Work results of subject 2*

The first step subject 2 is the same as that done by subject 1 by simplifying the theorem into a form of mathematical logic so that it appears that the theorem takes the form of implications. Next subject 2 writes a proof plan using direct verification techniques. This can be seen from (d) in the second row, namely subject 2 takes everything x of element $A \subseteq B$ henceforth it will be shown that member x $D\backslash B \subseteq D\backslash A$. Next subject 2 focuses on the conclusion of the given theorem. This shows that subject 2 writes the definition of a subset that is in conclusion. But at this stage, the use of the subset is not by following the definition of the subset even though subject 2 focuses on conclusions in the form of the subset. This results in the next step of proof is invalid. The process of proof conducted by subject 2 indicates that subject 2 also conducts a pre-analytical thinking process. This can be seen from figure 3 the process of proving that is done by subject 2 is divided into several parts. The first part is subject 2 writes a proof plan according to the theorem to be proven. The plan of proof is seen from the second row in Figure 3. The theorem in the form of implications and plans for proof of subject 2 also follows the form of the theorem by processing the premise given to prove the conclusion. The second part is subject 2 intends to prove the theorem by paying attention to the conclusions of the theorem. This results in no continuity between part one and part two. In this second part subject 2 also does not pay attention to the subset. This can be seen from the definition of the subset used. Subject 2 defines the subset as the conjunction operation in the set. Even though the subset does not include set operations but the relationship between sets. So in this case subject 2 still does the verification process even though the definition used is not appropriate. This is what causes subject 2 also to do the process of pre-analytical thinking.

3. Subject 3

The results of the subject 3 work are presented in the picture below.

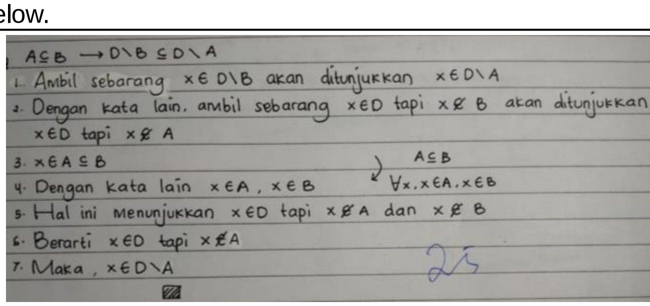

Fig. 4*. Work results of subject 3*

The first step subject 3 is to simplify the theorem given by mathematical logic. The second step of subject 3 seems to have focused on the conclusion of the theorem given. this can be seen from the steps of the two subjects 3 who planned proof by following the conclusions of the given theorem which is defining the subset correctly. Subject 3 understands the definition of the subset well. This shows that subject 3 is right in determining which premise and conclusion will be proven. Then continue the process of proof until the theorem given is proven. Based on the evidentiary process carried out, subject 3 can manipulate the verification process by taking into account the conclusions of the given theorem. Subject 3 can define the subset correctly so that subject 3 can determine the premise and conclusion that must be proven. This causes subject 3 to use the process of analytical thinking. This is by following what was stated by [20] that analytic thinking is a match between solving problems and problems given. Besides analytical thinking also means students have found the core solution to the problem given [18]. Another thing that supports subject 3 includes the thinking category of analysis is the use of appropriate algorithms, logical flow of thinking and the existence of statements that underlie each stage of the process of proof. The use of the right algorithm can be seen from the plan of proof carried out by subject 3. Although the form of the theorem to be proven is in the form of implications, subject 3 observes the conclusions of the theorem. The conclusion form of the theorem is also an implication, so subject 3 does not take the premise of the original form but takes the premise from the form of the theorem conclusion. So that the process of proof becomes simpler. The subject's logical thinking flow 3 is seen from the next step of proving. Subject 3 uses the master premise to support the subsequent verification process. This can be seen from step three, namely subject 3 uses $A \subseteq B$ to be used at the next verification stage. Seen in the results of his work sub-class 3 always gives reasons for each step of proving. Suppose that between step two and step three there is an arrow that leads from stage 2 to stage three. It explains that there is an underlying statement from step two to step three. The following is a table of differences in subjects 1,2 and 3 in terms of the process of analytical thinking.

