On Rough Interval Multi-Criteria Group Decision Making

Hegazy Zaher, Soha Mohamed

Abstract: Multi-Criteria rough interval group decision making is relatively a recent branch of multi-criteria decision making. In this paper, a new method is proposed based on Rough Interval Multi-Objective Optimization on the basis of Ratio Analysis method for considering the uncertainty in the Multi-Criteria Group Decision Making (RIMOORA-GDM) problems based on rough interval numbers. The proposed method is used for ranking the potential alternatives and selecting the most suitable one. The introduced method is very simple to implement. Finally, an illustrative example is given.

Keywords: Rough Interval, Rough Interval MOORA, Multi-Criteria Group Decision Making.

1 INTRODUCTION

When MCDM is done by more than one decision maker, it is called multi-criteria group decision making (MCGDM), each decision maker considers the same sets of alternatives and criteria [1]. A group decision situation involves multiple decision makers, each with different skills, experience and knowledge relating to different aspects (criteria) of the problem [2]. In brief, MCDM is the art of compromising conflicting criteria for a single decision maker. The group decision making is the art of compromising different opinions of group members, in a sense, group decision making can be a multi-dimensional MCDM problem, but more issues should be studied to treat the conflicting factors aforementioned [3]. Group decision making can be approached from two points of view. In the first approach, individual multi-criteria models are developed based on individual’s preferences. Each decision maker formulates a multi-criteria problem defining the parameters according to these preferences and solves the problem getting an individual solution set. Next, the separate solutions are aggregated by aggregation of operations resulting in the group solution. In the second approach, each decision maker provides a set of parameters which are aggregated by appropriate operators, providing finally a set of group parameters. Upon this set the multi-criteria method is applied and the solution expresses group preference [4]. There are few publications in literature used applications of MCDM methods based on rough numbers. Song et. al., [5] used a rough TOPSIS approach for failure mode and effects analysis in uncertain environments. Pamucar and Cirovie [6] Combination of rough AHP and MABAC. Roy et. al., [7] presented a hybrid rough-AHP and MABAC methods for selection of medical tourism sites. Gigovie et. al., [8] Combination of interval rough AHP and GIS is proposed for flood hazard mapping. Zaher et. al. [9] introduced three types of multi-criteria decision making methods based on the rough interval concept.

2 MULTI OBJECTIVE AND OPTIMIZATION ON THE BASIS OF RATIO ANALYSIS (MOORA) METHOD

MOORA method was firstly proposed by Brauers and Zavadskas [10], Considers both beneficial and non-beneficial objectives (criteria) for ranking or selecting one or more alternatives from a set of available options. The MOORA method starts with a decision matrix. The procedural steps of MOORA method are described as follows [11]:

Step 1: Decision matrix is normalized with Eq. (1).

\[ x_{ij}^n = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{n} x_{ij}^2}} \quad i = 1, 2, ..., m \text{ and } j = 1, 2, ..., n \]

Where, \( x_{ij}^n \) is a dimensionless number which belongs to the interval \([0, 1]\) representing the performance of \(i\)-th alternative on \(j\)-th attribute.

Step 2: Weighted normalized decision matrix is formed with the help of Eq. (2).

\[ w_{ij} = w_j \times x_{ij}^n \]

Where \( w_j \) shows the weight of the \(j\)-th criterion.

Step 3: Final preference \((y_{ij}^r)\) values obtained by using Eq. (3). Here \( j = 1, 2, ..., g \) indicates the criteria to be beneficial attributes and \( j = g+1, g+2, ..., n \) shows the criteria to be non-beneficial Attributes (cost).

\[ y_{ij}^r = \frac{\sum_{j=1}^{g} w_{ij} - \sum_{j=g+1}^{n} w_{ij}}{\sum_{j=1}^{g} w_{ij}} \]
Step 4: Ranking of the alternatives are obtained by ranking \( y_i^* \) values in descending order. Thus, the best alternative has the highest \( y_i^* \) value.

