Some Results On Homomorphism Of Vague Ideal Of A Gamma-Nearing

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Abstract: In this research paper, we invent the notion of homomorphism of Vague Ideal of a Γ-Nearring and prove that the Homomorphic image and pre-Homomorphic image of a Vague Ideal of a Γ-Nearring of M is also a Vague Ideal of a Γ-Nearring of M. Also we introduce and define Vague characteristic-set and prove that vague characteristic-set of a Ideal of a Γ-Nearring is a Vague Ideal of a Γ-Nearring.

Keywords: Vaguset, Vague characteristic-set, Vague Ideal of a Γ-Nearring, vague homomorphism of a Γ- Nearring. Mathematics Subject Classification: 08A72, 20N25, 03E72

1. INTRODUCTION

The theory of fuzzy Γ-Nearrings has been introduced and developed by Bh.Satyanarayana[7] and G.L.Booth. Later D.J.Buehrer and W.L.Gau [21] introduced the theory of Vague Sets in approximating the real time situations. Actually, Vaguessets are higher order fuzzy sets. By the definition, A Vaguset P is a pair (tP,fP), where tP,fP are fuzzy subsets of universe of discourse U with tP(u)+fP(u) ≤ 1, ∀ u ∈ U . Further S.Ragamayi ([15],[17],[18],[20]) has introduced and inspected the structures of Vague Γ-Nearring and Vague Ideal of a Γ-Nearring. Earlier, Ranjit Biswas had introduced the idea of applying vague homomorphism on groups. Later S.Ragamayi has implemented the similar idea on vague Γ-Nearrings. Now, we invent the notion of homomorphism on Vague Ideal of a Γ-Nearring. In this research article, we develop a 1-1 correspondence between crisp set and vague characteristic-set of S ≠ φ. Further we prove that the homomorphic pre-image, homomorphic image of vague-ideal of a Γ-Nearring, M are vague-ideals of Γ-Nearring, M. Furthermore we show that Homomorphic-image and pre-Homomorphic-image of a Normal Vague Ideal of a Γ-Nearring, M is again Normal Vague Ideal of a Γ-Nearring, M. Also, we prove that pre-Homomorphic-image and Homomorphic-image of a Vague Ideal of a Γ-Nearring with Sup. property is also Vague Ideal of a Γ-Nearring with Sup. property.

2. PRELIMINARIES

Definition 2.1: A triple (G, +, Γ) is known to be Γ-Nearring if

i) G is group with “+”

ii) (G, +, α1) is a nearring for each α1 ∈ Γ where G is a non-empty set of binary operators on G.

iii) rα1(sβ1)t = (rα1s)β1t for all r, s, t ∈ G and α1, β1 ∈ Γ.

A Γ-Nearring is known to be Zero-Symmetric if rα10 = 0 for every r ∈ G, α1 ∈ Γ.

Definition 2.2: A Vaguset P in U as a pair (tP, fP), tP, fP : U → [0, 1] are mappings such that tP(r) + fP(r) ≤ 1, ∀ r ∈ U. The functions tP represents true membership and fP represents false membership functions respectively.

Definition 2.3: The vague value of r in P , Vf (r) is defined as an interval [tP (r), 1 − fP (r)].

Definition 2.4: We say P ⊆ Q, for two given Vaguessets if and only if Vf (r) ≤ Vf (r) i.e., tP(r) ≤ tQ(r) and 1 − fP(r) ≤ 1 − fQ(r), ∀ r ∈ U.

Definition 2.5: Two Vaguessets L and N , their union is represented by R = L ∪ N , whose false membership and truth membership functions are 1 − fR = max{1 − fL, 1 − fN} = 1 − max(fL, fN) and tR = max{tL, tN}.

Definition 2.6: Two Vaguessets L and N and their intersection is a Vaguset R = L ∩ N , whose truth membership function tR = min{tL, tN} and false membership function 1 − fR = min{1 − fL, 1 − fN} = 1 − max(fL, fN).

Definition 2.7: The intersection and union of a collection of {Di/ i ∈ A} of vaguessets of a set U are defined by

1) ∀ r ∈ U, i ∈ A

2) ∀ r ∈ U, i ∈ A

Definition 2.8: A Vaguset P = (tP, fP) of G is known to be Vf-NR if ∀ r, s ∈ G; α1 ∈ Γ,

i) Vf (r + s) ≥ min{Vf (r), Vf (s)} and

ii) Vf (rα1s) ≥ min{Vf (r), Vf (s)}

i.e.,

(i). tP (r + s) ≥ min{tP (r), tP (s)},

1 − fP (r + s) ≥ min{1 − fP (r), 1 − fP (s)} and

(ii). tP (rα1s) ≥ min{tP (r), tP (s)},

1 − fP (rα1s) ≥ min{1 − fP (r), 1 − fP (s)}.

