Mathematical Model For Three-Dimensional Flow In Porous Media For Confined Aquifer In Mixing Zone Modeling To Study Saltwater Intrusion In Costal Aquifer

Cordy Jourvel Itoua-Tsele, Christian Tathy, David Mumo Malonza

Abstract: This paper focuses on the development of a mathematical model for saltwater intrusion modeling in a coastal aquifer and. At first, we used the mass conservation and Darcy's law to derive the governing equation in terms of hydraulic head. We chose mixing zone approach or variable-density and dispersion approach; the latter consists of coupling the flow and transport equations using a state equation that relates the solute density to its concentration with the following assumptions: • The temporal variations of the density are very important compared to its spatial variations; • neglecting the density variation in the solute transport equation. Finally, we proposed a mathematical model to study the behavior of the saltwater intrusion in heterogeneous anisotropic aquifer in unsteady state and it is also use for the optimal use of groundwater of an aquifer.

Index Terms: Confined aquifer, Anisotropic, Saltwater intrusion, Mixing zone modeling, Darcy's law, Mass conservation, Costal aquifer.

1. INTRODUCTION

The salinity of aquifer waters has been a well-known phenomenon for several decades. This is a natural phenomenon due to the hydraulic link between groundwater and seawater (the heaviest fluid is normally below the lighter) and is defined as the penetration of salt water into inland freshwater aquifers by pushing the saltwater/freshwater interface landward and/or upward. The density difference between freshwater and saltwater causes this inland flow from high-density seawater to low density freshwater. The intrusion of salt water into coastal aquifers takes the form of a bevel that dives under the freshwater aquifer. Hence his nickname salted bevel. Saltwater intrusion is a contamination source of freshwater resources when concentrations of dissolved solids exceed drinking and/or irrigation water standards [6], [7]. The intensity of the intrusion depends on several factors, such as the nature of the groundwater reservoir. It can be amplified by taking underground water that is to say by pumping operations and by modifying the water level, for example linked to climate change. An aquifer is a geological formation which (i) contains water and (ii) permits significant amounts of water to move through it under ordinary field conditions [9]. There are two types of aquifers: Confined aquifer and Unconfined aquifer. Saltwater intrusion and water-quality degradation have become two of the major constraints in groundwater management problems of coastal aquifers. Saltwater intrusion degrades water quality of production wells and consequently the wells have to be abandoned. To avoid saltwater-intrusion problems, adequate natural recharge of these aquifers should be maintained. Pumping rates and production-well locations are the two key parameters that can be optimized for adequate groundwater management [6]. In the case of salt intrusion, the numerical models are based on different concepts and can be categorized in two approaches as: sharp-interface approach and Variable-density and dispersion approach. This is a new approach model for seawater intrusion phenomenon's in confined and unconfined aquifers that has recently been developed by Choquet [10]. Sharp-diffuse interfaces approach. We propose in this paper, Variable-density and dispersion approach, which has the advantage of respecting the physics of the problem while preserving the numerical efficiency. In this paper, we develop the flow equations of fluid (water) and the transport equation of dissolved salts in case of confined aquifers in saturated porous media using the mass conservation law in an Eulerian approach and Darcy's law written in a Cartesian coordinate system (x, y, z).

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2 MASS CONSERVATION AND DARCY'S LAW

Let $\Omega \subset \mathbb{R}^3$ be an open bounded domain on a saturated porous medium occupied by a fluid (water) and V any open regular sub-domain of Ω (see Fig.1).
The porous medium is saturated for confined aquifer, the mass of fluid contained in V at time t can be written as:

$$M(t) = \int_{\mathrm{V}} \rho(x,t)\phi(x,t)dx \qquad (1)$$

Where ρ is the density of fluid (water) and ϕ the porosity of the medium.