3 ROUGH SET THEORY

Rough set theory, first proposed by Pawlak [12], generally, a rough interval comprises two parts: an upper approximation interval (UAi) and a lower approximation interval (LAi). A rough interval data formulation is a simple and intuitive way to represent uncertainty, which is typical of real decision problems.

3.1 Rough Interval

Definition 1. Let \( x \) denote a closed and bounded set of real numbers. A rough interval \( x^R \) is defined as an interval with known lower and upper bounds but unknown distribution information for \( x \):

\[
x^R = [x^\text{UAi}, x^\text{LAI}]
\]

Where \( x^\text{UAi} \) and \( x^\text{LAI} \) are upper and lower approximation intervals of \( x^R \), respectively, \( x^\text{UAi} \) and \( x^\text{LAI} \) are conventional intervals, and \( x^\text{LAI} \in x^\text{UAi} \). When \( x^\text{UAi} = x^\text{LAI} \), \( x^R \) becomes a conventional interval, i.e. \( x^R = x^\text{UAi} = x^\text{LAI} \) [13].

Definition 2. Let \( \ast \in \{+,-, \times, \div \} \) be a binary operation on rough intervals. For rough intervals \( x^R \) and \( y^R \) when \( x^R \geq 0 \) and \( y^R \geq 0 \), we have:

\[
x^R + y^R = [(x^\text{UAi}) + (y^\text{UAi})] : [(x^\text{LAI}) + (y^\text{LAI})]
\]

(4)

\[
x^R - y^R = [(x^\text{UAi}) - (y^\text{UAi})] : [(x^\text{LAI}) - (y^\text{LAI})]
\]

(5)

\[
x^R \times y^R = [(x^\text{UAi}) \times y^\text{UAi}] : [(x^\text{LAI}) \times y^\text{LAI}]
\]

(6)

\[
x^R \div y^R = [(x^\text{UAi}) \div y^\text{UAi}] : [(x^\text{LAI}) \div y^\text{LAI}]
\]

(7)

As \( x^\text{UAi}, x^\text{LAI}, y^\text{UAi} \) and \( y^\text{LAI} \) are conventional intervals, the above operations can be further transferred to the following functions if letting \( x^\text{UAi} = x^\text{UAj} = [x^\text{UAi}, x^\text{LAI}], \ x^\text{LAI} = [x^\text{LAI}, x^\text{LAI}] \) and \( y^\text{UAi} = y^\text{UAj} = [y^\text{UAi}, y^\text{LAI}], \ y^\text{LAI} = [y^\text{LAI}, y^\text{LAI}] \), where \( x^\text{UAi}, x^\text{LAI}, y^\text{UAi} \) and \( y^\text{LAI} \) are deterministic numbers denoting the lower and upper bounds of \( x^\text{UAi}, x^\text{LAI}, y^\text{UAi} \) and \( y^\text{LAI} \) [14].

4 PROPOSED TECHNIQUE (RIMOORA-GDM METHOD FOR GROUP DECISION MAKING)

A proposed method called RIMOORA in order to solve Group Decision Making problems. The proposed method determining the most preferable alternative among all possible alternatives, when performance ratings are described by rough interval. This method is very suitable for solving the group decision-making problem under rough interval environment. RIMOORA-GDM Method for Group Decision Making is shown in Fig. 1.
approximation and $x_{ij}^{-UK}$, $x_{ij}^{+UK}$ are the normalized upper approximation performance ratings for each decision maker, $K$—decision maker, $k=1,2,\ldots, K$.

Step 3: Calculate the weighted normalized rough interval decision-making matrix for all decision makers, $v_{ij}^{W} = \{ v_{ij}^{-UK}, v_{ij}^{+UK} | v_{ij}^{L}, v_{ij}^{U} \}$.

The Lower approximation and the Upper approximation bounds of a rough interval for each decision maker can be determined using the following formulæ:

$$v_{ij}^{Lk} = x_{ij}^{Lk} \times w_j^k,$$

(13)

$$v_{ij}^{uk} = x_{ij}^{uk} \times w_j^k,$$

(14)

$$v_{ij}^{-UK} = x_{ij}^{-UK} \times w_j^k,$$

(15)

$$v_{ij}^{+UK} = x_{ij}^{+UK} \times w_j^k.$$  

(16)

Where $v_{ij}^{Lk}$, $v_{ij}^{uk}$ is the lower approximation and $v_{ij}^{-UK}$, $v_{ij}^{+UK}$ the upper approximation of weighted normalized for each decision maker.

