Integrating A Graph Theory In A School Math Curriculum Of Indonesia Under Realistic Mathematics Education

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Abstract: Graph Theory is one of a branch of Discrete Mathematic which has been grown rapidly. Recently the application of graph theory can be found in all aspects of real life as well as in natural and social science study. Many real life problems can be modeled by means of a graph. Especially in designing a large network, it has become an issue of growing interest due to the recent advances of very large scale integrated technology and a social network such as the recent booming of social media service sites. Furthermore, when we teach graph theory to the university students, many graph theory problems solving can be a trigger for the rise of students higher order thinking skill which is now very important and as a key success to survive in the 21st century. However, in Indonesia, a graph theory has not been taught to school students. Why and how does it happen? How hard to introduce a graph theory in school level? By literature review method, this paper will show that there are many interesting simple models of graph theory which can be introduced in school level to foster the development of the students higher order thinking skills. The results show that it is feasible to introduce a graph theory in a school math curriculum level of Indonesia by showing some simple problem solving such as a spanning tree, layered diagram, dual graph, shortest path problem, graph coloring, rainbow connection, dominating set and other applications.

Index Terms: An elementary graph theory, school math curriculum, Realistic Mathematic Education.

1. INTRODUCTION

Discrete Mathematics is a branch of mathematics that studies a discrete model. The model is called discrete if it consists of a finite number of elements that are not continuous. The opposite of discrete is continuous so that in mathematics we recognize discrete and continuous models. The set of integers is considered as a discrete object, while the set of real numbers is considered as a continuous object. In discrete mathematics, there is a study that has the most applications in real life, namely Combinatorics and Graph Theory. This study is growing up quickly in the last decade. Even with the development of information and communication technology, and the development of other social dynamics, the use of combinatorics and graph theory are increasingly taken into account. Thus, the study of this theory is almost found in every branch of science. The Seven Bridges of Königsberg is a historically notable problem in mathematics. Its negative resolution by Leonhard Euler in 1736 laid the foundations of graph theory and prefigured the idea of topology. Now the issue has turned into a problem of Eulerian Circuits. Approximately one hundred years after the proposed of the Eulerian circuits, there was a significant progress with regard to the graph theory. Later in 1847, GR Kirchhoff (1824 - 1887) succeeded in developing a tree theory which was used in the electrical network problems. Ten years later, A. Cayley (1821 - 1895) also used the concept of a tree to explain the chemical problems, namely hydrocarbons. In the Kirchoff and Cayley era, it was also been born two important things in graph theory. One of them with regards to the four-color conjecture. It states that given any separation of a plane into contiguous regions, producing a figure called a map, no more than four colors are required to color the regions of the map so that no two adjacent regions have the same color. Another important point to be discussed in connection with the development of graph theory is proposed by Sir WR Hamilton (1805-1865). In 1859, he developed a game that was then sold to a toy factory in Dublin. The game is made of regular dodecahedron formed by a polyhedron of 12 faces and 20 corners. Each face is formed by the regular pentagon and each corner is formed by three different sides. Each corner of the dodecahedron is associated with well-known cities such as London, New York, Paris, and others. The problem in this game is "we are asked to find a route through the sides of the dodecahedron so that each city of 20 cities can be traversed exactly once". This problem nowadays is considered as a Hamiltonian Path / Cycle problem.

Approximately a half of century after Hamilton problem, the activity in the field of graph theory were relatively few. In the 1920s, these activities reappeared which was pioneered by D. Konig. Konig. He attempted to collect the results of mathematicians taught on graph theory, including the results of his own ideas into a book published in 1936. The book is considered to be the first book on graph theory. Since then, and the last thirty years the development graph theory is very rapidly growing up. The graph is studied in both aspects either in the perspective science for science or science for application. Many studies have been carried out by a group of researchers, thousands and even millions of articles have been published, and a wide of various books of graph theory can be now easily found from the highly theoretical to the practical one. However, it is a quite ironical as the graph theory in Indonesia is not introduced in the mathematics subject from primary school up to high school. In fact, we have known that the learning of graph theory has given a growth the rise of analytical skill, critical and creative students skills which is required to survive in the 21st century [3]. We also know that the uniqueness in the study of combinatorics and graph theory is the development of a theorem, lemma, corollary and conjecture, and the proof of them and also the simulation of computer programming. They obviously contribute to the development of 4C skill, namely Creativity Skill, Critical Thinking Skill, Communications skill and Collaboration skill the 21st Century Skills. For more basic concept of higher order

