Enhancement Of Optimal Dispatch By Latest Technological Advancements In Modern Power Systems

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Abstract: One of the fundamental issues in power systems operation is the need to achieve a balance between the most economical mean of operation and the highest level of reliability. Great achievement have been realized so far on the economic dispatch in conventional power, but the fast development of smart grid applications, distributed generations, renewable energy and the enhancement of co-generation power plants have brought new challenges in the field to run the system more economically, with options to reduce the effect on the environment as well. This paper discusses the recently proposed optimization control schemes in power systems, after it provides discussion on the optimal dispatch in general. Also, the paper analyzes the new approaches in power system generation that could reduce the total cost of both generation and operation of the power in the system. Co-generation principles will be presented to evaluate potential contribution to the optimal dispatch. In addition, the paper goes through the principles of the distributed generation and the latest research on this field to examine the effect of defining better locations to install the distributed generatorsto reach the desired economic dispatch. Finally, a discussion follows to assess the ability of wind energy, the most developed source of renewables, in making critical decisions in reducing the total power generation. The paper concludes with a conclusion evaluating the use of the mentioned resources as an alternative way for a better, more reliable and economic power system.

Keywords: Optimal dispatch, Distributed generation, nonlinear optimization, constrained parameter optimization, co-generation power, wind power, economic dispatch, thermal power plant optimization.

1. Introduction to Optimal Power Flow (OPF)

One of the main challenges in power systems is to find real and reactive power scheduling for each power plant in a way that provides the most economic operating cost of an interconnected grid. This is called as Optimal Dispatch (OD), or usually Optimal Power Flow (OPF). While some have tried to distinguish the concept of economic dispatch from OPF concepts, they all agree that both of the principles serve one goal which is run the power system in the most economic and reliable way. The optimal power flow could be achieved by taking into account minimizing the selected objective functions while maintaining an acceptable system performance in terms of generator capability limits and the output of the compensating devices.

1.1 Nonlinear Function Optimization

Unconstrained Parameter Optimization

The nonlinear function optimization is an important tool used in the computational methods for minimizing the overall cost of generation. Its basic goal is to reduce some nonlinear objective cost functions subjected to nonlinear equality and inequality constraints. The main idea is setting the partial derivatives of an unconstrained minimum of a function to be equal to zero and solving for the parameters values, with respect to the parameters that may vary. If there is a single local minimum, then there would also be a global minimum. The work would then be the evaluation of the cost function of each minimum to obtain the most suitable global minimum (1).

Constrained Parameter Optimization

Equality Constraints

The functions that have dependencies among their parameters are called constrained parameters. The problem is to minimize the cost function

\[ f(x_1, x_2, \ldots, x_n) \]

Subject to equality constraints

\[ g_i(x_1, x_2, \ldots, x_n) = 0 \quad i = 1, 2, \ldots k \]

The Lagrange Multiplier method is used to solve such functions. This would provide an augmented cost function by introducing k-vector \( \lambda \) of undetermined quantities. Thus, the unconstrained cost functions become

\[ L = f + \sum_{i=1}^{k} \lambda_i g_i \]

The constrained local minima of \( L \) are the following

\[ \frac{dL}{dx_i} = \frac{df}{dx_i} + \sum_{i=1}^{k} \lambda_i \frac{dg_i}{dx_i} = 0 \]

\[ \frac{dL}{d\lambda_i} = g_i = 0 \]

Inequality Constraints

In practical real-life problems, inequality constraints are included in solving the optimal power flow for a plant. The goal is to minimize the cost function

\[ f(x_1, x_2, \ldots, x_n) \]

Subject to equality constraints

\[ g_i(x_1, x_2, \ldots, x_n) = 0 \quad i = 1, 2, \ldots k \]

and to the inequality constraints

\[ U_i(x_1, x_2, \ldots, x_n) \leq 0 \quad i = 1, 2, \ldots m \]
The inequality constraints could be added to Lagrange multiplier by introducing the m-vector \( \mu \) of undetermined quantities. Thus, the unconstrained cost function becomes:

\[
L = f + \sum_{i=1}^{k} \lambda_i g_i + \sum_{j=1}^{m} \mu_j U_j
\]