Definition 2.9: A Vaguset P = (tP, fP) of G is known to be vague ideal( both left (resp. right)) of Γ-NR,G if ∀ k, m, a, b ∈ G; γ1 ∈ Γ,

1) Vf (k − m) ≥ min{Vf (k), Vf (m)}

2) Vf (m + k − m) ≥ Vf (k)

3) Vf (ay1(k + b) − ay1b) ≥ Vf (k)(resp. Vf (ky1a) ≥ Vf (k)) i.e.,

1) tP (k − m) ≥ min{tP (k), tP (m)}

2) tP (m + k − m) ≥ tP (k)

3) tP (ay1(k + b) − ay1b) ≥ tP (k)(resp. tP (ky1a) ≥ tP (k))
and
\[1] - f_o (k - m) \geq \min\{1 - f_o (k), 1 - f_p (m)\}
\[2] 1 - f_p (m + k - m) \geq 1 - f_p (k)
\[3] 1 - f_o (a_y (k + b) - a_y (b)) \geq 1 - f_p (k)
\]
\( (1 - f_o (k), a_y (m)) \geq 1 - f_p (k) \)

If \( P \) is both right and left Vague ideal of a \( \Gamma \)-NR, \( G \), then \( P \) is called a \( \Gamma \)-VIF-NR,G.

**Definition 2.10:** A Vaguset \( P = \{t_p, f_p\} \) of \( G \) is known to be normal, if \( f_p (0) = 1, t_p (0) = 1 \).

**Definition 2.11:** Let \( h : X \to Y \). Let \( P \) be a Vaguset on \( X \) whose vaguevalue \( V \). Then the image, \( g(P) \) of \( P \) is the Vaguset on \( Y \) described as
\[
V_{h^{-1}}(r) = \text{Sup}_{r \in h^{-1}(m)\text{ if } h^{-1}(m) \neq \emptyset}
\]
otherwise
\[
\emptyset \in Y, h^{-1}(m) = \{ k \text{ suchthat } h(k) = m \}.
\]

Let \( Q \) be a Vaguset in \( Y \). The inverse image \( h^{-1}(Q) \) of \( Q \), is the Vaguset on \( X \) by
\[
V_{h^{-1}}(r) = V_Q(h(k)) , \forall k \in X.
\]

**Definition 2.12:** A Vaguset \( P \) of \( X \) is known to had the Supr. property if forevery subset \( S \) of \( X, S, k_0 \) of \( S \) suchthat
\[
V_{D}(k_0) = \text{Sup}_{r \in V}(k) \text{ for every } k \in S.
\]

Notations: Throught out the following section, we use the following notations.
\( \Gamma \)-NR stands for \( \Gamma \)-Nearrings.
\( \Gamma \)-NR stands for Vague \( \Gamma \)-Nearrings.
\( \Gamma \)-NR stands for ordered Zero-Symmetric \( \Gamma \)-Nearrings.
\( \Gamma \)-NR stands for Sub \( \Gamma \)-Nearrings.
\( \Gamma \)-NR stands for Vague Ideal of a \( \Gamma \)-Nearrings.
\( \Gamma \)-NR stands for Normal Vague Ideal of a \( \Gamma \)-Nearrings.
\( \Gamma \)-NR stands for Ideal of \( \Gamma \)-Nearrings.

### 3. HOMOMORPHISM OF VAGUE IDEALS OF A \( \Gamma \)-NEARRING

In the following section we develop a one-one correspondence between crisp characteristic set and vague caracteristic set of subset \( S \). Later we show that inverse Homomorphic-image and Homomorphic-image of a \( \Gamma \)-VIF-NR M are \( \Gamma \)-VIF-NR M. Also, we show that pre-Homomorphic-image and Homomorphic-image of NVIF-NR M are again NVIF-NR M.

