Quad Difference Labeling For Degree Splitting Of Cycle Related Graphs

K. Bhuvaneswari, G. Subashini

Abstract: In this work, we discuss the context of Quad difference labeling (QDL) for degree splitting graphs like $P_n \odot K_1$, C_n, $B_{n,n}$, $C_n \odot K_1$, T_n, Y_{r+1}- tree graphs.

Keywords: QDL, Quad difference graph (QDG), degree splitting (DS).
AMS subject classification: 05C78

1. INTRODUCTION

The graphs taken in this paper are finite, simple and undirected. For notations and terminology, we refer [1]. The concept of degree splitting graph was introduced by R. Ponraj and S. Somasundaram [3]. In [2] have proved that degree splitting of some well-known graphs in I - Cordial and some regular graph are not I – Cordial. A splitting graph of a graph was introduced by E. Sampath kumar and H. B. Waliker[4].

2. DEFINITIONS

Definition 2.1

Let G be a graph and is said to be QDL if there exist a one to one and onto function from vertices to {0,1,..,p-1} such that f induces the mapping $f^*: E(G) \to N$ is given by

$f^*(uv) = \left|[f(u)]^4 - [f(v)]^4\right|$ is injective.

Definition 2.2

Degree splitting graph: For a graph (V, E), the degree Splitting graph DS(G) is obtained from G by adding a new vertex w_i for each partition V_i that contains atleast two vertices and joining w_i to each vertex of V_i.

- Department of Mathematics
- Associate Professor, Mother Teresa Women's University, Kodaikanal.
- Research Scholar, Mother Teresa Women's University, Kodaikanal.
- Assistant Professor, Bharath Institute of Higher Education and Research, Selaiyur.
- Corresponding author mail id: suba.thulam@gmail.com

Proof: Let $G = DS(P_n \odot K_1)$ be a graph with $2n+3$ vertices and $4n-1$ edges.

Define the vertex labeling function $f: V \to \{0,1,2,\dots,2n+2\}$ as follows:

For $i = 1\ to\ n$,

$f(u_i) = 2i$, $f(v_i) = 2i-1$, $f(v) = 2n$, $f(u) = 2n+1$, $f(w) = 2(n+1)$ for $n > 3$.

Introduce an annexed function f^* for edge labeling as$1 \le i \le n$,

$f^*(u_i v_i) = |u_i^4 - v_i^4|$

$f^*(u_i u_{i+1}) = |u_i^4 - u_{i+1}^4|$

$f^*(v v_i) = |v^4 - v_i^4|$

$f^*(u u_i) = |u^4 - u_i^4|$

$f^*(w u_1) = w^4 - u_1^4$

$f^*(w u_n) = w^4 - u_n^4$

All the edge labeling defined are distinct and are in increasing sequence and hence f^* defined is bijective. Thus $DS(P_n \odot K_1)$ proves QDG.

Example 3.1. $DS(P_5 \odot K_1)$ is illustrated below in figure 3.1.

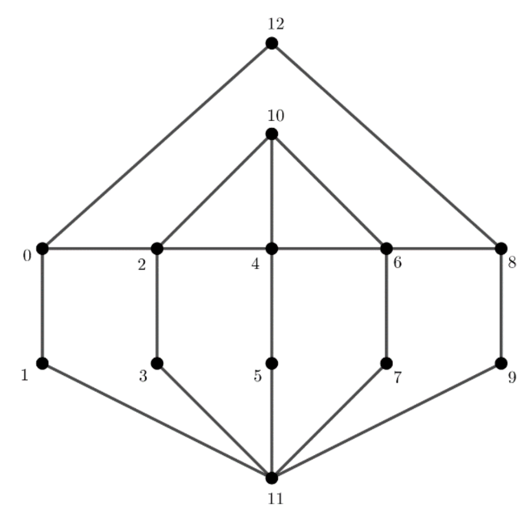

Figure 3.1

Theorem 3.2

The degree splitting of P_n, $n > 3$ admits QDL.

Proof:

Let G = $DS(P_n)$, $n > 3$ $|V(P_n)| = n + 2$ and $|E(P_n)| = 2n - 1$. The vertex and edge set are stated as follows $V(G) = \{u_i, v, w: 1 \leq i \leq n\}$ and $E(G) = \{u_1 u, u_1 u_2, u_9 v, u_9 u_8, u_i w, u_i u_{i+1}\}$ respectively.