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thinking skills, see Krathwol [15]. Actually, since the curriculum 1994 was launched in Indonesia, an introduction topic in graph theory was included in the syllabus of school mathematics, but unfortunately, when the curriculum has been changed several times until the latest curriculum 2013, the graph theory is not included and introduced in school mathematics anymore. School mathematics, especially in secondary school only covers the followings: (1) Exponents and logarithms, (2) Linear and nonlinear equations, (3) The system of linear equation and nonlinear equations, (4) matrix, (5) Relations and Functions, (6) Sequence and series, (7) Quadratic equations and functions, (8) Trigonometry, (9) Geometry, (10) Limit Functions, (11) Statistics and, (12) Probability [4]. Combinatorics and graph theory as part of Discrete Mathematics is only introduced in term of permutations, and combinations [4]. On the contrary, lately with the discovery of a supercomputer that can process terabytes data, the use of statistics tool is getting less useful. Scientists or researchers prefer to analyze the population data instead of analyzing the sampling data. It is due to the accuracy level. If the data analysis is obtained by the entire population then the inferential statistical is no longer needed.

2 SURVEY OF THE INTEGRATION OF GRAPH THEORY IN SCHOOL MATH

Since NCTM released a report on the changes of math curriculum which recommended that discrete mathematics should be taught in schools from grades 7-12, many countries start the integration of discrete mathematics in school mathematics curriculum. The following are some countries overview which describes some countries which already taken discrete mathematics into their school math. Thailand. It has already incorporated the Discrete Mathematics: Combinatorics and Graph Theory in their math syllabus in the secondary school. Singapore. Khioh Quek Seng, et al [21], in their paper, entitled Teaching of Discrete Mathematics at Advanced Level in Singapore: Teachers Perspectives explains how important to teach discrete mathematics in the schools. Singapore schools have taught Combinatorics and Graph Theory to their students of higher level 3 (H3) in addition to teaching the differential equations and geometry topics. Australia. Education authorities of the Australian Department of Education require the school students at the level of 9-12 should study discrete mathematics. The curriculum should include topics in discrete mathematics such as combinatorics and graph theory. This aims that student can solve some present problems by discrete structures such as infinite graphs, the matrix of adjacencies, neighbourhood, sequence and recursive relationship problems that arise in computer algorithms. Hungary. Hungarian curriculum has even inserted a little bit earlier the graph theory in their curriculum. They teach discrete math starting from grade 8 to grade 12. Italy. Likewise, with schools in Hungary, graph theory has been taught to students so that students are able to use it in their daily life. United State. Since the NCTM issued a guideline for the curriculum change in 1991, the integration of discrete mathematics written in a special book as a guideline for the implementation of it in their math curriculum. The book describes that some of the topics of graph theory that should be taught in school math are methods of enumeration, recursion, iteration, algorithm, and an introduction to graph theory. The school received a grant support from the National Council of Teachers of Mathematics (NCTM) to implement this concept since 2000. To realize this implementation, since 2000 NCTM has published a similar document to distribute discrete mathematics syllabus as a standard syllabus of school math curriculum. In conjunction with this effort, NCTM authority explains “As an active branch of contemporary mathematics that is widely used in business and industry, discrete mathematics should be an integral part of the school mathematics curriculum...” see [10]

3 RME AND GRAPH MODELLING

Realistic Mathematic Education (RME) is a learning theory developed by Hans Freudenthal in the Netherlands since the 1970s. RME is a learning theory comes from real things or student experiences. This theory is emphasized in process, discussion and collaboration, and debate skills so students can solve problems with their own efforts, which in turn they can use mathematics to solve their daily life problems both individually and in groups. RME starts the learning process by using real and student phenomena application, where the problem given is a contextual problem. In solving this problem, students are constructively guided by the teacher until they are understand the concepts of what they are learning, by rediscovering concepts and formulas mathematics. This rediscovery was carried out by investigative activities. This type the learning process can attract students’ attention in learning mathematics and develop their creativity about the benefits of mathematics in their life activities (like problem solving tool). If students do not study mathematics well, then mathematical material will not be useful in their daily lives. Can graph theory be taught in schools? How difficult is that graph theory? In this section, we will discuss some cases in daily life that lead to the utilization of graph theory. Unintentionally, school students in Indonesia have been affected by graph theory from an early age, i.e., when they are introduced in the food chain, these cycles of biological reactions and production cycles, problems preferences, scheduling matches, the organizational structure of student, interconnecting road, even nowadays the existing social media such as friendships on Facebook also indicates the use of graph theory. However, students in Indonesia have never studied formally graph theory so that they do not realize that it is actually they have been taught a graph element. The following examples illustrate how the graph theory can play in important role in modeling our simple daily life phenomenon into a graph theory model.