The constrained local minima of \( L \) could be found by the following:

\[
\frac{dL}{dx_i} = 0 \quad i = 1, \ldots, n
\]

\[
\frac{dL}{d\lambda_i} = g_i = 0 \quad i = 1, \ldots, k
\]

\[
\frac{dL}{d\mu_j} = U_j \leq 0 \quad j = 1, \ldots, m
\]

\[
\mu_j U_j = 0 \quad \mu_j > 0 \quad j = 1, \ldots, m
\]

2. Economic Dispatch in Power Grids

There are many approaches that calculate the economic dispatch of a power system. Some of them neglect transmission losses; especially if the transmission distances are very small; however, in a large interconnected network where power is transmitted over long distance with low load density areas, transmission losses are considered a major factor and ultimately affect the optimum dispatch of generation. Thus, considering losses are more related to real-life power operation. The losses could be found by the Kron’s loss formula as follow:

\[
PL = \sum_{i=1}^{n} Pi \cdot B_{ij} \cdot Pj + \sum_{i=1}^{n} Boi + Pi + Boo
\]

where \( Pi \) is the real power at the ith bus, \( B_{ij} \) is the nxn matrix of quadratic loss coefficients, \( Boi \) is the dimensionless vector of linear “loss coefficient and Boo is the constant loss coefficient. The coefficients \( B \) are called the “loss coefficients”, or simply “B-coefficients” which are always assumed to be constant. The optimal dispatch of a power system is to minimize the overall generating cost \( C_i \), which is the function of the plant output

\[
Ct = \sum_{i=1}^{ng} C_i
\]

\[
= \sum_{i=1}^{n} a_i + \beta Pgi + \gamma Pgi^2
\]

Subject to the famous power system equation, where the generation must meet the load plus the losses in the network

\[
\sum_{i=1}^{ng} Pi = Pd + PL
\]

Satisfying the inequality constraints

\[
Pi_{min} \leq Pi(t) \leq Pi_{max}
\]

where \( Pi_{min} \) and \( Pi_{max} \) are the minimum and maximum generating limits of generating unit i. It should be mentioned that the loss formula is expressed as

\[
PL = \sum_{i=1}^{ng} Boi + Pi^2
\]

The following equation has been reached which is used to find the optimum cost

\[
Pi^k = (\lambda^k - \beta i)/2(\gamma i + \lambda^k Boi)
\]

3. Thermal (Conventional) Plants

3.1 Operating Cost of a Thermal Plant

To achieve the economic dispatch of a conventional-generation system, it is necessary to take into consideration three major influencing factors: transmission losses, fuel cost, and operating efficiencies of the system’s generators (1). The criterion usually used is to place the generator with the highest efficiency in an area where the fuel cost is very high. In addition, the distance between the plant and the load center must be carefully studied to ensure there are no high transmission losses as well. Thus, it is better to determine the generation of different plants in order to have the most economic operating cost in real-life operation, not only in theory.

The input of any thermal plant, derived by gas, oil or coal, is measured in Btu/h, while the output is always measured in MW. The heat rate of the fuel is what is giving away the amount of power produced. Converting this heat value from Btu/h to $/h leads to the fuel-cost curve shown in Fig. 1. The fuel cost of generator i can be represented as a quadratic function of real power generation

\[
Ci (Pgi) = ai + \beta Pgi + \gamma Pgi^2 \quad $/hr (fuel cost)
\]

The derivation of the previous equation in terms of real power results in the incremental fuel-cost, given by the following equation, and shown in Figure 2

