Stability Analysis Of Single Neuron System With Levy Noise

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Abstract: This article addresses the asymptotic stability of single neuron system with neutral delay and Levy noise. Sufficient conditions are derived to ensure that the considered system with Levy noise is asymptotic stable by means of the linear matrix inequality (LMI) approach together with a Lyapunov-Krasovskii functional and stochastic analysis theory. This work provides two examples of application of stability analysis in numerical formulation about the impact of Levy noise on neutral type single neuron model.

Index Terms: Stochastic differential equations, Neuron system, Neutral equations, Asymptotic stability.

1. INTRODUCTION

For several decades, stability approach of Neural Networks (NNs) have been widely examined and found a lot of fruitful applications in realizing associative memory, imaging processing, solving engineering optimization problems, pattern classification, signal processing and so on [1, 2]. This kinds of applications motivated researchers to focus the study of stability analysis of NNs. In both practice and theory, the stability problem for neural network is too strong and more significant. Every time, external disturbances reduce the practical system performances. Stochastic theory is developed to exclude the negative effects of external disturbances. Hence, stochastic systems are suitable models for various phenomena [3] and several inspiring problems for stability criteria of stochastic systems have been described in the literatures [4, 5]. At this instant, there has been a rapidly increasing research attention has been focused to the issue of noise in neuroscience [6, 7]. The noise disturbance generally arises in biological networks based on environmental uncertainties. In the occurrence of noise, the neural networks modeled will be stronger and explore many states. Based on the progress of research about noise, its positive characteristic has been broadly exposed in some fields [8, 9].

The stochastic resonance pointedly improves its information processing abilities once it applied to a single neuron model [10, 11]. This type of neuron model characterizes the activities of stochastic system which is intently associated to the natural circumstance. So, it is essential to study the stability issues of single neuron system and several results for these systems with neutral-type and time delay has received considerable attention [12, 13].

It is pointing out that, in the previous literature, there are numerous results on the stability analysis of stochastic differential equations with a Brownian motion. But, a Brownian motion cannot be utilized to explain the stochastic disturbances in various real systems. For instance, abrupt environment fluctuations, this type of systems are very problematic and their paths may not be continuous. So, stochastic systems with Levy noise are quite appropriate to explain such discontinuous systems. The theory of Levy processes established in 1930s. Levy processes are stochastic processes with independent and stationary increases and it represents the motion of a point whose successive movements are random, independent and statistically equal over diverse time intervals of similar length. Levy processes have a wider kind of applications in such different areas as stochastic control, quantum field theory and complex biological and neuronal circumstance etc [14, 15]. Park and Kwon [16] discussed the stability approach of definite nonlinear differential equation. Asymptotic stability approach of definite neutral differential equations by using descriptor system method is investigated in [17]. Nam and Phat [18] discussed an improved stability analysis for neutral differential equations. Asymptotic stability for stochastic differential equations involving Levy noise is examined in [19]. Zhu [20] studied the stability approach of stochastic delay differential equations involving Levy noise. Inspired on the above, the asymptotic stability of neutral type stochastic single neuron system is discussed. The noise term in the single neuron model is considered as Levy type noise which consist of both Poisson random measures and Brownian motion. To the authors' knowledge, there is no work reported on asymptotic stability result for a neutral type single neuron system involving Levy noise, which has motivated us for this study. So, it is valuable to analyze the influence of Levy noise on single neuron model.

2. PROBLEM DESCRIPTION

The notations used throughout this article are standard. The matrix $X \geq 0$ and $X > 0$ denote positive semi-definite and positive definite. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space with filtration $\{\mathcal{F}_t\}_{t \geq 0}$. The operator $E$ denotes the mathematical expectation.

Consider the following neutral type single neuron system with Levy noise

$$d[x(t) + px(t - \tau)] = [-ax(t) + b \tan h x(t - \sigma)]dt + g(t, x(t), x(t - \tau), x(t - \sigma))dL(t), t \geq 0.$$ (1)

Where $x(t)$ is state vector with initial condition $x(\theta) = \varphi(\theta), \theta \in [-r, 0]$. $r = \max\{\tau, \sigma\}$. $a$ and $b$ are positive real numbers and $|p| < 1$. $\tau$ and $\sigma$ are neutral and state delays.

