Weighted Estimator Of Population Mean Under Stratified Random Sampling

Sarbjit Singh Brar, Ravinder Kumar

Abstract: In this paper, an unbiased weighted estimator of population mean is introduced in stratified random sampling which uses the information of mean square of each stratum at the estimation stage. It is shown that the proposed estimator is better than usual estimator of population mean under arbitrary and proportional allocation in stratified random sampling. Further, under certain conditions, it is proved that the proposed estimator under proportional allocation is better than usual estimator under Neyman's allocation (Optimum allocation) and both are equally efficient if each of the stratum has equal coefficient of variation. A simulation study is carried out to verify the proposed results.

Index Terms: Stratified Random Sampling, Proportional Allocation, Optimum Allocation, Weighted Estimator.

1. INTRODUCTION

THE main focus of sampling is to extract maximum information from the sample with minimum cost. Stratified random sampling is one such procedure which is widely used to take sample from a population. In stratified random sampling, the whole population, consists N units, is divided into L strata. These strata of sizes $N_1, N_2, N_3 \ldots\ldots N_L$ respectively, such that $= \sum_{h=1}^{L} N_h$. Then a simple random sample of size n_h; $h = 1,2,3 \ldots L$ is drawn from each of the stratum independently which constitutes a sample of size $n = \sum_{h=1}^{L} n_h$. The weighted mean of strata sample means is an unbiased estimator of population mean. The precision of the estimator of population mean in stratified random sampling depends upon the method of allocation of total sample size n for each stratum which can be fixed by surveyor. The allocation of sample size among different stratum should be made in such a manner as to estimate the population mean with desired precision for minimum cost or with maximum precision with given cost. Different approaches of allocation of sample size n among different strata are available. Bowley [1] proposed Proportional Allocation in which the total sample size n is allocated among various strata such that the sample fraction

$$\frac{n_1}{N_1} = \frac{n_2}{N_2} = \ldots. = \frac{n_k}{N_k} = \frac{n}{N}.$$

In other words, it implies that a small sample is selected from a small stratum and a large sample from large stratum. Another allocation of the total sample size n to different stratum is called minimum variance allocation or Neyman allocation and is due to [2]. This result was first discovered by [5] but remained unknown. This allocation takes into account both the stratum size and stratum variance and also assumes that the sampling cost per unit among different strata is same. For more details see [3].

1.1 Notations:-

Let y be the study variable, then following notations are used through out the paper.

y_{hi}, be the Value of i^{th} unit in h^{th} stratum where $h = 1,2,\ldots,L$ and $i = 1,2,3\ldots,N_h$.

$\bar{Y}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} y_{hi}$, be the population mean of h^{th} stratum for study variable y.

$\bar{Y} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} y_{hi} = \frac{1}{N}\sum_{h=1}^{L} N_h.\bar{Y}_h = \sum_{h=1}^{L} W_h.\bar{Y}_h$,

be population mean of y, where $W_h = \frac{N_h}{N}$.

$S_h^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}(y_{hi} - \bar{Y}_h)^2$, be the mean square of population in h^{th} stratum.

$C_{vh} = \frac{S_h}{\bar{Y}_h}$, be the coefficient of variation in the h^{th} stratum.

2 EXISTING ESTIMATOR AND VARIANCES

The usual unbiased estimator of population mean in stratified random sampling is defined as

$$\bar{y}_{st} = \sum_{h=1}^{L} W_h \bar{y}_h;$$

and expression of variance of $\bar{y}_{st}$ under stratified random sampling is given below

$$V(\bar{y}_{st}) = \sum_{h=1}^{L}\left(\frac{1}{n_h} - \frac{1}{N_h}\right) W_h^2 S_h^2. \quad (1)$$

Under proportional allocation, n_h is proportional to N_h and under this allocation, expression for variance of $\bar{y}_{st}$ for a given sample size can be written as

$$V_{prop}(\bar{y}_{st}) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h S_h^2. \quad (2)$$

Under optimum allocation, n_h is proportional to $S_h W_h$ and expression for variance of $\bar{y}_{st}$ for given sample size can be written as

$$V_{opt}(\bar{y}_{st}) = \frac{1}{n}\left(\sum_{h=1}^{L} W_h S_h\right)^2 - \frac{1}{N}\left(\sum_{h=1}^{L} W_h S_h^2\right). \quad (3)$$

Sometimes, cost of the survey is given in advance and cost function may be defined as below

$C = \sum_{h=1}^{L} c_h.n_h$, where C is the total cost of the survey, (4)

c_h be the cost per unit in the h^{th} stratum.