\[
ICi (Pgi) = \frac{dCi (Pgi)}{dPgi} = \beta + 2 \gamma Pgi \quad $/MWh
\]
3.2 Control Scheme for Thermal Power Plant Optimization
Fei (2011) proposed new power plant control architecture based on plantwide control where main job is to achieve the economic distribution of the power units while maintaining the voltage quality in the system. It adopts the integrated idea, which makes the acquisition of information and the execution of commands more fluent and achieves the active power's economic dispatch and the reactive power's optimization distribution as well. In order to achieve and improve plantwide control and optimization for the thermal power plant, the proposed control scheme would use another approach in the regular dispatch mode used in the power plants. This approach is the regular order signals sent to the generation units to change its generation level in order to have a more decision-making in dispatching the total active power order and voltage target, which are received from the dispatch center of the grid. To enhance the power supply voltage level and to improve the quality of the regional power grid, a voltage and reactive power control system (AVC) is considered in the control scheme in order to receive the bus voltage target orders transferred through the central scheduling in real-time. AVC would combine the local bus voltage measures in a way that allows the generation units to dispatch the rational allocation of reactive power to the appropriate units in a fast manner. It would also provide accurate tracking of the voltage target, which would improve the bus voltage levels in the grid. Figure 3 shows the configuration of the thermal power plant. The proposed plant wide control scheme could be separated into four control levels by the way of top-down analysis. The first (upper) level of the system is the dispatching control level, which belongs to the power grid cooperation. It also includes the installed automatic generation control software and the automatic voltage control software which is used to achieve automatically the needed adjustment of the active power and reactive power for the whole power grid. The second level is the Electric Network Control System (NCS) which is a supervisory control level responsible for receiving the active power and the bus voltage orders from the first control level, while acquiring the thermal power units' data for optimization and control purposes.

![Diagram](image-url)

Figure 3: Integrated optimization of power plant control structure

While the third level contains both the plant-level optimal load distribution and the voltage and reactive power control systems, the fourth level is the regular control level, which consists of the unit coordinate control system and the excitation system, where main job is to enhance the control accuracy and security of the AVC. The research of the plant-level optimal load distribution system is based on the coal consumption curve of each unit. It is very important to have the coal consumption curve updated regularly, since the accuracy of the curve will affect both the optimal result of the distribution, and the overall units operation. The formula used to calculate the standard coal consumption curve is:

\[
\begin{align*}
  b_i & = \frac{q_i}{Q_0 \times n_i} \\
  B_i & = p_i \times b_i = \frac{p_i \times q_i}{Q_0 \times n_i}
\end{align*}
\]

where \( q_i \) is the heat consumption rate of the power generating unit; \( Q_0 \) is the low temperature fever heat of the standard coal; \( n_i \) is the boiler efficiency. It is very important to send the real burning coal quantity of each power unit to NCS. In the same time, the environmental protection data listed are taken into account so that the plant-level optimal distribution system can formulate better and more accurate unit distribution policy (2).

3.3 Implementation of Co-generation Power System
The term Co-generation refers to the plant that produces, besides electric power, a considerable amount of heat that could be exploited in specific purposes to insure more revenue. Mostly there are three different types of cogeneration plants: the conventional plant that consists of...
a combustion chamber boiler to run steam turbines; the second configuration which consists of a gas turbine and a heat recovery steam generator that is used to extract heat from the turbine to produce steam for commercial usage; and the configuration which is famously known as a "combined cycle plant". The latest type benefits from the heat generated from the gas turbine to pass it through a heat recovery system in order to produce a sufficient amount of steam which could be effectively used to increase the generated power (3). This has helped in increasing the efficiency of the steam-based thermal plant to be around 50%.

Figure 4: Combined cycle unit scheme

The combined cycle (CC) units represent the majority of the new thermal plant installations around the world (4). This type of plant in the utility industry has been gaining in significance lately. For cogeneration units, the heat production capacities depend mainly on the power generation and vice versa (5). The heat-power mutual dependencies of the cogeneration units introduce a complication in their integration into the power system economic dispatch. Specifically, the combined heat and power economic dispatch problem could be viewed as a composition of the heat dispatch and the power dispatch sub-problems which are connected together via heat-power feasible region constraints of cogeneration units, shown in Figure 5. These constraints are mathematically described by Lagrangian function by multipliers that can be defined and interpreted. The multipliers influence the solution process by changing the incremental costs of the sub-problems, which would lead to an optimal power production (5).

