Image Compression Using Haar Transform And Modified Fast Haar Wavelet Transform

Avinash Ghorpade, Priyanka Katkar

Abstract: Wavelet Transform has been proved to be a very useful tool for image processing in recent years. Digital images require large amounts of memory to store and, when retrieved from the internet, can take a considerable amount of time to download. Compression makes it possible for creating file sizes of manageable, storable and transmittable dimensions The Haar wavelet transform provides a method of compressing image data so that it takes up less memory. The most distinctive feature of Haar Transform lies in the fact that it lends itself easily to simple manual calculations. Modified Fast Haar Wavelet Transform is one of the algorithms which can reduce the calculation work in Haar Transform. The present paper attempts to describe the algorithm for image compression using MFHWT.

Key words: Compression, Wavelet transform, Haar Wavelet Transform, Modified Fast Haar Wavelet Transform, MSE, PSNR, Compression Ratio

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I. INTRODUCTION

As computers have become more and more powerful, the temptation to use digital images has become irresistible. Image compression plays a vital role in several important and diverse applications, including tele-video conferencing, remote sensing, medical imaging and many more. Data compression is playing a significant role in saving storage space efficiently. It also helps in accelerating transmission speed. These requirements are not fulfilled with old techniques of compression like Fourier Transform, Hadamard and Cosine Transform etc. due to large mean square error occurring between original and reconstructed images. The wavelet transform approach serves the purpose very efficiently. The basic idea behind the image compression is that in most of the images we find that their neighboring pixels are highly correlated and have redundant information. It is therefore, necessary to find a less correlated representation of the image and it can be done by removing redundancy and irrelevancy. Redundancy reduction removes duplication in image and irrelevancy reduction omits that part of the signal which is not noticed by Human Visual System. Since the Haar Transform is memory efficient, exactly reversible without the edge effects, it is fast and simple. Modified Fast Haar Wavelet Transform is one of the algorithms which can reduce the tedious work of calculations.

II. HAAR TRANSFORM

The Haar Transform is one of the simplest and basic transformations from the space domain to a local frequency domain. A HT decomposes each signal into two components, one is called average (approximation) or trend and the other is known as difference (detail) or fluctuation. A precise formula for the values of first average sub signal, a = (a1, a2, …., aN / 2), at one level for a signal of length N i.e. f= (f_1, f2,….., fN) is

$a_n = (f_{2n-1}+f_{2n})/\text{sqrt}(2), n=1,2,3,..., / 2,$

and the first detail sub signal $d^1=(d_1, d_2,\ldots.d_{N/2})$ at the same level is given as:

$d_n=(f_{2n-1}-f_{2n})/\text{sqrt}(2), n=1,2,3,..., / 2$

In order to transform a matrix representing an image using the Haar wavelet transform, we will first discuss the method of transforming vectors called averaging and differencing. First, we start out with an arbitrary vector representing one row of an 8 x 8 image matrix:

y = (448 768 704 640 1280 1408 1600 1600)

Because the data string has length $8 = 2^3$ there will be three steps to the transform process. If the string were 2k long, there would be k steps in the process. For the first step our data string becomes:

y1 = (608 672 1344 1600 **-160 32 -64 0**)

We get this by averaging first four pairs of numbers (448 & 768, 704 & 640, etc) and the results become the first four entries of modified string y1. These are known as approximation coefficients. Next we subtract these averages from the first member of each pair. These answers become the last four entries of y1, known as detail coefficients. The detail coefficients are repeated in each subsequent transformation of this data string. We will proceed with the second step which changes our data string to:

y2 = (640 1472 **-32 -128 -160 32 -64 0**)

Once again we average the pairs (608 & 672, 1344 & 1600) and the results become the first two entries of y2 and are

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new approximation coefficients. We then subtract the averages from the first element of each pair. The results become the third and fourth elements of y2. The last four elements in y2 are identical to y1. For the last step our data string becomes:

y3 = (1056 - **416 -32 -128 -160 32 -64 0**)

This time we obtain the first entry of y3 by averaging the pair (640 & 1472) and second element by subtracting the average from the first element of the pair. This is final detail coefficient followed by detail coefficients from y2. The Haar wavelet does this transformation to each row and column of image matrix. The resulting matrix is known as the Haar wavelet transform of the original matrix. The point of doing Haar wavelet transform is that the areas of the original matrix that contain little variation will end up as zero elements in the transformed matrix. A matrix is considered as sparse if it has a "high proportion of zero entries". Sparse matrices take much less memory to store. Since the transformed matrices are expect to be not always sparse, we decide on a non-negative threshold value known as ε, and then let any entry in the transformed matrix whose absolute value is less than ε to be reset to zero, this will give us results with a kind of sparse matrix. This process is entirely reversible so as to retrieve the original image.