Step 4: Calculate the rough interval composite score ($S_{i}^{RIK}$) for all decision makers by using Eqs. (13 to 16). Calculate the $S_{i}^{RIK}$ overall ratings of the rough interval beneficial and non-beneficial criteria for each alternative.

$$S_{i}^{RIK} = \sum_{j=1}^{g} v_{ij}^{R} - \sum_{g+1}^{n} v_{ij}^{R},$$

(17)

Where $\sum_{j=1}^{g} v_{ij}^{R}$ and $\sum_{g+1}^{n} v_{ij}^{R}$ are for the rough interval beneficial and non-beneficial (cost) criteria for all decision makers, respectively. Here $j = 1,2,\ldots,g$ indicates the criteria to be beneficial and $j = g+1,g+2,\ldots,n$ shows the criteria to be non-beneficial.

a. beneficial criteria, the overall ratings of an alternative for lower and the upper approximation values of the rough intervals for each decision maker based on equation (17) are computed as follows:

$$S_{i}^{+}\text{Rik} = \{ (S_{i}^{+}\text{LAik}), (S_{i}^{+}\text{UAik}) \},$$

$$S_{i}^{-}\text{Rik} = \{ (S_{i}^{-}\text{LAik}), (S_{i}^{-}\text{UAik}) \};$$

$$v_{ij}^{-UK} = \{ v_{ij}^{-UK}, v_{ij}^{+UK} | v_{ij}^{L}, v_{ij}^{U} \}.$$  

(18)

(19)

(20)

(21)

Where ($S_{i}^{-}\text{LAik}$, ($S_{i}^{+}\text{LAik}$, ($S_{i}^{-}\text{UAik}$ and ($S_{i}^{+}\text{UAik}$ are the benefit values of lower and upper approximation intervals for each decision maker.

b. Similarly, calculate the overall rating of non-beneficial criteria:

$$S_{i}^{-}\text{LAik}, S_{i}^{+}\text{LAik}, S_{i}^{-}\text{UAik} and S_{i}^{+}\text{UAik}$$

the lower and upper approximation for each decision maker intervals based on equation (17) are calculated using the following formulæ:

$$S_{i}^{-}\text{LAik} = \sum_{j=g+1}^{n} v_{ij}^{L} (j \in j_{\text{min}}),$$

(22)

$$S_{i}^{+}\text{LAik} = \sum_{j=g+1}^{n} v_{ij}^{L} (j \in j_{\text{min}}),$$

(23)

$$S_{i}^{-}\text{UAik} = \sum_{j=g+1}^{n} v_{ij}^{L} (j \in j_{\text{min}}),$$

(24)

$$S_{i}^{+}\text{UAik} = \sum_{j=g+1}^{n} v_{ij}^{L} (j \in j_{\text{min}}),$$

(25)

Where ($S_{i}^{-}\text{LAik}$, ($S_{i}^{+}\text{LAik}$, ($S_{i}^{-}\text{UAik}$ and ($S_{i}^{+}\text{UAik}$ are the non-beneficial (cost) values of lower and upper approximation intervals for each decision maker.

Step 5: Calculate the crisp composite score for each alternative. As a result of performing the previous steps. For this, the crisp values of the overall performance ratings obtained based on rough intervals beneficial and non-beneficial for each alternative.

a. Calculate the crisp composite score $S_{i}^{W}$, for each $k$—decision maker is given as:

$$S_{i}^{W} = \frac{1}{k} \left[ (S_{i}^{+}\text{LAik}) + (S_{i}^{+}\text{UAik}) + (S_{i}^{-}\text{LAik}) + (S_{i}^{-}\text{UAik}) \right] - \frac{1}{4} (S_{i}^{+}\text{LAik}) + (S_{i}^{+}\text{UAik}) - (S_{i}^{-}\text{LAik}) + (S_{i}^{-}\text{UAik}) \right].$$  

(26)

Where $S_{i}^{W}$ crisp valued overall performance index for each decision maker.

b. Calculate the crisp composite score $S_{i}^{*}$, for all alternatives under all decision makers.