Figure 5 shows the heat-power feasible operation region of a combined cycle cogeneration unit. This feasible region is enclosed by boundary curve ABCDEF. As the heat capacity increases, the power generation would decrease. The use of the cogeneration system in reducing the cost of generation can be done by determining the unit heat and production so that the power generation cost is reduced while the heat and power demand and other constraints are met. Mathematically, this could be done by the following function

\[
\sum_{i=1}^{np} Ci(P_i) + \sum_{j=1}^{nc} Ci(h_i, P_j) + \sum_{k=1}^{nh} Ck(h_k)
\]

Subject to

\[
\begin{align*}
Pi_{\text{min}} &\leq P_i & i = 1, \ldots, np \\
Pi_{\text{min}} (h_j) &\leq P_j & j = 1, \ldots, nc \\
hj_{\text{min}} (P_j) &\leq h_j & j = 1, \ldots, nc \\
hk_{\text{min}} &\leq h_k & k = 1, \ldots, nh
\end{align*}
\]

c is the unit production cost; 
\(P_i\) is the unit power generation; 
\(h_i\) is the unit heat production; 
\(h_j\) and \(P_j\) are the system heat and power demands; 
\(i\), \(j\), \(k\) are the indices of conventional power units, CO 
\(np\), \(nc\) and \(n_h\) are the numbers of generation units and heat-only units respectively; mentioned above; 
\(P_{\text{min}}\) and \(P_{\text{max}}\) are the unit power capacity limits. 
\(h_{\text{min}}\) and \(h_{\text{max}}\) are the unit heat capacity limits.

The regular process of the technique involves adjusting \(h\) iteratively until the power demand is met. The initial value of \(h\) should be large so that all units operate at maximum power capacities. After that, the unit with the highest power incremental cost would be selected and adjusted to reduce its power generation. The reduction continues until the demand is met. Details of the algorithm are described in following steps:

1. Initialize the unit power generations to their maximum capacities;
2. Calculate the unit heat capacity limits of cogeneration units by looking up their heat-power feasible regions;
3. Solve the heat dispatch in the following equations
\[
\sum_{j=1}^{nc} C_i(h_i, P_i) + \sum_{k=1}^{nh} C_k(h_k)
\]

\[
\sum_{j=1}^{nc} h_j + \sum_{k=1}^{nh} h_k = hd
\]

\[
h_{\text{min}}(P_j) \leq h_j \leq h_{\text{max}}(P_j), \quad j = 1, \ldots, nc
\]

\[
h_{\text{min}} \leq h_k \leq h_{\text{max}}, \quad k = 1, \ldots, nh
\]

Obtain the system heat marginal cost \( \lambda_h \); set \( \lambda_h \) to 0 or a sufficiently large number if deficit or surplus of heat production occurs;

4. If the total power generation meets the power demand, exit else continue;

5. Calculate the unit power incremental costs using the following equations for cogeneration units

\[
\Delta P = \frac{dC_j(h_j, P_i)}{P_i}
\]

and using \( \frac{dC_j(P_i)}{dP_i} \) for conventional power units;

6. Select the unit with the highest power incremental cost but its power generation within its power limits and reduce its power production by \( \Delta P \);

7. Go back to Step 2.

4. Distributed Generation

4.1 The Technological advancement of Distributed Generation

Distributed generation (DG), which also known as "decentralized" or "on-site generation", is the idea of having the electricity generated locally from small energy resources, such as small wind farm or solar system, diesel generators, fuel cells, micro combined heat and power …etc. This kind of generation can sometimes provide more economic solutions to load growth than building new substations and lines. Furthermore, a distributed generator could contribute effectively in eliminating a cable overload, or enhance the low voltage in the system due to the load growth, and sometimes, it is considered a very economical solution to be an alternative for underground cables (6). In addition, distributed generators increase the reliability, efficiency, and capacity of a distribution circuit, not to mention it is counted as an environmental-friendly option. All these features have made DG a new hot topic in power system, and thus further studies and research are being conducted nowadays to make DG plays important rules in power system in the near future. One of these studies that have been conducting is to define the optimal locations and sizes of DG so that it could contribute to the whole economic flow of the grid, and lately optimal power flow (OPF) is successfully applied in identifying the best network location for DG (7). Harrison and Wallace (2008) used OPF to maximize the capacity of DG and identify available headroom modeling fixed-power factor DG as negative load, but also presents a method combining OPF and genetic algorithm (GA) together to find the best available sites within a distribution network for connecting the distributed generators required for reaching the targeted optimal power flow. It is also very important to take into account the economic or technical indices as objective functions, whereas the trade-off between incentives for DG developers and distribution network operator (DNO) is defined. Yet, there are some up-now facts that the DG connection to distribution network may reduce the utilization of the existing network and bring a great deal of cost sunk. The economic potential for DG connecting to distribution network has not been evaluated. These planning models only concentrate on the economic benefits of either DNO or the developer, instead of the improvement of the social energy efficiency, which might cause problems due to the fact that this is against the principle of integrated resource planning at present. It should be noted that it is well known that DG could diminished (or delay) the investment cost of electrical network (7).