Mathematical Model 1. (Favorite Fruit) In the fruit basket there are several kinds of fruits, including Apples, Mango, Banana, Rambutan, Watermelon. Five children are interviewed for their fruits preference. The obtained data show that Andi likes Mango, Banana, and Watermelon. Fina likes Bananas and Mango. Rahman likes Apple, Rambutan, and Watermelon. Shinta likes Apples and Watermelon, and Herman likes Bananas, Rambutan and Watermelon. The question is which the most favorite fruit is chosen by children? Solution. If the above data is modeled in a graph theoretical means, we will have Figure 1.
From this figure, it can be seen easily that the most favourite fruit is watermelon, which is preferably by four children namely Andi Rahman, Shinta and Herman. Figure 1 shows an example of a graph. Hartsfield and Ringel [1] state that a simple graph is formed by some dots called vertices, and some lines that connect the dots, called edges. Mathematically, a graph G is defined as the set of pairs (V,E), where V is not an empty set of vertices V = {v1,v2,..., vn} and E is the set of edges connecting a pair of vertices E = {e1,e2,..., en} in a graph, V can not be empty set, while E may be empty (Dafik, 2007). The vertex in graph can be assigned by letters of natural numbers, or by using combination of letters and numbers. Let vi and vj be two vertices of a graph, then the edge connecting vertices vi and vj is represented by the pair vivj or denoted by symbol e1 e2 e3,.., en.

Mathematical Model 3. (Intercity Connection Route Distribution)Local Transport District Authority (LTDA) intends to offer an intercity connection route in a region to bus transport company. A number of companies sign up and want every bus to visit all the cities in the area. Additionally, passengers who use this transport service must be served to go and return from one city to another. LTDA does not want some companies to operate the bus transport on the same route. How to distribute the route?

Solution. Solving directly the above problem will be a bit confusing, but if the above problem is represented in the form of layered diagram of graphs [8], see Figure 3, then the problem would be relatively simple.

The steps can be represented by a vertex, while the condition terms are described by an edge. The steps that do not require the completion of the other steps can be carried out at the same time. Note that the number of workers can’t reduce the number of days, as the number of days is measured from the amount of the minimum day required by the theoretical concept of building strength. By considering the graph above, the number of days required is 5 + 7 + 22 + ((5 + 19) - (5 + 19)) + ((15 + 5 + 7) - (15 + 5 + 7)) + (6 + 9 + 19) = 5+7+22+34 = 34+34 = 68 days. The number of days that first required is 5 + 7 + 22, the job D, E, H can be done simultaneously as the condition to finish those jobs are the same, i.e. the completion of the work C. The number of days (5 + 19) and (15 + 5 + 7) are not calculated as it has been included in the calculation of (6 + 9 + 19) days. It is easy to see that it is not hard for students in school level to understand the solution of the problem by means of graph theory.
Solution. Given that a province consists of 11 cities with the number of roads connect all cities is 22 roads, see Figure 4.

Mathematical Model 2. (House Construction Process)

To build a house requires the following steps: (A) Preparation for the architectural construction drawing takes 5 days, (B) Excavation and base construction takes 7 days provided that A is finished, (C) Frame and closure construction of main structure takes 22 days provided that B is finished, (D) Installation of water pipes takes 5 days provided that C is finished, (E) Cable installation takes 6 days provided that C is finished, (F) Installation of cooler and heater will takes 9 days provided that E is finished, (G) Insulation and wall banishing will take 19 days provided that D, F are finished, (H) Installation of boards, decorations and exterior painting takes 15 days provided that C is finished, (I) Completion and interior painting of the house takes 11 days provided that G is finished, (J) Floor ceramic process will take 8 days provided that I is finished, (K) Garden development will take 5 days provided that H is finished, (L) Fence development will take 7 days provided that K is finished. The question is how many days minimum is needed to complete the job?

Solution. Suppose a first floor of the hotel has room doors structure connecting among the doors shown in Figure 7(a), then the graph representation is needed to analyze the connectivity structure of the hotel room. If a room is represented by a vertex and an accessibility is represented by and edge then its graph model can be given in Figure 7(b).
Through the model of the graph, we can see which room is the closest or longest to the main door. From the graph, it can be seen that the closest and longest distance are respectively 1 and 4 for Graph(a) and 1, and 5 for Graph(b). In Graph(a) does not have a cycle that is quite worrying if there is a problem in the hotel, while Graph(b) has a cycle so that when there are problems with the hotel building, there is an alternative way to get out to the main door. Isn’t it easy for students?

Completion. Given that a road map is presented in Figure 8. By considering the intersection as a vertex and the road as an edge, the graph representation can be presented in Figure 9.