**Definition 3.1:** Let \( h : M \to N \). Let \( P \) be a Vaguset in \( M \) whose vaguevalue \( V \).

Then the image of \( P \), \( h(P) \) is the Vaguset in \( N \) defined by
\[
V_{h(P)}(m) = \text{Sup}_{r \in h^{-1}(m)\text{ if } h^{-1}(m) \neq \emptyset}
\]
\( \text{Sup}_{r \in h^{-1}(m)\text{ if } h^{-1}(m) \neq \emptyset} = \emptyset \in N, h^{-1}(m) = \{ k \text{ suchthat } h(k) = m \}.
\]

Let \( Q \) be a Vaguset on \( N \). The inverse image \( h^{-1}(Q) \) of \( Q \), is the Vaguset on \( M \) by
\[
V_{h^{-1}}(r) = V_Q(h(k)) , \forall k \in M.
\]

**Definition 3.2:** A Vaguset \( P \) of \( M \) is possessing Supr. property if for any subset \( S \) of \( M \), such that \( k_0 \text{ of } S \) suchthat
\[
V_{P}(k_0) = \text{Sup}_{r \in V}(k) \text{ where } k \in S.
\]

**Definition 3.3:** Let \( h \) be homomorphism from \( M \) into itself then
\[
h(ky, m) = h(k)g(y, m), k, m \in M; y, k \in G.
\]

**Theorem 3.4:** Let \( E \neq \emptyset \) and \( E \subseteq M \). Then \( E \) is a \( \Gamma \)-NR,M if and only if vague caracteristic-set \( \delta \) of \( E \) is a \( \Gamma \)-VIF-NR,M.

Proof. : Let \( E \) be a \( \Gamma \)-VIF-NR,M.

For each \( y \in Y \), take \( k, m \in E; a, b \in M \).
2. \( V_{\delta E}(m + k - m) \geq \min\{V_{\delta E}(k), V_{\delta E}(m)\} = [1, 1] \)
3. \( V_{\delta E}(a_y (k + b) - a_y (b)) \geq \min\{V_{\delta E}(k), V_{\delta E}(m)\} = [1, 1] \)

Hence \( E \) is a \( \Gamma \)-NR-M.

By Converse, consider \( E \) be a \( \Gamma \)-NR-M.

Take \( k, m \in E \) and \( y \in Y \).

\[
V_{\delta E}(k - m) \geq \min\{V_{\delta E}(k), V_{\delta E}(m)\} = [1, 1]
\]

Hence \( E \) is a \( \Gamma \)-NR-M.
a ∈ h−1(k) and V_P (m_0) = \text{sup} V_P (b) where b ∈ h−1(m) Then

1. \text{V}_{\text{h}^{-1}}(k - m) = \text{sup} V_P (\xi) where \xi ∈ h^{-1}(k-m)
\geq V_P (\xi), \xi ∈ h^{-1}(k - m)
= \text{VP} (k_0 + m_0)
= \min\{V_P (k_0), V_P (m_0)\}
= \text{min}\{V_{\text{h}^{-1}}(k), V_{\text{h}^{-1}}(m)\}

2. \text{V}_{\text{h}^{-1}}(m + k - m) = \text{sup} V_P (\xi) where \xi ∈ h^{-1}(m+k-m)
\geq V_P (\xi), \xi ∈ h^{-1}(m + k - m)
= V_P (m_0 + k_0 - m_0)
= V_P (k_0)(\text{resp.} \geq V_P (m_0))
= \text{V}_{\text{h}^{-1}}(k)(\text{resp.} = \text{V}_{\text{h}^{-1}}(m))

3. \text{V}_{\text{h}^{-1}}(a\beta + b - a\beta b) = \text{sup} V_P (\xi) where \xi ∈ h^{-1}(a\beta + b - a\beta b)
\geq V_P (\xi), \xi ∈ h^{-1}(a\beta + b - a\beta b)
= V_P (a_\beta k_0 + b_0 - a_\beta b_0)
= V_P (k_0)
\geq V_P (k_0)
= \text{V}_{\text{h}^{-1}}(k)

Hence h_p is a \text{VIG-NR}, N_2.

Theorem 3.7: Let N_1 and N_2 be a two \text{Γ-Nearrings}. Let 'h' be a homomorphism from N_1 onto N_2. If Q is a \text{VIG-NR}, N_2, then the pre-Homomorphic-image \Gamma^{-1}(Q) of Q is \text{VIG-NR}, N_1.