4.2 Control of DG for Optimal flow

One of the most feature of having DG used is that the utility customer can participate in the operation by installing DG to use it during the power outages or in the times when energy prices go up in the day during peak hours. Moreover, the customer will not need DG unless for a small fraction of the year; thus, the customer could provide the utility effectively if his/her DG equipment is correctly equipped. This requires both sides to install an agreement for a proper mutual benefits occurrence, and some utilities have started already to put such agreements into place (6).

Field Control Experience of DG

After reaching a decision in defining the best location for installing the DG, the control of this DG into the grid must be controlled. The control of DG includes both manual and automatic operation in order to reach the economic dispatch efficiently, as following:

1. **Manual Dispatch**: where operators are responsible for manually dispatching the DG. Dispatch decisions are made according to ambient temperature or circuit loading.

2. **Manual Dispatch Based on Model Recommendation**: this is a model-based control algorithm that makes dispatch suggestions. The algorithm may use load forecasting, ambient temperature, as well as violation criteria such as low customer level voltage to decide the dispatch amount. Aggregated model-based control can also suggest generation dispatch for independent System Operator (ISO) usage.

3. **Database Driven Automatic Dispatch**: generation is automatically dispatched when circuit loading reaches a predetermined level.

4. **Model-Based Automatic Dispatch**: which is a model-based control algorithm automatically
dispatches generation among the connected sources.

![Diagram](image)

**Figure 6:** A hierarchal, model-based, aggregated DG control scheme

Aggregated, hierarchal model-based control can tie together DG from around the system and dispatch them as a block of generation. Figure 6 describes a model-based, real-time DG control strategy, while Figure 7 shows a real-time, hierarchal distribution control scheme.

![Diagram](image)

**Figure 7:** A hierarchal, model-based, distribution control scheme

### 4.3 DG Connection Cost

Since the focus in this paper is to enhance the economic condition of running the power system, it is good to include a brief summary of a DG connection cost. In general, the DG connection cost is totally composed of the DG investment and operation, and the benefits from the DG energy source. The initial cost of installing a DG could be discounted from the annual capital cost. The DG connection cost per unit DG capacity in a year can be expressed in the following equation:

\[
C_g = f_g x CRF + (V_g - V_b - U_g)x 8760 CF
\]

Where:

- \( C_g \): the connection cost per unit DG capacity in a year.
- \( f_g \): fixed cost of DG per kW.

- \( V_g \): variable operating cost of DG per kWh.
- \( V_b \): price of distribution network operation (DNO) purchasing power from the main grid.
- \( U_g \): the governmental subsidies on energy-saving and environmental protection policies.
- 8760: the number of hours per year.
- \( CF \): the capacity factor of the DG, which is the total production of a DG to its potential production if operated constantly at full capacity.

### 5. Wind Turbines

#### 5.1 Wind Power Generation as a Modern Solution for Optimal Power Flow

Among all Renewable Energy categorizes, wind energy is considered as the most developed renewable, and the most-likely to make contribution in the next decade in producing power (8). With the rapid increase of fuel prices and environmental awareness due to thermal power generation, generating power from renewables have become a must-target in most of the countries, and laws and regulations have beenissued in this manner. Typically a fuel-free source, the integration of wind energy into power system would lead to a more economic and optimal power system flow. However, wind power is always considered as an issue in the power systems operation, due to its variability and hard predictability. Power system operators are not in favor in the inclusion of wind farms into OPF due to the detrimental effects on the power quality of the grid which would result from adding fluctuating and largely undispatchable power source like the wind energy (9). If the wind power generation is included, then the generation cost would be neglected since there is no cost for fuel in this case (free wind). After completing the interrelation between the wind turbines and thermal plants in the generation process, the total cost of wind power plant will be added to the total cost, leading to a much minimized generation cost and to more economic power flow (10).