The vertex labeling of a function mapping f from vertex to $\{0,1,, \ldots, p-1\}$ is as follows:

$f(u_i) = i - 1$ for $1 \leq i \leq n$.

$f(v) = n$, $f(w) = n + 1$.

Similarly the edge labeling is defined by an induced function $f^*: E[DS(P_n)]$ as follows:

$f^*(u_1 u) = v^4 - u_1^4$,

$f^*(u_1 u_2) = u_2^4 - u_1^4$

$f^*(u_9 v) = v^4 - u_9^4$

$f^*(u_9 u_8) = u_9^4 - u_8^4$

$f^*(u_i w) = w^4 - u_i^4$, $i = 2,3, \ldots, n-1$

$f^*(u_i u_{i+1}) = u_{i+1}^4 - u_i^4$ for $1 \leq i \leq n-1$.

All the edge labeling are distinct and f^* defined is bijective and hence theorem satisfies the defined labeling.

Example 3.2. Degree splitting of P_7 is show below

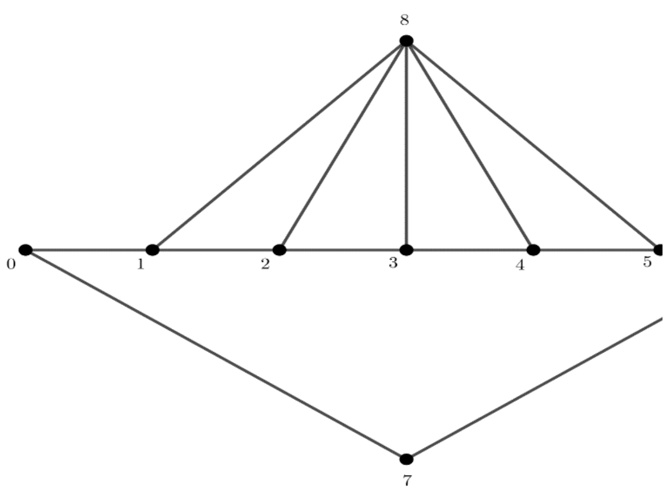

Figure 3.2

Theorem 3.3

The graph $DS(C_n)$, $n \geq 3$ is a QDG.

Proof:

Consider a graph $DS(C_n)$ with $2n + 1$ vertices and $2n$ edges.

Vertex and edge set is defined as

$V(G) = \{v_0, v_i \,/\, 1 \leq i \leq 2n\}$ and $E(G) = \{v_0\, v_i, v_i\, v_{i+1} \,/\, 1 \leq i \leq 2n\}$

Now define a vertex labeling of a function $f: V \rightarrow \{0,1,2,, \ldots, n\}$ as follows:

$f(y) = 0, f(x_i) = i, 1 \leq i \leq n.$

Define a edge labeling by an induced function f^* from an Edge to set of Natural numbers as follows.

For $1 \leq i \leq n$; $f^*(x_i y) = i^4$ and $f^*(x_i x_{i+1}) = x_{i+1}^4 - x_i^4$ for $i = 1 \; to \; n-1$.

Hence all edges labeled are distinct and f^* is injective as defined in the above labeling pattern.

Therefore $DS(C_n)$ satisfies QD graph.

Example 3.3. C_5 is illustrated below for QDL in figure 3.3.

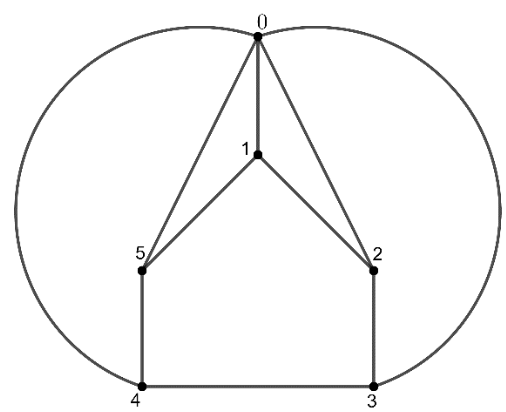

Figure 3.3

Theorem 3.4

Degree splitting of a bistar graph $B_{n,n}$ for $n \geq 2$ admits QDL.

Proof:

Let $G = Ds(B_{n,n})$ be a graph with $|V(G)| = 2(n + 2)$ and $|E(G)| = 4n + 3$ and its vertex and edge set are defined as:

$V = \{u, u_i, v, v_i, x, y \,/\, 1 \leq i \leq n\}$ and

$E = \{uv, uu_i, vv_i, uy, vy, u_i x, v_i x: i = 1 \; to \; n\}$ respectively.