The aggregation of the measure for the group measures of the composite score $S_{i}^{*}$

$$S_{i}^{*} = \frac{\sum_{k=1}^{K} S_{i}^{W}}{K}.$$  

(27)

Step 6: Ranking of the alternatives are obtained by ranking $S_{i}^{*}$ values in descending order, the alternative with the highest overall performance index is the best choice.

5. NUMERICAL EXAMPLE

The numerical example presented in Roszkowska [14] solved by the crisp method will be considered here under more general conditions using the rough interval method. The group involved three decision makers ($D_{1}, D_{2}$ and $D_{3}$) with four criteria ($C_{1}$, $C_{2}$, $C_{3}$ and $C_{4}$) to rank five alternatives ($A_{1}$, $A_{2}$, $A_{3}$, $A_{4}$ and $A_{5}$). The first two criteria ($C_{1}$ and $C_{2}$) are benefit criteria, greater values being better while the other two criteria are cost type (the minimum is better); the ratings are allocated by experts (decision makers) while the last criterion ($C_{3}$) is measured by fixed rating. As shown in Table 1. Three decision matrices are obtained and put into this augmented table. The allocated criteria weights for each decision maker are found in Table 2. Where UAI and LAI are the upper approximation interval and lower approximation interval respectively.

| Table 1. The Rough Interval Decision Matrix for Three Decision Makers |
|------------------------|------------------------|------------------------|------------------------|
| **Criteria**           | **Optimization**       | **max**                | **max**                | **min**                | **min**                |
| Alternatives           | **c_{1}**              | **c_{2}**              | **c_{3}**              | **c_{4}**              |
| **UAI**                | **LAI**                | **UAI**                | **LAI**                | **UAI**                | **LAI**                |
| $D_{1}$                | $[1.7, [3.5]]$         | $[19.25, [21.23]]$     | $[2, 10, [6, 8]]$      | $[4, 17, [11, 14]]$    |
| $A_{1}$                | $[7,14], [10,12]$      | $[3,5, [8,9]]$         | $[6, 15, [9,13]]$      | $[5, 91, [6, 7]]$      |
| $A_{2}$                | $[14, 21], [16,19]]$   | $[24, 36, [28,33]]$    | $[3,14, [5,9]]$        | $[9, 18, [13, 16]]$    |

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5.1 Results and Discussion
By applying the procedure of RIMOORA-GDM, we obtain the beneficial and non-beneficial criteria for each decision maker in tables 3 to 5, by Eqs. (18 to 25). Composite scores for all decision makers are listed in table 6 for all alternatives are calculated by Eqs. (26 and 27). The $A_4$ alternative should be selected because it has the maximum composite score.

### Table 2. Criteria Weights for the Three Decision Makers

<table>
<thead>
<tr>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>0.196</td>
<td>0.225</td>
</tr>
<tr>
<td>$c_2$</td>
<td>0.413</td>
<td>0.475</td>
</tr>
<tr>
<td>$c_3$</td>
<td>0.239</td>
<td>0.125</td>
</tr>
<tr>
<td>$c_4$</td>
<td>0.152</td>
<td>0.175</td>
</tr>
</tbody>
</table>

Source: Roszkowska [14].