Roy (2012) suggests that it is possible to aggregate the effect of short duration variations of wind turbines on the total power output. Moreover, Roy affirms the possibility to correct the output power of Wind Energy Conversion Systems (WECS) prior to its inclusion in the conventional economic dispatch of the system. There are many reasons that led to Roy’s conclusion; the first is that the conventional units are not entirely free of minor dynamics problems. The second is that the load itself is always changeable due to consumers need, and that the attempts to meet the load are similar to aggregates rather than precisely instantaneous values. Short durations wind variations primarily include two types of aerodynamic non-idealities; turbulence and gusts. Turbulence includes random fluctuations based on short-duration-stable mean value \( u \), while gusts related with identifying operational issues of wind turbines such as amplitude, rise time, peak, and lapse time. As the loss coefficients have been used in the conventional (thermal) economic dispatch to model aggregate systems’ losses as function of the MW generation levels, Roy believes the same concept could be extended to systems with significant wind power output if only an acceptable range of aggregation of WECS output power is used. If this is not the case (in the presence of long duration wind variations); then
the overall energy loss must be evaluated across time horizons (11).

5.2 The Problem Formulation Including Wind Turbines
The economic dispatch of a power system incorporating wind power plant involves the allocation of generation among wind and thermal plants so it can minimize the total generation costs while satisfying various constraints. As mentioned earlier, the generation cost via wind turbines is ignored in the optimization process since there is no fossil fuel cost in the process. The objective of the economic dispatch in this case is to minimize the total generation cost within a defined interval (i.e. 1 hour) while satisfying the other various constraints. The economic dispatch problem including wind turbines could be expressed as

Minimize \( Ft = \sum_{i=1}^{n} F_i (P_i) \)

where \( n \) is the total number of wind turbines units, \( Ft \) is the total generation cost, and \( F_i \) is the power generation cost of the \( i \)th wind unit. As mentioned earlier in this paper, the cost of the thermal generation can be expressed as a second order polynomial function:

\[ C_i (P_i) = \alpha i + \beta P_i + \gamma P_i^2 \]

The following constraints are also considered in this case

- **Power Loss Constraints**

\[ P_l = \sum_{i=1}^{n} P_i B_{ij} P_j + \sum_{i=1}^{n} B_{oi} + P_i + Boo \]

The illustration of these variables has been dissected earlier in this paper, in the economic dispatch section.

- **Unit Capacity Constraint:**

\[ P_i \text{min} \leq P_i (t) \leq P_i \text{max} \]

where \( P_i(t) \) is the present output power, \( P_i(\text{min}) \) and \( P_i(\text{max}) \) are both the minimum and maximum power outputs of the \( i \)th wind turbine, respectively.

6. Conclusion
Establishing the optimal flow in the system is not an easy task; it took tens of studies and research in order to reach a more economic power flow. Recently, the hope of obtaining more optimal dispatch have been the concern of many researchers and scientists, due to the technological advancement in power system generation approaches that include many option that would affect the pricing the operators are paying for generating, transmitting and distributing power. After a brief introduction on power dispatch concept, with illustrations and derivation of formulas used in this manner, the paper concentrated on the new approaches that are currently being considered in reducing the total cost of the system. New control schemes have been proposed in order to make the acquired information and executed commends more fluent, which lead to a more economic dispatch of the grid. Also, the concept of using combined heat and power in minimizing the total cost has been introduced in this paper. Cogeneration is currently considered the best option in power systems to generate power with more efficient output. After that, the paper illustrated the approaches that distributed generators can play in improving the reliability and capacity of distribution. However, it is too early to call a decision whether distributed generators can be competitive with fossil fuel in insuring the availability of power for consumers, since some suggested it not clear whether it is economically feasible to install DGs in such manner. Lastly, the advancement of wind turbine operation capability has made this type of renewable energy a very viable one, especially that some researches have proposed ideas in overcoming the variability of short duration winds and thus have the wind power as a valuable, free energy source. In conclusion, it is clear that the latest technologies in power generation are more welcomed more than ever to participate more effectively in the generation process to reach a more economic generation scheduling, in an era not only the economics are important, but also the preserving of our nature as well.