A vertex labeling of a mapping from f to the vertex set $\{0,1,2, \ldots, 2n + 3\}$ which is defined as follows:

$f(u) = 0$, $f(u_i) = 2i$, $f(v) = 1$, $f(v_i) = 2i + 1$ for $i = 1 \; to \; n$,

$f(x) = 2(n + 1)$, $f(y) = 2n + 3$.

Now, we define an edge labeling by introducing an induced function f^* from edges to the natural numbers as:

$f^*(uv) = 1$

$f^*(uu_i) = (2i)^4$

$f^*(vv_i) = (2i+1)^4 - 1$

$f^*(uy) = (2n+3)^4$

$f^*(u_ix) = (2(n+1))^4 - (2i)^4$

$f^*(v_ix) = (2(n+1))^4 - (2i+1)^4$

Hence all edge labeling defined are distinct.

Therefore $f^*$is one to one function. Thus the theorem is proved.

Example 3.4 : QDL for $B_{4,4}$ graph is shown below in figure 3.4

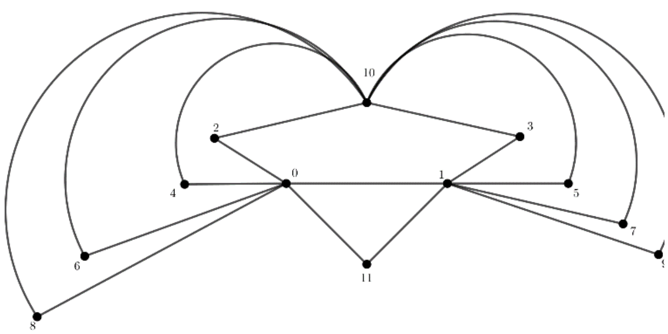

Figure 3.4

Theorem 3.5

The graph $DS(C_n \odot K_1)$ is a QD graph.

Proof:

Consider degree splitting of crown graph with $2(n+1)$ vertices and $2n$ edges.

Define a vertex set of a graph is $V = \{u_i, u_i', x, y; 1 \leq i \leq n\}$ and the edge set of a graph is defined as $E = \{u_iu_{i+1}, u_iu_i', u_i'y, u_ix; i = 1 \; to \; n\}$.

Now we define a vertex labeling function f mapping from a vertex of $DS(C_n \odot K_1)$ to $\{0,1,2,\dots,2(n+1)-1\}$ is as follows: For $1 \leq i \leq n$

$f(u_i) = 2(i-1)$, $f(u_i') = 2i-1$, $f(x) = 2n$ and $f(y) = 2n+1$

Now define edge labeling by introducing a induced function on edge as for $1 \leq i \leq n$

$f^*(u_iu_{i+1}) = (2i)^4 - [2(i-1)]^4$

$f^*(u_iu_i') = (2i-1)^4 - [2(i-1)]^4$

$f^*(u_ix) = 2n - [2(i-1)]^4$

$f^*(u_i'y) = (2n+1) - (2i-1)^4$

We observe that all edge labeling are in increasing sequence and are distinct. Hence f^* is injective. Therefore the theorem proves that the given graph G is QDG.

Example 3.5 Degree splitting of C_4 corona K_1 depicts QD labeling as given below:

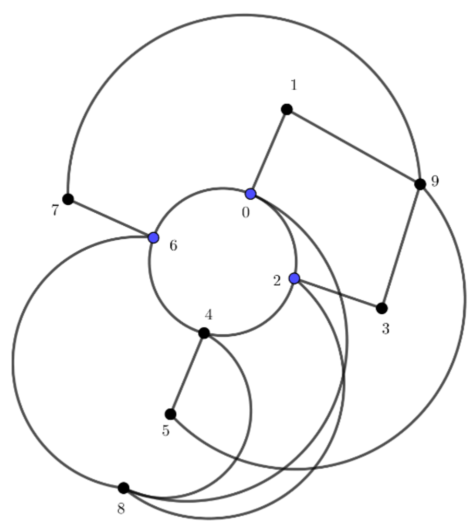

Figure 3.5

Theorem 3.6

A graph $DS(T_n)$ for n > 3 admits QDL

Proof :

Let $G = DS(T_n)$ be a graph with $|v| = 2n+1$ & $|E| = 5n$-4.