Proof: From theorem 3.5, h^{-1}(Q) is a \text{VIG-NR}, N_1.
Since Q is normal, we have
V_2(0) = \{1, 1\}, where '0' is zero element in N_2.
we have V h^{-1}(Q)(0) = V_0(h(0)) = V_0(0) = \{1, 1\}.
Thus h^{-1}(Q) is a \text{VIG-NR}, N_1.

Theorem 3.8: Let N_1 and N_2 be a two \text{Γ-Nearrings}. Let 'h' be a homomorphism from N_1 onto N_2. If P is a \text{VIG-NR}, N_1 with Supp. property, then the Homomorphic-image of P , h(P ) is a \text{VIG-NR}, N_2.

Proof: By 3.6-theorem: h(P ) is a \text{VIG-NR}, N_2.
As P is normal → h(0) = \{1, 1\}.
Since 'h' is epimorphism, \exists 0 ∈ N_2 such that f(0) = 0.
we have V h^{-1}(0) = sup V_P (\xi), where \xi ∈ h^{-1}(0)
= sup V_P (0) where \xi ∈ h^{-1}(0) = V_0(0) = \{1, 1\}.
Thus h_P is a \text{VIG-NR}, N_2.

Definition 3.9: Let P = (l_0, f_0) and Q = (l_0, f_0) be \text{VIG-NRs} of M. If \exists \phi \in \text{Aut}(M) such that V_P (k) = V_0(\phi(k)), ∀ k ∈ M, i.e., l_0(k) = l_0(\phi(k)) and f_0(k) = f_0(\phi(k)), then we say that P and Q are homologous \text{VIG-NRs} of M.
If P, Q are homologous, then Q, P are also homologous.

Theorem 3.10: Let Q = (l_0, f_0) be a \text{VIG-NR} of M and \phi ∈ \text{Aut}(M). If P = (l_0, f_0) is a Vague set of M such that V_P (k) = V_0(\phi(k)), ∀ k ∈ M, then P and Q are homologous \text{VIG-NRs} of M.

Proof: Let k, \xi ∈ \Gamma.
1. V_P (k - n) = V_0(\phi(k - n))
= V_0(\phi(k) - \phi(n))
\geq \text{min}(V_0(\phi(k)), V_0(\phi(n))}
\geq \text{V}_{\text{h}^{-1}}(\phi(k))
2. V_P (n + k - n) = V_0(\phi(n + k - n))
= V_0(\phi(n) + \phi(k) - \phi(n))
= V_0(\phi(n) + V_0(\phi(k)) - V_0(\phi(n))}
\geq V_0(\phi(k))

3. V_P (a_\theta (k + b) - a_\theta b) = V_0(\phi(a_\theta (k + b) - a_\theta b))
= V_0(\phi(a_\theta)(\phi(k) + \phi(b)) - \phi(a_\theta b))
= V_0(\phi(a_\theta)(\phi(k)) + V_0(\phi(b)) - V_0(\phi(a_\theta b))}
\geq V_0(\phi(k))

Therefore P is a \text{VIG-NR} of M.

Hence P and Q are homologous \text{VIG-NRs} of M.

Theorem 3.11: Let P = (l_0, f_0) be a \text{VIG-NR} of M. Consider h: M → M be onto homomorphism. The Vague set P^h = (l_0^h, f_0^h) defined by V_P h(k) = V_0(h(k)), ∀ k ∈ M is a \text{VIG-NR} of M.

Proof: Let k, m ∈ M; \gamma, \xi ∈ \Gamma.
1. V_P^h (k - m) = V_0(h(k - m))
= V_0(h(k) - h(m))
\geq \text{min}(V_0(h(k)), V_0(h(m))}
\geq \text{min}(V_0^h (k), V_0^h (m))
2. V_P^h (m + k - m) = V_0(h(m + k - m))
= V_0(h(m) + h(k))
\geq V_0^h (k)

3. V_P^h (a_\theta (k + b) - a_\theta b) = V_0(h(a_\theta (k + b) - a_\theta b))
= V_0(h(a_\theta)(h(k) + h(b)) - h(a_\theta b))
\geq V_0^h (k)

Hence P^h is a \text{VIG-NR} of M.

4 CONCLUSION
In this research paper, we inspected the notion of homomorphism of a Vague Ideal of a \text{Γ-Nearring} and proved that the homomorphic pre-image and image of a Vague Ideal of a \text{Γ-Nearring} of M is a \text{VIG-NR} of M. Later we introduced Vague characteristic-set and proved that vague characteristic-set of a IF^h-NR is a \text{VIG-NR}.

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