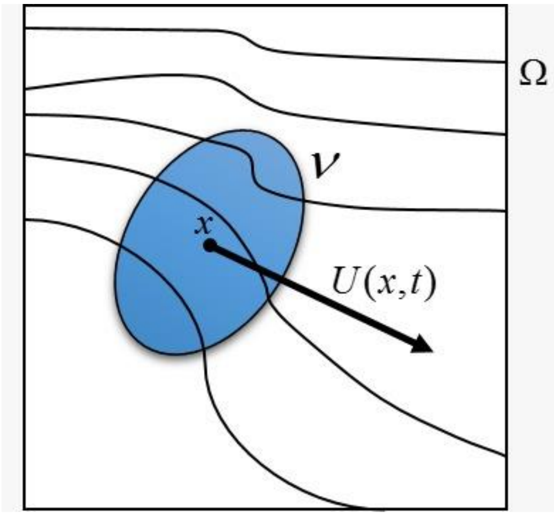

Fig. 1: Flow in domain Ω

According to the conservation mass law, we have

$$\frac{d}{dt}\left(M(t)\right)=\int_{\mathrm{V}}\frac{\partial\rho\phi}{\partial t}dx-\int_{\partial\mathrm{V}}\rho(x,t)\phi(x,t)\mathbf{u}(x,t)\mathbf{n}(x)dS+\int_{\mathrm{V}}\rho(x,t)Q(x,t)dx \quad (2)$$

Where $\mathbf{n}(x)$ is the outward unit normal to V at a point $x\in\partial\mathrm{V}, \mathbf{u}(x,t)$ is the interstitial velocity of the fluid at point x at time t, Q the mass produced (or withdrawn) per unit volume and S an oriented surface whose contour is $\partial\mathrm{V}$.

The Stokes formula

$$\int_{\partial\mathrm{V}}\mathbf{v}.\mathbf{n}dS=\int_{\partial\mathrm{V}}\nabla.\mathbf{v}dx, \qquad \forall\mathbf{v}\in\left(H^1(\mathrm{V})\right)^3 \quad (3)$$

applied to the right hand side of the equation (2) with $\mathbf{v}=\rho(.,t)\phi(.,t)\mathbf{u}(.,t)$ gives after permutation of the derivation with respect to time and the sum on V on the left hand side:

$$\int_{\mathrm{V}}\left(\frac{\partial\rho\phi}{\partial t}+div(\rho\phi\mathbf{u})-\rho Q\right)dx=0 \quad (4)$$

This equation is true for every neighborhood V of any point x and at time $t\in]0,T[$, we therefore obtain a locale expression of mass conservation:

$$\frac{\partial\rho\phi}{\partial t}+\nabla.(\rho\phi\mathbf{u})=\rho Q \quad (5)$$

The interstitial velocity $\mathbf{u}$ in a porous medium being related to the flow density $\mathbf{q}$ (also called Darcy velocity) by the relation

$$\mathbf{q}=\phi\mathbf{u} \quad (6)$$

Hence, the mass conservation equation in term of flux density in general can be written as:

$$\frac{\partial\rho\phi}{\partial t}+\nabla.(\rho\mathbf{q})=\rho Q \quad (7)$$

Now, the flux density or specific discharge $\mathbf{q}$ is a fictitious macroscopic rate of a flux of water passing through a unit area. Discovered experimentally for a homogeneous isotropic porous medium by Darcy in 1856, Darcy's law is an equation that describes the flow of a fluid through a porous medium [5]. It is used to define the relation between specific flow $\mathbf{q}$ and hydraulic head h which can be measured. But it has been generalized to saturated and unsaturated flows in heterogeneous and anisotropic media.

$$\mathbf{q}=-\frac{k}{\mu}\left[\nabla p+\rho g.\nabla z\right] \quad (8)$$

Where k is the intrinsic permeability tensor (m^2), a property of the porous medium; μ is dynamic viscosity $(kg/m/s)$ of the groundwater; p is fluid pressure $(kg/m/s^2)$, ρ is fluid density (kg/m^3) and $\mathbf{g}$ is the gravitational acceleration (m/s^2).

The hydraulic head h is defined as:

$$h=\frac{p}{\rho g}+z \quad (9)$$

Where z is the elevation of the point at which the piezometric head is being considered, above some datum level, p and ρ are the fluid's pressure and mass density, respectively, and g is the gravity acceleration.