By this graph, further analysis can be done for which crossroads occurs a potential congestion and for which intersections occurs a very low traffic flow. It is easy to consider in-degree (number of inflows) and out-degree (number outflow) from each vertex. At the intersection 4 occurs a congestion due to the in-degree is greater than the out-degree. At the intersection 5 and 8, the traffic flow will be low due to the in-degree is less than the out-degree. Many other studies can be done, such as the connectivity, see [6,18]. Are all the crossroads reachable from any other intersections? The answer is by observing the existence of a path between a vertex to the other vertex of the graph. This example also easy to introduce to a students in school math level.
This type of problem can be easily found in daily life, such as a train scheduling, interview schedule, airport scheduling and many other scheduling can use this type of coloring theory. For the simple case such the problem above is also able to introduce to a students in school level.

Solution. These issues is considered as the issue of an optimal graph, where the understanding of the cardinality and graph characteristics are needed such as order, size, degree, girth, diameter, radius, Hamiltonian cycles, Eulerian cycle, bridges and cut vertex. By the given road map, we can develop a graph representation well-known ad J-Graph (Justified Graph) in the theory of Space Syntax introduced by Bill Hiller [14]. In this case, the road (not the intersection) is considered as a vertex and the accessibility across the road is modelled as an edge.

Mathematical Model 7. (Examinations Schedule) Police officer will analysis the city road route. Given a roadmap of the city as shown in Figure 11. The simple problem is whether all roads are reachable from every street? What is the longest or closest distance to cross from one path to another? How is the balance of road access between entrance and exit? In which ways does the potential traffic congestion occur? Where is the right position to place a police post?

Ridho et.al [19] construct J-graph of the given road map by using construction techniques line digraph [13] as shown in Figure 11, the labels at the vertex are obtained by the street names. By understanding the resulting J-graph in Figure 12, it is easy to answers the above problems. We clearly understand that all the roads are accessible since the availability of in-degree and out-degree at each vertex of the graph representation. The longest and the closest distance between any two roads can be determined by obtaining the diameter and radius of a graph. Note that the resulting graph is not regular since the road access between in and outflow are not the same. Thus the potential congestion occurs on Street 12 and Street 5. Their in-degree of the graph are respectively 9, 8 which exceed their out-degree 2, 2. Furthermore, the police post can easily be placed in a yellow, see Arigroffo [1]. It is also interesting to describe in front of school students.

Mathematical Model 8. (Distribution of National Examination Documents) The Educational District Authority will distribute national exam documents to all schools in a district. This document is absolutely confidential and it requires an independent team to watch the documents delivery process. Given that the road connection to reach each school is depicted in Figure 13. How to develop the secure delivery system but the minimal number of a team is required as a constraint.

Solution. To answer the above problem, we need a concept of a rainbow coloring. Li and Sun [16] defines that a coloring $c : E(G) \rightarrow \{1,2,...,k\}$, $k \in \mathbb{N}$. A path in an edge colored graph is said to be a rainbow path if no two edges on the path have the same color. The rainbow connection number of a graph $G$, denoted by $rc(G)$, is the smallest number of $k$ colors required to edge color the graph such that the graph is rainbow connected. Rearrange Figure 13, we have Figure 14.
The next step is to assign color on each edge so that the properties meet the rainbow coloring, see Figure 14. In this case $rc(G) = 5$, and the corresponding theorem of the upper and lower bound of $rc(G)$ is $k(G) \leq rc(G) \leq k(G) + 1$, where $k(G)$ is the diameter of $G$ [2]. It can be concluded that it takes five teams to watch the delivery process of national exam documents. Rainbow coloring is a new variance of edge coloring. The concept was first introduced by Chartrand et. al [6] to assess the occurrence of terrorist attacks to the WTC US on September 9, 2001. Elspas [11] in his book "A matter of security" made a simple observation as follows: "An unanticipated aftermath of Reviews those deadly attacks was the realization that intelligence agencies could not communicate with each other through reviews their regular channels, from radio systems to databases. The technologies utilized a prohibited shared access, meaning that there was no way for officers and agents to cross-check information between various organization". Lately, the study of rainbow connection has become a major concern of graph researchers, and many new results obtained and all results have been well documented in the dynamic book written by Xueliang Li, Yuefang Sun [16] entitled "Rainbow Connection of Graphs - A Survey". The study now is increasing quickly such that there are many types of rainbow coloring arises such as a strong rainbow coloring, a vertex rainbow coloring and the latest one is $k$-connected rainbow coloring. All those topic can be studied in theoretical or application point of view. In the school level, the easiest one is to consider the application of rainbow connection in students daily life. It is also feasible to deliver it in school mathematics classroom. Solution. The above problem is considered to be "a shortest path problem", see [13]. Suppose we choose the path CBAFEDG of red color in Figure 15, the total distance is 113 km. Suppose we choose the path AGFEDCB of red color in Figure 16, the total distance is 85 km, however it has not been the shortest one. Now, if we choose the path BCDGAEF in Figure 17, then we obtain the shortest one, it is only 84 km. Thus, the shortest path that can be used by salesmen is BCDGAEF. This problem holds frequently in daily life. People sometimes face such problem but they do not know how to solve it effectively. By means of graph theory, this problem can be solved easily. The problems become complex when the number of areas (vertex in graph theory perspective) is getting bigger and bigger. The concept of the shortest path problem will be then useful. For the basic applications of the shortest path problem are easy, it will be very much easy to introduce it to the students in school level as well.
4 DISCUSSION