Now, the expression of variance of y_{st} under proportional allocation for fixed cost can be defined as below

$$V_{prop(c)}(\bar{y}_{st}) = \sum_{h=1}^{L}\left(\frac{\sum_{h=1}^{L} W_h c_h}{W_h C} - \frac{1}{N_h}\right) W_h^2.S_h^2 \quad (5)$$

and expression of variance under optimum allocation for fixed cost can be written as

$$V_{opt(c)}(\bar{y}_{st}) = \left(\sum_{h=1}^{L}\sqrt{\frac{c_h}{C}} W_h.S_h\right)^2 - \frac{1}{N}\sum_{h=1}^{L} W_h.S_h^2. \quad (6)$$

- *Sarbjit Singh Brar, Department of Statistics, Punjabi University, INDIA. E-mail: sarbjitbrar@pbi.ac.in*
- *Ravinder kumar, Department of Statistics, Punjabi University, INDIA. E-mail: 8ravinder@gmail.com**
  (corresponding author)*

Although, Neyman allocation is most efficient but it has some limitations. There are cases where Neyman allocation deviates from optimization. In all such cases, this deviation increases the variance of the estimator of population mean.

1. In case of multiple variables, Neyman allocation in different stratum is performed on the basis of single variable which leads to other variables deviated from Neyman allocation/optimum allocation.
2. Neyman allocation requires the knowledge of stratum standard deviations i.e. $S_i's$ which usually are unknown. To overcome this problem, [4] proposed the idea of preliminary sample to estimate $S_i's$. However, this approach increases the variance of the estimator of population mean as well as cost of the survey as compared to proportional allocation.
3. In case of Neyman allocation, the optimum values of sample sizes are obtained. But these sample sizes can assume only integral values. Sometimes, Neyman formula gives non-integral values, so in such cases, the investigator uses generally nearest integer values as sample sizes. This approach again deviates Neyman allocation. However, this problem can be tackled with optimal sample allocation discussed in a series of papers by [6], [7], [8].
4. There are also some cases when Neyman formula gives larger sample size n_h than corresponding stratum size N_h which is impossible in practical situations.

3 PROPOSED ESTIMATOR

Let us consider an unbiased estimator of $\bar{Y}$

$$\bar{y}'_{st} = \sum_{h=1}^{L} k_h \bar{y}_h; \quad (7)$$

where k_h is a pre-assigned constant, such that

$$E(\bar{y}'_{st}) = \bar{Y}$$

$$\text{i.e.} \quad \sum_{h=1}^{L} k_h \bar{Y}_h = \bar{Y}. \quad (8)$$

Now, we shall find the variance of the proposed estimator,

$$V(\bar{y}'_{st}) = V\left(\sum_{h=1}^{L} k_h \bar{y}_h\right)$$

$$= \sum_{h=1}^{L} k_h^2 V(\bar{y}_h) + \sum_{h=1}^{L} \sum_{l(\neq h)=1}^{L} k_h k_l Cov(\bar{y}_h, \bar{y}_l).$$

Since samples are taken independently from each stratum,

$$Cov(\bar{y}_h, \bar{y}_l) = 0, \quad h \neq l.$$

$$\text{Therefore,} \quad V(\bar{y}'_{st}) = \sum_{h=1}^{L} k_h^2 V(\bar{y}_h). \quad (9)$$

We also have,

$$V(\bar{y}_h) = \left(\frac{1}{n_h} - \frac{1}{N_h}\right) S_h^2, \quad (10)$$

from (9) and (10), we have

$$V(\bar{y}'_{st}) = \sum_{h=1}^{L} k_h^2 \left(\frac{1}{n_h} - \frac{1}{N_h}\right) S_h^2. \quad (11)$$

Now, we shall find the value of k_h which minimize the variance of proposed estimator w.r.t condition (8). The Langrange's method will be employed for this purpose.

Let us consider a function,

$$\phi = \sum_{h=1}^{L} k_h^2 \left(\frac{1}{n_h} - \frac{1}{N_h}\right) S_h^2 + \lambda\left(\bar{Y} - \sum_{h=1}^{L} k_h \bar{Y}_h\right),$$

where λ is a Langrange's multiplier.