Here we define the vertex labeling function $f: V \rightarrow \{0,1,2,\dots,2n\}$ by

$f(v_i) = i-1, 1 \leq i \leq n, f(x) = 2n, f(y) = 2n-1.$

The edge labeling are introduced by an procured function $f^*: E(G) \rightarrow N$ as

$f^*(v_i v_{i+1}) = v_{i+1}^4 - v_i^4$

$f^*(v_1 y) = y^4 - v_1^4$

$f^*(v_{2n-1} y) = y^4 - v_{2n-1}^4$

$f^*(v_{2i} v_{2i+1}) = v_{2i+1}^4 - v_{2i}^4$

$f^*(v_{2i+1} x) = x^4 - v_{2i+1}^4$

$f^*(v_{2i} y) = y^4 - v_{2i}^4$

$f^*(uv) = |f(u)^4 - f(v)^4|$ for every uv ϵ E(G) is injective. Hence the graph proves the theorem.

Example 3.6. $DS(T_4)$ illustrates the Quad difference labeling

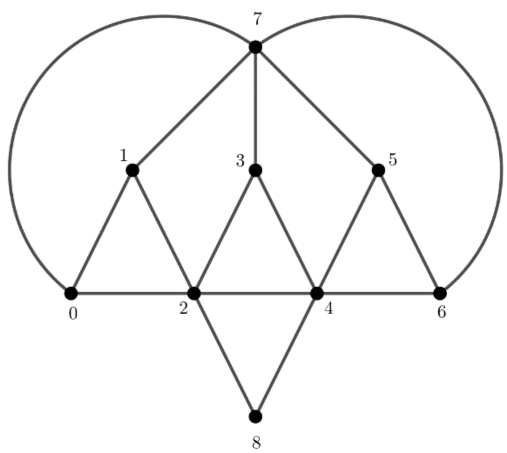

Figure 3.6

Theorem3.7

$DS(Y_{r+1})$ – tree for $r > 4$ admits QDL.

Proof:

Consider the degree splitting graph of Y_{r+1} – tree graph with number of vertices $r + 3$ and edges $2r$.

Define a labeling $f: v \to \{0,1,2, \ldots\ldots, v-1\}$ as follows

$f(x_i) = (i-1), f(w) = r, f(m) = r+1, f(n) = r+2$ for $1 \le i \le r$.

The induced for f^* on edge is defined as follows

$f^*(x_i x_{i+1}) = x_{i+1}^4 - x_i^4; 1 \le i \le r$

$f^*(w x_2) = w^4 - x_2^4$

$f^*(n x_1) = (r+2)^4$

$f^*(n x_r) = (r+2)^4 - x_r^4$

$f^*(w_n) = (r+2)^4 - r^4$

$f^*(x_{i+2} m) = (r+1)^4 - x_{i+2}^4$

Now all the edge labeling defined are distinct and hence f^* is injective. Thus proves the theorem.

Example 3.7: $DS(Y_7)$ – tree graph is given below satisfies the QD labeling.

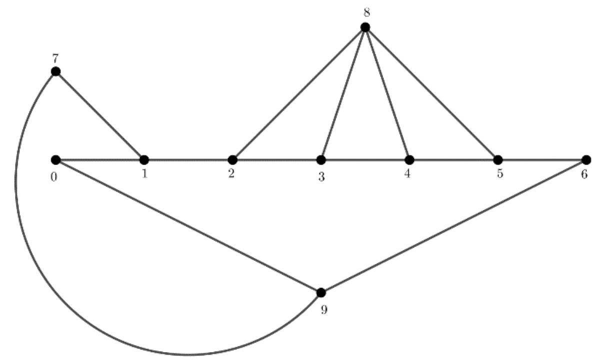

Figure 3.7

CONCLUSION

Here we have investigated the behavior of degree splitting of some graphs like $P_n \odot K_1$, C_n, $B_{n,n}$, $C_n \odot K_1$, T_n, Y_{r+1}- tree graphs are QD labeling.

REFERENCES:

[1] Gallian JA, A dynamics survey of graph labeling. The electronic journal of combinatorics 2009
[2] P. Maya, T. Nicholar, "Degree Splitting of Some I Cordial Graphs", International Journal of Research in Advent Technology, Vol.6, No.8, August 2018.
[3] Ponraj, Sathish Narayanan. S and Kala. R, 2013, "Difference Cordial Labeling of Graphs", Global Journal of Mathematical Sciences: Theory and Practical, 3, 192-201.
[4] E. Sampath Kumarand H.B. Waliker, "On Splitting graph of a graph", J. Karnatak univ. Sci, 25(1980) 13-16.