Appendix A
Defining the Best Location and Sizing of DG for a Better Economic Dispatch
Empowering and encouraging the development of DG at suitable sites would have a great impact in having high economic results. Thus, many research and studies have been conducted to relocate the best places to install the DGs. One of these studies have reached to a heuristic approach to identify the optimal location and sizing of DG, illustrated in the following steps:

**Step 1:** Construct the set of candidate nodes for DG, \( S_g \), and set the incremental DG capacity, \( \Delta kW \);

**Step 2:** Calculate the initial incremental cost of all branches according to expression;

**Step 3:** Calculate the deviation of the total capacity cost resulting from the injection of \( \Delta kW \) DG into each bus in \( S_g \). The candidate bus with the highest deviation would be selected.

**Step 4:** Check if \( \Delta C_{mw,km} \) is positive. If so, continue; otherwise, stop;

**Step 5:** Check if the constraint of the power flow direction is satisfied when adding \( \Delta kW \) DG into bus km. If so, installed \( \Delta kW \) DG into bus km; otherwise, remove node km from \( S_g \) and turn step 7;

**Step 6:** Update the incremental cost of branches upstream of bus km considering the effect of DG expanding capacity and turn step 3;

**Step 7:** Check if \( S_g \) is empty. If so, stop. Otherwise, turn step 3.

Where \( \Delta C_{mw,km} \) is the deviation of the total capacity cost resulting from the injection of \( \Delta kW \) DG into bus k at the tth time; \( S_g \) is the set of candidates nodes (location) for
installing the DG. Figure 8 shows the flowchart of the identification process of the best location for DGs.

**Objective**
- Calculate the optimal power flow for the system.
- Find the optimal generation for each unit.
- Compare the cost of the generation with the existing system.

**The work**
The main idea of the optimal power flow study is that the output of any generator should not exceed its rating nor should it be below that necessary for stable boiler operation. Thus, the generators are restricted to lie within given minimum and maximum limits. Usually in optimal power flow, the system transmission line losses are neglected, and the total demand $P_d$ should equal the sum of all the real power generation. The equation that used in this manner is the following:

$$C_{total} = \sum_{i=1}^{N_g} C_i = A_i + B_i P_{gi} + C_i (P_{gi})^2 + D_i (P_{gi})^3 + (F_c);$$

whereas

- $A_i$: Fuel cost dependent value, Mbtu/hr.
- $B_i, C_i, D_i$: Cubic cost model, Mbtu/hr.
- $P_{gi}$: The real power generation.
- $F_c$: Unit fuel cost, $/Mbtu.$

In this set of problem, and since we have unknown type of fuel in this existing system, I will assume $F_c$ to be equal to one, for more simplicity. Figure 1 shows the existing system of seven buses with five generators attached to them. The operating cost is nearly 7700 $/hr, with a total load of 760 Mw connected to this network. The optimal power flow calculation should meet this load with the most economic cost, which is expected to be around this given cost.

**Step 1:** The first step is retrieving the constants of the generating unit in the system in order to apply them in the equation. As we did in one of the earlier sets, we can do this by two ways: either by the generator dialog information, or by simply choosing case information → network → generator/load cost model → generator cubic cost models. Figure 10 shows the values of the models constants.

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**Appendix B**

**PowerWorld Optimal Power Flow Calculations Introduction**

Optimal power flow study is an important tool involving numerical analysis applied to the electrical network. This tool is very critical in planning future expansions as well as in determining the best operation of the existing system. This set of problems tries to implement the optimal power flow studies in order to determine the economic dispatch of the system via optimal power flow study. The OPF should lead us to the most suitable amount of generation for each generating unit in the system to reach the optimal flow of the system.
Figure 10: the models values of the generators

Step 2: Start the calculation by inserting the models values into the fuel-cost functions $C_i$. Since we have five generators, we would have five equations:

- $C_1 = 373.50 + 7.62 P_1 + 0.0013 P_1^2$
- $C_2 = 403.61 + 7.519 P_2 + 0.0014 P_2^2$
- $C_3 = 253.24 + 7.836 P_3 + 0.0013 P_3^2$
- $C_4 = 388.9 + 7.57 P_4 + 0.0013 P_4^2$
- $C_5 = 194.2 + 7.77 P_5 + 0.0019 P_5^2$