After solving following equations,

$$\frac{\partial \phi}{\partial k_h} = 0, \quad h = 1,2,\ldots,L,$$

we get,

$$2\left(\frac{1}{n_h} - \frac{1}{N_h}\right) k_h S_h^2 + \lambda(-\bar{Y}_h) = 0, \quad h = 1,2,\ldots,L$$

$$\text{or} \quad k_h = \frac{\lambda \bar{Y}_h}{2\left(\frac{1}{n_h} - \frac{1}{N_h}\right) S_h^2}; \quad h = 1,2,\ldots,L. \quad (12)$$

Using (8) and (12), we have

$$\lambda = \frac{\bar{Y}}{\sum_{h=1}^{L} \frac{\bar{Y}_h^2}{2\left(\frac{1}{n_h} - \frac{1}{N_h}\right) S_h^2}}.$$

Now, put this value of λ in (12) we get,

$$k_h = \frac{\bar{Y}_h}{\left(\frac{1}{n_h} - \frac{1}{N_h}\right) S_h^2} \cdot \frac{\bar{Y}}{\sum_{h=1}^{L} \frac{\bar{Y}_h^2}{\left(\frac{1}{n_h} - \frac{1}{N_h}\right) S_h^2}} = k_h^*(say), \quad h = 1,2,\ldots,L, \quad (13)$$

which is required optimum value of k_h^*.

Therefore, proposed estimator and its variance can be written as

$$\bar{y}'_{st}(opt) = \sum_{h=1}^{L} k_h^* \bar{y}_h,$$

$$\text{or} \quad \bar{y}'_{st}(opt) = \frac{\bar{Y}}{\sum_{h=1}^{L} \frac{\bar{Y}_h^2}{\left(\frac{1}{n_h} - \frac{1}{N_h}\right) S_h^2}} \sum_{h=1}^{L} \frac{\bar{Y}_h \cdot \bar{y}_h}{\left(\frac{1}{n_h} - \frac{1}{N_h}\right) S_h^2}.$$

After using optimum value of k_h in (11), we get

$$V_{min}(\bar{y}'_{st}) = \frac{\bar{Y}^2}{\sum_{h=1}^{L} \frac{\bar{Y}_h^2}{\left(\frac{1}{n_h} - \frac{1}{N_h}\right) S_h^2}}. \quad (14)$$

Note: Above value of k_h^* depends upon unknown population parameters, so estimated value of k_h^* may be used in practical situations which can be obtained by replacing these unknown parameters with their respective estimators, the modified proposed estimator can be written as

$$\bar{y}'_{st(mod)} = \sum_{h=1}^{L} \hat{k}_h^* \bar{y}_h.$$

$$\text{where,} \quad \hat{k}_h^* = \frac{\bar{y}_{st}}{\sum_{h=1}^{L} \frac{\bar{y}_h^2 - \left(\frac{1}{n_h} - \frac{1}{N_h}\right) s_h^2}{\left(\frac{1}{n_h} - \frac{1}{N_h}\right) s_h^2}} \frac{\bar{y}_h}{\left(\frac{1}{n_h} - \frac{1}{N_h}\right) s_h^2}$$

4 EFFICIENCY COMPARISONS

We shall now compare the efficiency of proposed estimator with usual unbiased estimator $\bar{y}_{st}$ of population mean under stratified random sampling.

Consider, using (1) and (14),

$$\frac{V(\bar{y}_{st})}{V_{min}(\bar{y}'_{st})} = \frac{\sum_{h=1}^{L} \left(\frac{1}{n_h} - \frac{1}{N_h}\right) W_h^2 S_h^2 \quad \sum_{h=1}^{L} \frac{\bar{Y}_h^2}{\left(\frac{1}{n_h} - \frac{1}{N_h}\right) S_h^2}}{\bar{Y}^2},$$

using Cauchy-Schwarz's inequality,

$$\frac{V(\bar{y}_{st})}{V(\bar{y}'_{st})} \geq \frac{\left(\sum_{h=1}^{L} \left(\frac{1}{n_h} - \frac{1}{N_h}\right)^{1/2} W_h S_h \quad \frac{\bar{Y}_h}{\left(\frac{1}{n_h} - \frac{1}{N_h}\right)^{1/2} S_h}\right)^2}{\bar{Y}^2} = 1,$$

$$\Rightarrow \quad V(\bar{y}_{st}) \geq V(\bar{y}'_{st}).$$

Hence, under an arbitrary allocation, the proposed estimator is

more efficient than $\bar{y}_{st}$.
Now, we shall compare the efficiency of proposed estimator with that of $\bar{y}_{st}$ in stratified random sampling under proportional allocation.
We know, under proportional allocation