Substituting equation (9) into equation (8), we therefore obtain the Darcy's law in term of hydraulic head h

$$\mathbf{q}=-\mathbf{K}\cdot\nabla h \quad (10)$$

Where $K=\dfrac{\rho g k}{\mu}$ is hydraulic conductivity tensor of the fluid in a porous medium and $\mathbf{q}$ is the specific flux vector. The components of the tensor $\mathbf{K}$ in a three-dimensional space, can be written in the matrix form

$$\mathbf{K}=\begin{bmatrix}K_{xx} & K_{xy} & K_{xz}\\ K_{yx} & K_{yy} & K_{yz}\\ K_{zx} & K_{zy} & K_{zz}\end{bmatrix}$$

The specific discharge $\mathbf{q}$ with the components q_x, q_y, q_z in the direction of the Cartesian x, y, z is given by

$$\begin{cases} q_x=-\dfrac{K_{xx}}{\mu}\dfrac{\partial p}{\partial x}-\dfrac{K_{xy}}{\mu}\dfrac{\partial p}{\partial x}-\dfrac{K_{xz}}{\mu}\left(\rho g+\dfrac{\partial p}{\partial z}\right)\\ q_y=-\dfrac{K_{yx}}{\mu}\dfrac{\partial p}{\partial x}-\dfrac{K_{yy}}{\mu}\dfrac{\partial p}{\partial x}-\dfrac{K_{yz}}{\mu}\left(\rho g+\dfrac{\partial p}{\partial z}\right)\\ q_z=-\dfrac{K_{zx}}{\mu}\dfrac{\partial p}{\partial x}-\dfrac{K_{zy}}{\mu}\dfrac{\partial p}{\partial x}-\dfrac{K_{zz}}{\mu}\left(\rho g+\dfrac{\partial p}{\partial z}\right)\end{cases}$$

The hydraulic conductivity tensor is symmetric, that is

$K_{xy}=K_{yx}, K_{xz}=K_{zx}$, $K_{yz}=K_{zy}$.

3 DENSITY DEPENDENT FLOW MODEL FOR SATURATED POROUS MEDIUM

3.1 Equation of state for fluid compressibility

To model the compressibility of the fluid, we assume that the pressure p is related to the density ρ by the following equation of state:

$$\frac{d\rho}{\rho} = \beta dp \Leftrightarrow \rho = \rho_0 \exp[\beta(p - p_0)] \qquad (11)$$

Where β is the coefficient of fluid compressibility, defined as [1]

$$\beta = \frac{1}{\rho}\frac{\partial p}{\partial t} \qquad (12)$$

Resulting from the variation of the pressure and p_0 is the reference pressure. If, $\beta = 0$, then the fluid is incompressible.

3.2 Equation of state for soil compressibility

Let us assume that we are dealing with relatively small volume changes, so that the soil is assumed to behave as an elastic material, with a constant coefficient of soil compressibility, α defined by [1]

$$\alpha = \frac{1}{1-\phi}\frac{\partial \phi}{\partial p} \qquad (13)$$

3.3 Different assumptions for our model

In unsaturated porous medium, the compressibility of water and the porous medium are negligible. The density of water ρ is constant and the porosity ϕ does not depend on the pressure.

- Assumption on fluid compressibility

Now, water in confined aquifer in a porous medium is compressible, although this compressibility is small, that is, $\beta \ll 1$ and a reduction of water pressure by pumping from a well results in an increase in the head borne by the solid skeleton of the aquifer.

- Assumption on soil compressibility

The solid matrix of the aquifer is elastic and not rigid. We assume that the elasticity of the solid or the solid particles is much smaller (relative to the solid matrix as a whole), that is, $\alpha \ll 1$ so that their volume remains unchanged.

- Bear assumption

Bear assumption [2] will be interpreted as the spatial variations in the density are much smaller than the temporal ones, that is, $\left|\mathbf{q}\cdot\nabla\rho\right| \ll \left|\phi\frac{\partial\rho}{\partial t}\right|$

3.4 Continuity equation: pressure-dependent density

Substituting the Darcy's law (10) into the mass conservation equation (7) we obtain the flow equation governing the hydraulic head:

$$\frac{\partial\rho\phi}{\partial t} = -\nabla\cdot\left(\rho\mathbf{K}\cdot\nabla h\right) + \rho Q \qquad (14)$$

The term $\frac{\partial\rho\phi}{\partial t}$ in (14) expresses the rate of increase of the mass of water per unit volume of porous medium. On the right hand side of (14) the divergence describes the water mass flux (remember $\mathbf{q} = \phi\mathbf{u}$) that is the excess of inflow over outflow of water mass per unit, volume of porous medium.