Several mathematical models above illustrate how wide the application of a discrete mathematics, especially graph theory in day life. Gross [13] stated that a simple graph formed by dots called vertices, and the lines that connect the vertices is called an edge. Mathematically, a graph G is defined as the set of pairs (V,E), where V is non-empty set of all vertices \(V = \{v_1, v_2, ..., v_n\}\) and E is the set of edges connecting a pair of vertices \(E = \{e_1, e_2, ..., e_m\}\). Understanding the above definition, it is clearly understood that the vertices and edges are not a representation of geometric shape, thus a graph is not part of geometry study. It indicates that teaching geometry in school mathematics curriculum does not cover the graph theory topics. The set of a vertex in a graph may be elements of computers, fruits, children, vehicles, trains, planes, subjects courses, company, or a set of other elements such as train track, road, runway, and place. While the relations between the set of elements and the others indicate the edges of a graph. Therefore a graph is totally different with a geometry. Graphs can be used to model many types of relations and processes in physical, biological, social and information systems, see [11]. Many practical problems can be represented by graphs. Emphasizing their application to real-world systems, the term of network where we are in this era now, is defined to mean a graph. In computer science, graphs are used to represent networks of communication, data organization, computational devices, the flow of computation, etc. Graph-theoretic methods have proven particularly useful in linguistics, since natural language often lends itself well to discrete structure. Traditionally, syntax and compositional semantics follow tree-based structures, whose expressive power lies in the principle of compositionality, modeled in a hierarchical graph. More contemporary approaches such as head-driven phrase structure grammar model the syntax of natural language using typed feature structures, which are directed acyclic graphs. Graph theory is also used to study molecules in chemistry and physics see [20]. In condensed matter physics, the three-dimensional structure of complicated simulated atomic structures can be studied quantitatively by gathering statistics on graph-theoretic properties related to the topology of the atoms. In chemistry, a graph makes a natural model for a molecule, where vertices represent atoms and edges bonds. This approach is especially used in computer processing of molecular structures, ranging from chemical editors to database searching. Similarly, in computational neuroscience graphs can be used to represent functional connections between brain areas that interact to give rise to various cognitive processes, where the vertices represent different areas of the brain and the edges represent the connections between those areas. Graphs are also used to represent the micro-scale channels of porous media, in which the vertices represent the pores and the edges represent the smaller channels connecting the pores. Likewise, graph theory is useful in biology and conservation efforts where a vertex can represent regions where certain species exist (or inhabit) and the edges represent migration paths or movement between the regions see [7]. This information is important when looking at breeding patterns or tracking the spread of disease, parasites or how changes to the movement can affect other species. Considering those facts, the study of graph theory in school mathematics especially in secondary schools from grades 7-12 is very important. It concludes that we need to reconstruct our math curriculum as the problem in daily life is changing rapidly and the application of graph can’t be avoidable, and graph theory is needed anywhere, any place and any time.

5 CONCLUSION

In this paper we have shown how important graph theory nowadays in real life. Its application can be found in any aspect of real life. However, school math curriculum in Indonesian does not consider the involvement of discrete
mathematics especially combinatorics and graph theory in school mathematic topics. In fact, since the NCTM recommended the integration of discrete mathematics in United State math curriculum, there are a lot of countries have taught graph theory in their school mathematic class. Furthermore, it is also understood that almost all higher education subject course has taken graph theory study into account. Therefore, researchers propose the following suggestions for further research studies:

1. What are the obstacles and challenges of integrating the graph theory in school mathematic curriculum of Indonesia?
2. How is the readiness of teacher professional competence in teaching graph theory at the school level of Indonesia?
3. In which part can we integrate graph theory into school mathematic curriculum of Indonesia?

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