$$n_h = \frac{n}{N} N_h = nW_h,$$

therefore, variance of proposed estimator under proportional allocation can be written as

$$V_{prop}(\overline{y'}_{st}) = \frac{\bar{Y}^2}{\sum_{h=1}^{L} \frac{\bar{Y}_h^2}{\left(\frac{1}{nW_h} - \frac{1}{NW_h}\right) S_h^2}}$$

$$V_{prop}(\overline{y'}_{st}) = \left(\frac{1}{n} - \frac{1}{N}\right) \frac{\bar{Y}^2}{\sum_{h=1}^{L} \frac{\bar{Y}_h^2 W_h}{S_h^2}}; \qquad (15)$$

using (2) and (15),

$$\frac{V_{prop}(\bar{y}_{st})}{V_{prop}(\overline{y'}_{st})} = \frac{\sum_{h=1}^{L} W_h S_h^2 \sum_{h=1}^{L} \frac{\bar{Y}_h^2 W_h}{S_h^2}}{\bar{Y}^2},$$

using Cauchy-Schwarz's inequality,

$$\frac{V_{prop}(\bar{y}_{st})}{V_{prop}(\overline{y'}_{st})} \geq \frac{\left(\sum_{h=1}^{L} W_h^{1/2} S_h \frac{\bar{Y}_h W_h^{1/2}}{S_h}\right)^2}{\bar{Y}^2} = 1.$$

From the above comparison, we have

$$V_{prop}(\bar{y}_{st}) \geq V_{prop}(\overline{y'}_{st}).$$

Therefore, proposed estimator is better than $\bar{y}_{st}$ under proportional allocation.
Now, we shall compare the efficiency of $\overline{y'}_{st}$ under proportional allocation with that of $\bar{y}_{st}$ under Neyman's allocation (Optimum Allocation)
Consider, using (3) and (15) for large N,

$$V_{opt}(\bar{y}_{st}) - V_{prop}(\overline{y'}_{st}) = \frac{1}{n}\left(\sum_{h=1}^{L} W_h S_h\right)^2 - \frac{1}{n}\frac{\bar{Y}^2}{\sum_{h=1}^{L} \frac{\bar{Y}_h^2 W_h}{S_h^2}}$$

$$= \frac{1}{n}\frac{\left[\left(\sum_{h=1}^{L} W_h . S_h\right)^2 . \sum_{h=1}^{L} \frac{\bar{Y}_h^2 W_h}{S_h^2} - \bar{Y}^2\right]}{\sum_{h=1}^{L} \frac{\bar{Y}_h^2 W_h}{S_h^2}}$$

$$= \frac{1}{n}\frac{\left[\left(\sum_{h=1}^{L} W_h . \bar{Y}_h C_{vh}\right)^2 . \sum_{h=1}^{L} \frac{W_h}{C_{vh}^2} - \bar{Y}^2\right]}{\sum_{h=1}^{L} \frac{W_h}{C_{vh}^2}}. \qquad (16)$$

Now,

$$V_{opt}(\bar{y}_{st}) - V_{prop}(\overline{y'}_{st}) > 0,$$

$$\text{iff} \quad \left[\left(\sum_{h=1}^{L} W_h \bar{Y}_h . C_{vh}\right)^2 . \sum_{h=1}^{L} \frac{W_h}{C_{vh}^2} - \bar{Y}^2\right] > 0,$$

$$\text{iff} \quad \sum_{h=1}^{L} \frac{W_h}{C_{vh}^2} > \frac{\bar{Y}^2}{\left(\sum_{h=1}^{L} W_h \bar{Y}_h C_{vh}\right)^2}. \qquad (17)$$

Hence, the performance of proposed estimator under proportional allocation is better than the performance of usual estimator under Neyman's allocation provided condition (17) satisfied.
Further, if $C_{vh} = C_v \quad \forall h$ then from (16), we have

$$V_{opt}(\bar{y}_{st}) = V_{prop}(\overline{y'}_{st}).$$

That is, in such cases performance of proposed estimator under proportional allocation is same as the performance of usual estimator under Neyman's allocation.