### Table 3. Beneficial and Non-Beneficial Criteria for First Decision Maker.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>beneficial</th>
<th>non-beneficial</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>0.13753, 0.219339</td>
<td>[0.1648, 0.192069]</td>
</tr>
<tr>
<td>$A_2$</td>
<td>0.198689, 0.301015</td>
<td>[0.239593, 0.273746]</td>
</tr>
<tr>
<td>$A_3$</td>
<td>0.259717, 0.389575</td>
<td>[0.300752, 0.355423]</td>
</tr>
<tr>
<td>$A_4$</td>
<td>0.152599, 0.334642</td>
<td>[0.246082, 0.300621]</td>
</tr>
<tr>
<td>$A_5$</td>
<td>0.192332, 0.355554</td>
<td>[0.253623, 0.31465]</td>
</tr>
</tbody>
</table>

### Table 4. Beneficial and Non-Beneficial Criteria for Second Decision Maker.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>beneficial</th>
<th>non-beneficial</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>0.133358, 0.187747</td>
<td>[0.150904, 0.168449]</td>
</tr>
<tr>
<td>$A_2$</td>
<td>0.224999, 0.286006</td>
<td>[0.256175, 0.28646]</td>
</tr>
<tr>
<td>$A_3$</td>
<td>0.249146, 0.347404</td>
<td>[0.278973, 0.313212]</td>
</tr>
<tr>
<td>$A_4$</td>
<td>0.28603, 0.396553</td>
<td>[0.326378, 0.371991]</td>
</tr>
<tr>
<td>$A_5$</td>
<td>0.301815, 0.477249</td>
<td>[0.387776, 0.459383]</td>
</tr>
</tbody>
</table>

### Table 5. Beneficial and Non-Beneficial Criteria for Third Decision Maker.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>beneficial</th>
<th>non-beneficial</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>0.202493, 0.305537</td>
<td>[0.23484, 0.274384]</td>
</tr>
<tr>
<td>$A_2$</td>
<td>0.17134, 0.209682</td>
<td>[0.188115, 0.198899]</td>
</tr>
<tr>
<td>$A_3$</td>
<td>0.306735, 0.354662</td>
<td>[0.328302, 0.339808]</td>
</tr>
<tr>
<td>$A_4$</td>
<td>0.341482, 0.394202</td>
<td>[0.367842, 0.383419]</td>
</tr>
<tr>
<td>$A_5$</td>
<td>0.243231, 0.36305</td>
<td>[0.274834, 0.341482]</td>
</tr>
</tbody>
</table>

### Table 6. Ranking using RIMOORA-GDM Method for Group Decision Makers

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>$S^1_{OMT}$</th>
<th>$S^1_{GDM}$</th>
<th>$S^1_{2OMT}$</th>
<th>$S^1_{2GDM}$</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>0.034515</td>
<td>0.06334</td>
<td>0.13756</td>
<td>0.07847</td>
<td>5</td>
</tr>
<tr>
<td>$A_2$</td>
<td>0.090107</td>
<td>0.151451</td>
<td>0.123344</td>
<td>0.121634</td>
<td>4</td>
</tr>
<tr>
<td>$A_3$</td>
<td>0.155846</td>
<td>0.112691</td>
<td>0.143825</td>
<td>0.137454</td>
<td>3</td>
</tr>
<tr>
<td>$A_4$</td>
<td>0.177204</td>
<td>0.295583</td>
<td>0.259982</td>
<td>0.242560</td>
<td>1</td>
</tr>
<tr>
<td>$A_5$</td>
<td>0.057679</td>
<td>0.251661</td>
<td>0.141687</td>
<td>0.150342</td>
<td>2</td>
</tr>
</tbody>
</table>
Then the final ranking of alternatives is obtained with RIMOORA-GDM method as $A_4 > A_3 > A_2 > A_1$ and the best alternative is $A_4$ with the overall performance index of 0.244256 and the worst alternative is $A_1$ with the result of 0.07847. Compared with the different MCDM methods the proposed method is very simple to understand and easy to implement especially in rough interval domain.

6. CONCLUSION
A new method so-called RIMOORA-GD is proposed for solving group decision-making problems under rough interval. This method selects the best alternative among all possible alternatives in group decision making problem, when performance ratings are described by rough interval. The proposed method is computationally simple and its underlying concept is logical and comprehensible, thus facilitating its implementation in a computer-based system. The performance rating values of alternatives versus conflicting are described by rough interval. This study shows that the proposed method effectively applied in group decision-making problems respecting rough interval.

REFERENCES