We can write the terms $\frac{\partial\rho\phi}{\partial t}$ and $\nabla.\rho\mathbf{q}$ of the equation (7) as

$$\frac{\partial\rho\phi}{\partial t} = \phi\frac{\partial\rho}{\partial t} + \rho\frac{\partial\phi}{\partial t} \qquad (15)$$

and

$$\nabla.\rho\mathbf{q} = \rho\nabla\cdot\mathbf{q} + \mathbf{q}\cdot\nabla\rho \qquad (16)$$

Now, taking into account the above assumptions, we can neglected thermal effect and for $\rho = \rho(p), \phi = \phi(p)$ only, the equation (15) becomes

$$\frac{\partial\rho\phi}{\partial t} = \phi\frac{\partial\rho}{\partial t} + \rho\frac{\partial\phi}{\partial t} = \rho\left(\frac{\partial\phi}{\partial p} + \frac{\phi}{\rho}\frac{\partial\rho}{\partial p}\right)\frac{\partial p}{\partial t} \qquad (17)$$

Substituting the equations (16) and (17) into the equation (7), then we obtain

$$\rho\left(\frac{\partial\phi}{\partial p} + \frac{\phi}{\rho}\frac{\partial\rho}{\partial p}\right)\frac{\partial p}{\partial t} + \rho\nabla\cdot\mathbf{q} + \mathbf{q}\cdot\nabla\rho = \rho Q \qquad (18)$$

By making use of (12), (13) and Bear assumption, we obtain from (18)

$$\rho[\alpha(1-\phi) + \beta\phi]\frac{\partial p}{\partial t} + \rho\nabla\cdot\mathbf{q} = \rho Q \qquad (19)$$

So, we obtain after dividing the equation (19) by $\rho \neq 0$

$$S_0\frac{\partial h}{\partial t} - \nabla\cdot(\mathbf{K}.\nabla\mathbf{h}) = Q \qquad (20)$$

Where S_o specific mass storativity relate to pressure changes defined by

$$S_o = \rho g[\alpha(1-\phi) + \beta\phi] \qquad (21)$$

Now for $\rho = \rho(p,c)$, i.e., the density depend on the pressure p, and solute concentration c, so we have

$$\frac{\partial\rho\phi}{\partial t} = \left(\rho\frac{\partial\phi}{\partial p} + \phi\frac{\partial\rho}{\partial p}\right)\frac{\partial p}{\partial t} + \phi\frac{\partial\rho}{\partial c}\frac{\partial c}{\partial t} \qquad (22)$$

Returning now to (7) and inserting in it $\frac{\partial\phi\rho}{\partial t}$ from (12), (13), (16), then we obtain

$$\rho[\alpha(1-\phi) + \phi\beta]\frac{\partial p}{\partial t} + \phi\frac{\partial\rho}{\partial c}\frac{\partial c}{\partial t} + \rho\nabla\cdot\mathbf{q} + \mathbf{q}\cdot\nabla\rho = \rho Q \qquad (23)$$

S_0 and $\mathbf{K}$ are unaffected by variations in ϕ due to matrix deformability. It is assumed that these variations are small relative to the initial ϕ. The same is true for ρ.

We finally obtain the local 3D continuity equation in the porous medium for a confined aquifer

$$S\frac{\partial h}{\partial t} + \frac{\phi}{\rho_0}\frac{\partial\rho}{\partial c}\frac{\partial c}{\partial t} = -\frac{1}{\rho_0}\nabla\cdot(\rho\mathbf{q}) + \frac{\rho}{\rho_0}Q \qquad (24)$$

Where $S = \rho g[\alpha(1-\phi) + \phi\beta]$ is the specific storage coefficient.

4 SOLUTE TRANSPORT EQUATION IN POROUS MEDIA

Solute mass transport in porous media is strongly correlated with pore fluid flow. The analysis of solute transport is an effective means for studying medium heterogeneities. The governing equation for the solute mass transport problem is the advection-dispersion equation. For fluid flow in a porous medium, solute transports are due to three important mechanisms: diffusion, dispersion, and advection.

- Advection: the displacement of the pollutant in the porous medium to the average fluid velocity. There is normally driven by gravity or pressure forces.
- Diffusion: this is a physical phenomenon modeled by Fick's law and which translates the transfer of molecules from a zone of high concentration to a zone of low concentration (Brownian motion).
- Dispersion: describes the phenomenon by which a substance propagates within the soil and spread is due to the displacement of the molecules under the effect of the molecular diffusion.