Step 3: After entering the values, we should try to find $h$, the incremental cost of the system, which should be one value for all of the generators. $h$ can be found by the following equation according to formula (7.33) in Power System Analysis, H. Sadaat, 2nd edition:

$$h = \frac{P_{demand} + \sum_{i=1}^{n} Bi \cdot (c_i)}{\sum_{i=1}^{n} \frac{1}{2(c_i)}}$$

$$h = \frac{760 + 7.62 \cdot 179.48 + 7.519 \cdot 202.85 + 7.836 \cdot 96.54 + 7.57 \cdot 198.85 + 7.77 \cdot 83.42}{384.62 (3013.85 + 2912 + 2044.74)} = \frac{14346.3}{1774.2} = 8.087 \$/Mwh.$$

Thus, the incremental cost ($h$) for all the generators system is equal to 8.087 $/mwh.$

Step 4: Using the formula (7.31), we find the optimal generation for each unit. The answer for each generator should be within its maximum and minimum limit. The formula is

$$P_i = \frac{h \cdot Bi}{2(c_i)}.$$

Applying this formula:

- $P_1 = \frac{8.087 - 7.62}{0.0026} = 179.48 \text{ Mw}.$
- $P_2 = \frac{8.087 - 7.519}{0.0028} = 202.85 \text{ Mw}.$
- $P_3 = \frac{8.087 - 7.836}{0.0026} = 96.54 \text{ Mw}.$
- $P_4 = \frac{8.087 - 7.57}{0.0026} = 198.85 \text{ Mw}.$
- $P_5 = \frac{8.087 - 7.77}{0.0038} = 83.42 \text{ Mw}.$

The results came right and all the equations gave us very reasonable numbers located between the minimum and maximum limits for each unit. These equations are known as coordination equations, and applying these values to our generation cost equation should give us the optimal generation for each unit, which eventually lead to the optimal power flow in the network.

Step 5: After finding the coordination results of the generators, I will insert them into the generation cost equations in step 2 to calculate the operating cost:

- $C_1 = 373.50 + 7.62 (179.48) + 0.0013 (179.48)^2 = 1783.04 \$/hr$
- $C_2 = 403.61 + 7.519 (202.85) + 0.0014 (202.85)^2 = 2286.45 \$/hr$
- $C_3 = 253.24 + 7.836 (96.54) + 0.0013 (96.54)^2 = 1021.84 \$/hr$
- $C_4 = 388.9 + 7.57 (198.85) + 0.0013 (198.85)^2 = 1946 \$/hr$
- $C_5 = 194.2 + 7.77 (83.42) + 0.0019 (83.42)^2 = 855.6 \$/hr$

The sum of all the costs above will give us the economic dispatch of the system:

$$\text{Cost} = 1783.04 + 2286.45 + 1021.84 + 1946 + 855.6 = 7894 \$/hr.$$

Applying these values to all the generators in the system shall give us the optimal power flow in system. Figure 11 shows the system after applying these values.

Figure 11: The total cost of the system after applying the changes on the units.

The cost nearly equals that one in the beginning in the question, and there is some $100 extra may apply due to calculation error, since we used the analytical method not the iteration method. Another issue came into surface is that the line between buses two and five got overloaded, and this may happened because we neglected the transmission lines losses in the system. Another thing to notice is that the total generation of all the generating units should be equal to the Power demand, especially that we
have no losses since we had neglected. \( P_g = 179.48 + 202.85 + 96.54 + 198.85 + 83.42 = 761 \text{ Mw} \) which is nearly equal to the \( P_{\text{demand}} \) and \( 760 \text{ Mw} \).

**Discussion**

Power optimal flow is very critical topic in power systems. It determines for us what values should we run the system’s generators for a better and more economical flow. It is has very various methods, and I only use to choose the basic optimal study which neglects the lines losses. The results came reasonable and all the answers are suggesting very realistic values to be inserted into the generating units to operate in order to achieve the optimal flow. The total cost came nearly equal to that one the simulator had given before changing the system. Besides, the total demand comes equal to the total optimal generation of all the units, which has made the calculations more accurate.

**References**


[2]. Fei, H., Quan, G. “Optimization and control for thermal power plant based on plantwidecontrol”, IEEE international conference on advanced power system automation and protection, 2011.