5 VARIANCE UNDER PROPORTIONAL ALLOCATION FOR FIXED COST

In this section, variance of proposed estimator under proportional allocation for fixed cost will be derived and compared with usual estimator of population mean under proportional and Optimum allocation for fixed cost.
Under proportional allocation, using $n_h = \mu N_h$ and (4), we have constant of proportionality as below

$$\mu = \frac{C}{\sum_{h=1}^{L} c_h N_h}.$$

And sample size for h^{th} stratum can be written as

$$n_h = \frac{W_h . C}{\sum_{h=1}^{L} c_h . W_h}, \qquad (18)$$

therefore, variance of proposed estimator for this sample size under proportional allocation can be written as

$$V_{prop(c)}(\overline{y'}_{st}) = \frac{\bar{Y}^2}{\sum_{h=1}^{L} \frac{\bar{Y}_h^2}{\left(\frac{\sum_{h=1}^{L} W_h c_h}{CW_h} - \frac{1}{NW_h}\right) S_h^2}}$$

and for large N,

$$V_{prop(c)}(\overline{y'}_{st}) = \frac{\bar{Y}^2 \sum_{h=1}^{L} W_h c_h}{\sum_{h=1}^{L} \frac{C\bar{Y}_h^2 . W_h}{S_h^2}}. \qquad (19)$$

Using (5), variance of usual estimator $\bar{y}_{st}$ under proportional allocation for fixed cost and large N can be written as

$$V_{prop(c)}(\bar{y}_{st}) = \frac{\sum_{h=1}^{L} W_h c_h \sum_{h=1}^{L} W_h S_h^2}{C}. \qquad (20)$$

Now consider,

$$\frac{V_{prop(c)}(\bar{y}_{st})}{V_{prop(c)}(\overline{y'}_{st})} = \frac{\sum_{h=1}^{L} W_h c_h \sum_{h=1}^{L} W_h S_h^2}{C} . \frac{\sum_{h=1}^{L} \frac{C\bar{Y}_h^2 . W_h}{S_h^2}}{\bar{Y}^2 \sum_{h=1}^{L} W_h c_h},$$

using Cauchy-Schwarz's inequality,

$$\frac{V_{prop(c)}(\bar{y}_{st})}{V_{prop(c)}(\overline{y'}_{st})} \geqq \frac{\left(\sum_{h=1}^{L} \sqrt{W_h}\sqrt{C}\bar{Y}_h . \sqrt{(W_h)}\right)^2}{C\bar{Y}^2} = 1,$$

from above, clearly,

$$V_{prop(c)}(\bar{y}_{st}) \geqq V_{prop(c)}(\overline{y'}_{st}).$$

Therefore, the proposed estimator is better than $\bar{y}_{st}$ under proportional allocation for fixed cost.
Now, we shall compare the efficiency of proposed estimator under proportional allocation with that of usual estimator of population mean under Neyman's allocation.
For large N, Using (6) and (19) consider,

$$V_{opt(c)}(\bar{y}_{st}) - V_{prop(c)}(\overline{y'}_{st}) = \left(\sum_{h=1}^{L} \sqrt{\frac{c_h}{C}} W_h . S_h\right)^2 - \frac{\bar{Y}^2 \sum_{h=1}^{L} W_h c_h}{\sum_{h=1}^{L} \frac{C\bar{Y}_h^2 . W_h}{S_h^2}}$$

$$= \frac{\left(\sum_{h=1}^{L} \sqrt{\frac{c_h}{C}} W_h . S_h\right)^2 . \sum_{h=1}^{L} \frac{C\bar{Y}_h^2 . W_h}{S_h^2} - \bar{Y}^2 \sum_{h=1}^{L} W_h . c_h}{\sum_{h=1}^{L} \frac{C\bar{Y}_h^2 . W_h}{S_h^2}}. \qquad (21)$$

Now,

$$V_{opt(c)}(\bar{y}_{st}) - V_{prop(c)}(\overline{y'}_{st}) > 0,$$

$$\text{iff}\quad \left(\sum_{h=1}^{L}\sqrt{c_h}.W_h.S_h\right)^2.\sum_{h=1}^{L}\frac{\bar{Y}_h^2.W_h}{S_h^2} \geq \bar{Y}^2\sum_{h=1}^{L}W_h c_h.$$

Again using Cauchy-Schwarz's inequality on the left side of above inequality,

$$\text{iff}\quad \sum_{h=1}^{L} c_h W_h.\sum_{h=1}^{L} W_h.S_h^2.\sum_{h=1}^{L}\frac{\bar{Y}_h^2.W_h}{S_h^2} \geq \bar{Y}^2\sum_{h=1}^{L} W_h c_h,$$

$$\text{or}\quad \sum_{h=1}^{L}\frac{\bar{Y}_h^2.W_h}{S_h^2} > \frac{\bar{Y}^2}{\sum_{h=1}^{L}W_h S_h^2}. \qquad (22)$$

Hence, for fixed cost, the performance of proposed estimator under proportional allocation is better than performance of usual estimator under Neyman's Allocation provided condition (22) is satisfied.