Also,

$$L(t) = W(t) + \int |H(x(u-), z)\tilde{N}(du,dz),$$

where, $W(t)$ represents the Levy noise which consist of both Poisson random measures and Brownian motion.
Where $W(t)$ is a scalar Brownian motion on $(\Omega, \mathcal{F}, P)$ and $N$ is a Poisson random measure with compensator $\tilde{N}$.

The following assumptions are used to drive the result.

(A1) The function $g(t)$ is locally Lipschitz continuous. Also, satisfies the linear growth condition. For $\rho_i > 0$, $i = 1, 2, 3$, such that

$$g^2(t,x(t),x(t-\tau),x(t-\sigma)) \leq \rho_1 x^2(t) + \rho_2 x^2(t-\tau) + \rho_3 x^2(t-\sigma).$$

(A2) For $k > 0$ such that

$$\int |H(x,z)|v(dz) \leq k|x|.$$  

After rewriting, system (1) becomes the following descriptor form

$$dy(t) = [-ax(t) + b \tanh x(t-\sigma)]dt + g(t,x(t),x(t-\tau),x(t-\sigma))dL(t),$$

$$0 = -y(t) + x(t) + px(t-\tau).$$

Let $f(t) = -ax(t) + b \tanh x(t-\sigma)$ and $g(t) = g(t,x(t),x(t-\tau),x(t-\sigma)).$

So, (2) becomes

$$dy(t) = f(t)dt + g(t)dL(t),$$

$$0 = -y(t) + x(t) + px(t-\tau).$$

(3)

### 3 Main Results

Theorem 3.1. For given positive constants $\rho_1, \rho_2, \rho_3$ and $k$, (1) is asymptotically stable, if there exist symmetric positive scalars $q_i, i = 1, 2, \ldots, 7$ and real numbers $m, n,$ then the following LMI holds:

$$\Pi = \begin{bmatrix} -2q_2 & \phi_1 & \phi_2 & q_3 & q_4 & q_5 & q_6 & q_7 & 0 & 0 & 0 \\ \phi_1 & \phi_2 & q_3 & q_4 & q_5 & q_6 & q_7 & 0 & 0 & 0 & 0 \\ \phi_2 & q_3 & q_4 & q_5 & q_6 & q_7 & 0 & 0 & 0 & 0 & 0 \\ \phi_3 & q_4 & q_5 & q_6 & q_7 & 0 & 0 & 0 & 0 & 0 & 0 \\ \phi_4 & q_5 & q_6 & q_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \phi_5 & q_6 & q_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \phi_6 & q_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \phi_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0,$$

where $\phi_1 = q_1 \rho_2 - q_3$ and

$$\phi_2 = 2q_1 + q_1 \rho_2 + q_4 + \tau^2 q_5 + q_6 + q_7 - 2ma.$$  

Proof. Construct the following Lyapunov-Krasovskii function (LKF) for system (1)

$$V(t,x(t)) = \sum_{i=1}^{4} V_i(t,x(t))$$

Here

$$V_1(t,x(t)) = \left[ y(t) \ x(t) \right] \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} q_1 & 0 \\ q_2 & q_3 \end{bmatrix} \begin{bmatrix} y(t) \\ x(t) \end{bmatrix}.$$  

$$V_2(t,x(t)) = \int_{t-\tau}^{t} q_4 x^2(s)ds$$

$$V_3(t,x(t)) = \tau \int_{t-\tau}^{t} q_5 x^2(\theta)d\theta$$

$$V_4(t,x(t)) = q_6 \int_{t-\sigma}^{t} \tanh^2 x(s)ds + q_7 \int_{t-\sigma}^{t} x^2(s)ds$$