The solute mass transport equation is based on the mass conservation equation in the presence of source terms:

$$\frac{\partial \phi \rho c}{\partial t} = -\nabla \cdot \vec{J} + \rho Q c_s, \qquad (25)$$

Where $\vec{J}$ is the solute mass flux, ϕ is porosity of the porous medium, ρ is fluid density, c is solute concentration, Q sinks and sources terms, c_s is concentration of solute as a mass fraction in the source fluid.

The flux $\vec{J}$ contains advection flux (J_{adv}), diffusion flux (J_{diff}) and dispersion flux (J_{disp}),

$$\begin{aligned} \vec{J} &= J_{adv} + J_{diff} + J_{disp} \\ &= \rho \mathbf{q} c - \rho\phi D_m \nabla c - \rho\phi D_{md} \nabla c \end{aligned} \qquad (25)$$

D_m is coefficient of molecular diffusion, D_{md} is coefficient of mechanical dispersion tensor, due to the heterogeneity of the velocity in longitudinal and transversal direction. Thus, the continuity equation of fluid flow coupled by solute concentration c transport with sources terms Q.

$$\frac{\partial \phi \rho c}{\partial t} = -\nabla \cdot (\rho \mathbf{q} c) + \nabla \cdot (\rho\phi D \cdot \nabla c) + \rho Q c_s. \qquad (26)$$

The hydrodynamic dispersion tensor, D, is defined as the sum of the coefficients of mechanical dispersion, D_{md} and of molecular diffusion in a porous medium, D_m

$$D = D_{md} + D_m \qquad (27)$$

By developing (26), then we obtain

$$\rho\phi \frac{\partial c}{\partial t} + c\frac{\partial \rho\phi}{\partial t} = -c\nabla \cdot (\rho \mathbf{q}) - \rho \mathbf{q} \nabla c + \nabla \cdot (\rho\phi D \cdot \nabla c) + \rho Q c_s. \qquad (28)$$

Multiplying the mass conservation equation (7) by c and substituting the result into the equation (28), we obtain the solute transport equation in confined aquifer:

$$\rho\phi \frac{\partial c}{\partial t} = \nabla \cdot (\rho\phi D \cdot \nabla c) - \rho \mathbf{q} \cdot \nabla c + \rho Q (c_s - c). \qquad (29)$$

4.1 Dispersion in porous media

Understanding of dispersion phenomenon and movement of fluids in porous media is very crucial in the study of the problem of saltwater intrusion; using the mixing zone approach. In this work, we will not study the issue in depth, but for more details, we refer the reader to the books of J.Bear: [1], [2], [3], etc.

Mechanical dispersion is responsible for the spreading caused by the velocity variations at the microscopic level.

Molecular diffusion is responsible for the spreading caused by the random movement of molecules in a fluid and also random movement under concentration gradient by molecular movement under concentration.

Hydrodynamic dispersion denotes the spreading (at the macroscopic level) that results from both mechanical dispersion and molecular dispersion.

4.2 Hydrodynamic dispersion tensor

The relationship between the hydrodynamic dispersion tensor D and porous matrix is given by [3] as:

$$D_{ij} = \alpha_{ijkm} \frac{\overline{v_k}\,\overline{v_m}}{\overline{v}} f(P_e, \delta) \qquad (30)$$

Where v is the average velocity, δ is the ratio of the length characterizing their cross-section, and $f(P_e, \delta)$ is a function to introduce the effect of molecular. For practical applications the function $f(P_e, \delta) = 1$.

The coefficient α_{ijkm} is the dispersivity of the medium (L). It is a parameter which accounts for the loss of information about the pore velocity fluctuation when passing from the macroscopic scale [6].

With the longitudinal dispersivity, α_L and the transverse dispersivity, α_T, the components of the dispersivity for an isotropic porous medium can be expressed in the form [3]:

$$\alpha_{ijkm} = \alpha_T \delta_{ij}\delta_{km} + \frac{\alpha_L + \alpha_T}{2}(\delta_{ik}\delta_{jm} + \delta_{im}\delta_{jk}), \qquad (31)$$

Where δ_{ij} denotes the Kronecker symbol (with $\delta_{ij} = 0$ for $i \neq j$ and $\delta_{ij} = 1$ for $i = j$).