Table 1

Table of Differences in Subjects 1,2 and 3

	Subject 1	Subject 2	Subject 3
Simplifying the problem	Simplifying the theorem becomes a form of the symbol of implication	Simplifying the theorem becomes a form of the symbol of implication	Simplifying the theorem becomes a form of the symbol of implication
Plan of proof	Just write down the premise and conclusion	The proof plan is by following the premise and conclusion of the theorem	The proof plan is adjusted to the conclusions of the theorem
Proving process	Carry out a verification process but it is not related to the premise and conclusions set	The process of proving that is done does not pay attention to the form of conclusions from the theorem so that the subject is not appropriate in processing the	Properly processing the premise so that it proves the conclusion from the conclusions of the theorem. The proving process is done with the right algorithm,

		premise. This results in invalid evidence.	logical and the existence of each step
Conclusion	Pre-analytical	Pre-analytical	Analitycal

4 CONCLUSION

Based on the results of the study, two categories of thinking processes were obtained in proving a mathematical argument, namely the process of pre-analytic thinking and analytic thinking. 2 subjects were using the pre-analytical thinking process and 1 subject used the analytical thinking process. The process of thinking pre-analytic subject 1 begins by changing the theorem to form a symbol of implication. The plan of proof is not explained by subject one. Subject 1 only describes the premise and conclusion without connecting the two and examining the conclusions.The process of thinking pre-analytical subject 2 begins with the simplification of the theorem to form a symbol of implication. Then subject 2 plans the process of proof by writing the premise and conclusion correctly. But subject 2 does not pay attention to the form of conclusion so that the process of proof is invalid. Subject 3 means the process of analytical thinking. beginning with the simplification of the form of the theorem, then planning the process of proof with prior attention to the form of conclusions. Subject 3 uses the premise from the conclusion as the premise of the proof process. The process of proof of subject 3 uses the right algorithm, the flow of logical thinking, and the explanation of each step.

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REFERENCES

[1] W. P. Thurston, "On proof and progress in mathematics," Bull. Am. Math. Soc., 1994.
[2] D. O. Tall, "The transition to formal thinking in mathematics," Math. Educ. Res. J., 2008.
[3] J. D. Depree, Introduction to Real Analysis Introduction to Real Analysis. 2012.
[4] R. Maya and U. Sumarmo, "Mathematical Understanding and Proving Abilities: Experiment With Undergraduate Student By Using Modified Moore Learning Approach," J. Math. Educ., vol. 2, no. 2, Nov. 2016.
[5] R. M. Dudley, Real analysis and probability. 2018.
[6] K. S. Perbowo and T. R. Pradipta, "Pemetaan Kemampuan Pembuktian Matematis Sebagai Prasyarat Mata Kuliah Analisis Real Mahasiswa Pendidikan Matematika," KALAMATIKA J. Pendidik. Mat., 2017.
[7] S. Syamsuri, P. Purwanto, S. Subanji, and S. Irawati, "Using APOS Theory Framework: Why Did Students Unable To Construct a Formal Proof?," Int. J. Emerg. Math. Educ., vol. 1, no. 2, p. 135, Aug. 2017.
[8] R. B. Ferri, "Mathematical thinking styles and their influence on teaching and learning mathematics," 12th Int. Congr. Math. …, 2012.
[9] S. Vinner, "The pseudo-conceptual and the pseudo-analytical thought processes in mathematics learning," Educ. Stud. Math., 1997.
[10] D. E. RUMELHART, "Psychology - Cognitive," Science (80-.)., vol. 198, no. 4319, pp. 816–817, 1977.
[11] J. K. Robbins, "Problem solving, reasoning, and analytical thinking in a classroom environment.," Behav. Anal. Today, 2014.
[12] S. Kita, "How representational gestures help speaking," in Language and Gesture, 2010.
[13] R. R. Greenwald and I. J. Quitadamo, "A mind of their own: Using inquiry-based teaching to build critical thinking skills and intellectual engagement in an undergraduate neuroanatomy course," J. Undergrad. Neurosci. Educ., 2014.
[14] R. E. Nisbett, I. Choi, K. Peng, and A. Norenzayan, "Culture and systems of thought: Holistic versus analytic cognition," Psychol. Rev., 2001.
[15] L. Sydonie, "Creative thinking," Learn. Disabil. Today, 2010.
[16] G. Hanna and M. de Villiers, "ICMI Study 19: Proof and proving in mathematics education," ZDM - Int. J. Math. Educ., 2008.
[17] L. Lamport, "How to Write a Proof," Am. Math. Mon., 2006.
[18] J. T. Kinard and A. Kozulin, Rigorous Mathematical Thinking. 2012.
[19] A. Selden, J. Selden, and A. Benkhalti, "Proof Frameworks: A Way to Get Started," PRIMUS, 2018.
[20] D. Hardman and L. Macchi, Thinking: Psychological Perspectives on Reasoning, Judgment and Decision Making. 2005.