6 SIMULATION STUDY

To judge the performance of the proposed estimator with that of usual estimator of population mean, a simulation study is conducted by using R software. An artificial population is generated as given below:-

TABLE 1

STATISTICS OF GENERATED POPULATION

	Stratum I	Stratum II	Stratum III	Overall
Size	400	800	1000	2200
Mean	110.84	55.71	30.72	54.08
S.D.	48.45	29.29	9.84	Var- 1630.79

The above artificial population has been generated in such a manner that the stratum size increases but the variance decreases. We have selected 100000 samples under each allocation procedure from above generated population and find out the variances of usual and proposed estimators of population mean.

TABLE 2

VALUES OF VARIANCES FOR VARIOUS ESTIMATORS

Sample Size	SRSWOR	Prop. Allocation	Opt. Allocation	No Allocation
	$V(\bar{y})$	$V(\bar{y}_{st})$ $V(\bar{y}'_{st})$	$V(\bar{y}_{st})$	$V(\bar{y}_{st})$ $V(\bar{y}'_{st})$
20	20	(4 7 9)	(7 9 4)	(7 7 6)
	79.37	38.02 22.98	28.37	30.50 24.69
30	30	(5 11 14)	(11 13 6)	(10 10 10)
	53.66	27.46 14.90	18.57	20.37 15.84
50	50	(9 18 23)	(18 229)	(17 17 16)
	32.21	15.06 8.73	11.16	11.81 9.40

From the above table, which shows that for sample size 20, the proposed estimator under proportional allocation has minimum variance as compared to the variance of usual estimator under various allocation schemes. Similarly, for sample size 30 and 50, the proposed estimator is efficient than usual estimator of population mean

TABLE 3

PERCENTAGE RELATIVE EFFICIENCY OF VARIOUS ESTIMATORS TO SRSWOR

Sample Size	SRSWOR	Prop. Allocation		Opt. Allocation	No Allocation	
	$\bar{y}_{srswor}$	$\bar{y}_{st}$	$\bar{y}'_{st}$	$\bar{y}_{st}$	$\bar{y}_{st}$	$\bar{y}'_{st}$
20	100	209	345	280	260	321
30	100	195	360	289	263	339
50	100	214	369	289	273	343

Above table shows that the largest gain in efficiency is due to the proposed estimator under proportional allocation over the usual estimator $\bar{y}_{srswor}$ and $\bar{y}_{st}$ under proportional and optimum allocation respectively.

7 CONCLUSION

The Proposed estimator is more beneficial than the usual estimator of population mean under stratified random sampling. The proposed estimator also overcomes the limitations of Neyman's allocation such as, multiple variables allocation, more than hundred percent sampling and allocation required non integral values of sample sizes. Simulation study also verified the theoretical results and this study has shown that the proposed estimator is more robust than the usual estimator because the variance of proposed estimator is less affected by the allocation schemes.

REFERENCES

[1] A. Bowley, "Measurement of the precision attained in sampling" Bull. Int. Stat. Inst. Amsterdam, 22:1–62, 1926.

[2] J. Neyman, "On the two different aspects of the representative method: the method of stratified sampling and the method of purposive selection" Journal of the Royal Statistical Society, 97(4):558–625, 1934.

[3] P. V. Sukhatme, B. Sukhatme, S. Sukhatme, and C. Asok. Sampling theory of surveys with applications 3d ed, Iowa state University Press, U.S.A. 1984.

[4] P. Sukhatme, "Contribution to the theory of the representative method" Supplement to the Journal of the Royal Statistical Society, 2(2):253–268, 1935.

[5] A. Tshuprow, "On the mathematical expectation of the moments of frequency distributions in the case of correlated observations" Metron, 2(4), 646-683, 1923.

[6] T. Wright, " The equivalence of neyman optimum allocation for sampling and equal proportions for apportioning the us house of representatives" The American Statistician, 66(4):217–224, 2012.

[7] T. Wright, "A simple method of exact optimal sample allocation under stratification with any mixed constraint patterns" Research Report series (Statistics 2014-17), Center for Statistical Research and Methodology, U.S. Bureau of the Census, Washington, D.C., 2014.

[8] T. Wright, "Exact optimal sample allocation: More efficient than neyman" Statistics & Probability Letters, 129:50–57, 2017.