In three-dimension, Cartesian coordinates, and velocity components v_x, v_y, v_z in $x, y,$ and z directions, we have:

$$\begin{aligned} D_{xx} &= \alpha_T v + (\alpha_L - \alpha_T) v_x^2 / v = [\alpha_T (v_y^2 + v_z^2) + \alpha_L v_x^2] / v, \\ D_{xy} &= (\alpha_L - \alpha_T) v_x v_y / v = D_{yx}, \\ D_{yy} &= \alpha_T v + (\alpha_L - \alpha_T) v_y^2 / v = [\alpha_T (v_x^2 + v_z^2) + \alpha_L v_y^2] / v, \\ D_{yz} &= (\alpha_L - \alpha_T) v_y v_z / v = D_{zy}, \\ D_{zz} &= \alpha_T v + (\alpha_L - \alpha_T) v_z^2 / v = [\alpha_T (v_x^2 + v_y^2) + \alpha_L v_z^2] / v. \end{aligned} \qquad (32)$$

5 Coupled density-dependent flow and solute transport

Mathematical modeling of saltwater intrusion, based on the principle of existence of a mixing zone can be formulated in terms of two partial differential equations. The first equation is used to describe flow of variable-density fluid (mixture of fresh water and seawater), and the second equation is used to describe the transport of dissolved salt. Coupling is done using a state equation that relates density to concentration and a constitutive equation that relates viscosity to concentration [4], [8], [10], [11].

5.1 State equation

The state equations are the relations between the state variables of a thermodynamic equilibrium system. The density is expressed in terms of concentration (at constant temperature). The density ρ in term of linear function of reference (freshwater) density ρ_0 and the concentration c

$$\rho = \rho_0(1+\frac{\varepsilon c}{c_s}) \qquad (33)$$

Where ρ_0 is the reference (freshwater) density; $\varepsilon = \frac{\rho_s - \rho_0}{\rho_0}$ is the density difference ratio; ρ_s is the density of maximum concentration.

We denote

$$\eta = \frac{\varepsilon}{c_s} \qquad (34)$$

Where c_s is the maximum concentration.

We express Darcy's velocity in term of hydraulic head and concentration as:

$$\mathbf{q} = -\frac{k\rho_0 g}{\mu}(\nabla h + \eta c \nabla z) \qquad (35)$$

For isothermal conditions and dilute salt concentration, the fluid density is expressed as (33), that is, $\rho = \rho_0(1+\eta c)$. Then, the equation (24) becomes

$$S\frac{\partial h}{\partial t} + \eta\phi\frac{\partial c}{\partial t} = \nabla\cdot\{\mathbf{K}(\nabla h + \eta c\nabla z)\} + (1+\eta c)Q \qquad (36)$$

Taking into account, Oberbeck-Bousinesq approximation which consists in neglecting the variation of water density compare to reference density in the solute transport equation and Darcy flux depends on the water density variation. Hence, the transport equation (29) becomes

$$\phi\frac{\partial c}{\partial t} = \nabla\cdot(\phi D\cdot\nabla c) - \mathbf{q}\cdot\nabla c + Q(c_s - c). \qquad (37)$$

5.2 Initial and boundary conditions

The saltwater intrusion into coastal aquifers, using transition zone approach, is mathematically describe by the couple of two partial differential equations, one describing the flow equation of water (mixture of freshwater and saltwater) and the another the solute transport flow equation. Thus, additional information and constrains are needed to define a particular problem. These are [3]:

- domain geometry in which the problem is under consideration and part of the boundary are at infinity;
- data about all relevant physical parameters and fluid properties $(\phi, \mathbf{K}, \alpha_T, c_s, etc.)$;
- initial conditions which describe the initial state of the unknowns in the study domain;
- boundary condition which describe how the fluid in the study domain interacts with its surroundings.

6 CONCLUSION

The aim of this paper was to develop a complete mathematical model to study the evolution of saltwater intrusion in the flow regime of coastal area which is basically the flow equation (36) and the transport equation (37). Several Benchmarking for density-coupled are used for testing the performance of saltwater intrusion models, one can use Salt-dome problem, or the most common Henry's problem (saltwater encroachment), etc. Moreover, most of the numerical method used to solve the mathematical model that simulate the behavior of saltwater intrusion can be found in the literature such as Finite Element Method, Finite Difference Method and Finite Volume Method, etc.

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