Now, define the following operator

$$\Phi V(t,x(t)) = V_1(t,x(t)) + V_2(t,x(t))f(x(t))$$

$$+ \frac{1}{2} \left[ g^T(x(t))V_3(t,x(t))g(x(t)) \right]$$

$$+ \int [V(x(t) + H(x(t),y)) - v(t,x(t)) \left. \right|_{|x|<c}$$

$$- H(x(t),y)V_4(t,x(t)))v(dy)$$

Now, we shall compute the following value by using assumptions

$$E[\Phi V(t,x(t))] = E[2q_1[-ax(t)y(t) + by(t)\tanh x(t-\sigma)]]$$

$$+ 2q_2[-y^2(t) + x(t)y(t) + py(t)x(t-\tau)]$$

$$+ 2q_3[-x(t)y(t) + x^2(t) + px(t)x(t-\tau)]$$

$$+ q_4[\rho_1 x^2(t) + \rho_2 x^2(t-\tau) + \rho_3 x^2(t-\sigma)]$$

$$+ q_5 k x^2(t)$$

and also

$$E[\Phi V_2(t,x(t))] = E[q_4 x^2(t) - q_5 x^2(t-\tau)]$$

$$E[\Phi V_3(t,x(t))] = E[\tau q_5 x^2(t) - q_5 \int_{t-\tau}^{t} x^2(s)ds]$$

Where

$$E[\Phi V_4(t,x(t))] = E[q_6 \tanh^2 x(t) - q_6 \tanh^2 x(t-\sigma)]$$

Now by combining the above equations (6)-(9), we obtain

$$E[\Phi V(t,x(t))] = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} q_1 & 0 \\ q_2 & q_3 \end{bmatrix} \begin{bmatrix} y(t) \\ x(t) \end{bmatrix}$$

with $\phi_1 = q_1 \rho_2 - q_3$ and

$$\phi_2 = 2q_1 + q_1 \rho_2 + q_4 + \tau^2 q_5 + q_6 + q_7 - 2ma.$$
\begin{align*}
\mathbb{E}[\Phi V(t, x(t))] \\
\leq \mathbb{E}[y^2(t)[-2q_2] + x^2(t)[2q_5 + q_1\rho_1 + q_1k + q_4] \\
+ \tau^2 q_5 + q_6 + q_7] + x^2(t - \tau)[q_1\rho_2 - q_4] \\
+ \tanh^2 x(t - \sigma)\{-q_6 + x^2(t - \sigma)\[q_1\rho_3 - q_7\] \\
+ \int_{t-\tau}^t x^2(s)ds \{-\tau q_5\} + y(t)x(t)[-2aq_1 + 2q_2 - 2q_3] \\
+ y(t)x(t - \tau)2q_3p + y(t)\tanh x(t - \sigma)2q_1b \\
+ x(t)x(t - \tau)2q_3p
\end{align*}

On the other hand, we have for any real numbers $m$ and $n$,

\[ [x(t) f(t)]^T [\begin{pmatrix}m & n \end{pmatrix}] [-ax(t) + b \tanh x(t - \sigma) - f(t)] = 0 \]

By simple computation

\[ \mathbb{E}[\Phi V(t, x(t))] \leq \mathbb{E}[\xi^T(t)\Pi\xi(t)] \]

where

\[ \xi(t) = [y(t) x(t) x(t - \tau) \tanh x(t - \sigma) x(t - \sigma)] \]

\[ \int_{t-\tau}^t x(s)ds \ f(t)^T \]

Therefore, if $\Pi < 0$ holds, we have

\[ \mathbb{E}[\Phi V(t, x(t))] < 0. \]

From Lyapunov-Krasovskii theory which indicates that the equilibrium point of the system is asymptotically stable.

4 NUMERICAL EXAMPLES

**Example 1.** In stochastic neutral system with Levy noise (1), we choose the following parameters

\[ p = 0.3, \ a = 1.5, \ b = 0.4, \ k = 0.1, \]

and also

\[ g(t) = 0.2x(t) + 0.2x(t - \tau) + 0.1x(t - \sigma) \]

The feasible solution of LMI (4) can be obtained by using the LMI toolbox in Matlab for large value of $\tau$ up to $10^{11}$. Thus, it follows from Theorem (3.1) that the system (1) is asymptotically stable.

**Example 2.** In the stochastic neutral system with Levy noise (1), we choose the following parameters

\[ p = 0.6, \ a = 0.6, \ \tau = 0.5, \ k = 0.2, \]

and also

\[ g(t) = 0.2x(t) + 0.2x(t - \tau) + 0.1x(t - \sigma) \]

The feasible solution of LMI (4) can be obtained by using the LMI toolbox in Matlab for maximum value of $b = 0.599$.

5 CONCLUSION

In this paper, we examined the asymptotic stability procedure for a neutral type single neuron system involving Levy noise based on the LMI approach together with a stochastic analysis theory and Lyapunov-Krasovskii functional. The efficiency of derived LMI-based conditions have been demonstrated by numerical examples.

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REFERENCES