III. MODIFIED FAST HAAR WAVELET TRANSFORM

In MFHWT, first average subsignal, $a^1 = (a1,a2,...,aN / 2)$, at one level for signal of length N i.e. $f= (f_1,f_2,.....f_N)$ is

$$a_m = \frac{f_{4m-3}+f_{4m-2}+f_{4m-1}+f_{4m}}{4},\ m=1,2,3,\ldots,N/4,$$

and first detail subsignal, at the same level is given as

$$d_m = \begin{cases} \dfrac{(f_{4m-3}+f_{4m-2})-(f_{4m-1}+f_{4m})}{4}, & m=1,2,3,\ldots,N/4, \\ 0 & ,\ m=N/2,\ldots,N. \end{cases}$$

Here four nodes are considered at a time instead of two nodes in Haar transform. The MFHWT is faster in comparison to HT and reduces the calculation work. In MFHWT, we get the values of approximation and detail coefficients one level ahead than the HT.

PROPOSED ALGORITHM OF MFHWT

A MFHWT can be done by performing the following steps

- Read the image as a matrix.
- Apply MFHWT, along row and column wise on entire matrix of the image.
- We get a transformed image matrix of one level of input image.
- The image is reconstructed.
- Calculate MSE, PSNR and number of zeros in reconstructed image.

In order to measure and evaluate the performance of our approach, commonly used metric such as the Mean Square Error (MSE) and Peak Signal to Noise Ratio (PSNR) which are the two error metrics used to compare image compression quality. The MSE represents the cumulative squared error between the compressed and the original image, whereas PSNR represents a measure of the peak error, the lower the value of MSE, the lower the error. The compression ratio is defined as the size of the original image divided by the size of the compressed image. The ratio provides a clue of how much compression is achieved for a particular mage.

$$MSE = \frac{\sum_{M,N}[I_1(m,n)-I_2(m,n)]^2}{M.N}$$

In the previous equation, M and N are the number of rows and columns in the input images, respectively. Then the block computes the PSNR using the following equation:

$$PSNR = 10\log_{10}\left(\frac{R^2}{MSE}\right) = 20\log_{10}\left(\frac{R}{\sqrt{MSE}}\right)$$

In the previous equation, R is the maximum fluctuation in the input image data type. For the color input image it has an 8-bit unsigned integer data type, so R is 255, the higher the PSNR the smaller is the difference between the reconstructed image and the original. In Modified fast Haar wavelet transform algorithm after a DWT transform, the image is divided into four corners, upper left corner of the original image, lower left corner of the vertical details, upper right corner of the horizontal details, lower right corner of the component of the original image detail (high frequency).

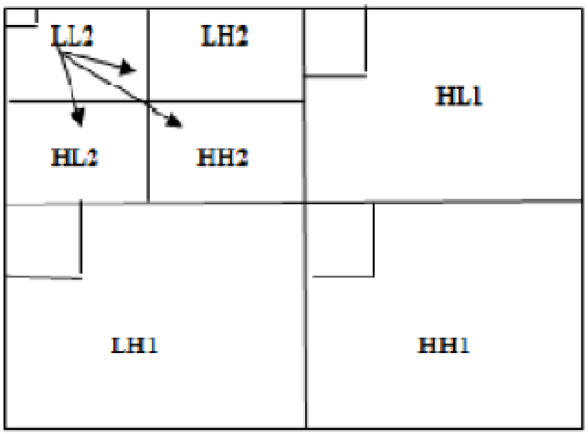

Fig. 1: Decomposition of Original Image in Matrix Form

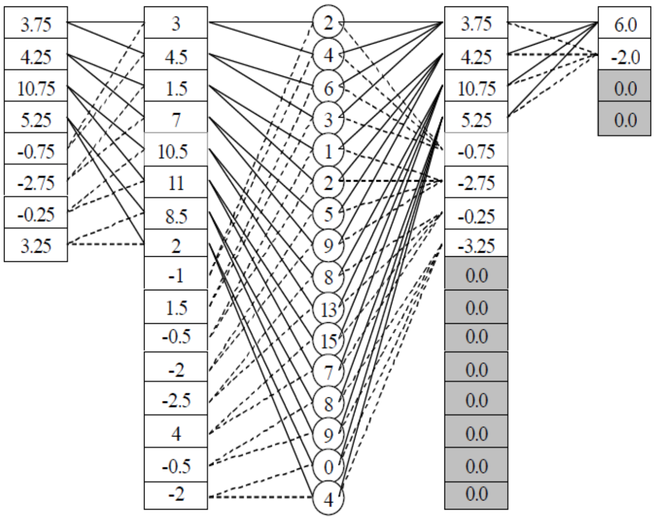

Fig. 2: Operations of MFHWT (right side) and HT (left side)

IV. RESULTS

Results obtained after compressing an image using haar transform are shown below:

Fig. 3: Original image

Fig. 4: Compressed image

MSE = 6.7500
PSNR = 39.8718 dB
Compression Ratio = 1.7628:1
Percent of zeros occurred in the compressed image: 88.9313%

Results obtained after compressing an image using MFHWT are shown below:

Fig. 5: Original image

Fig. 6: Compressed image

MSE = 1.015
PSNR = 48.1 dB
Compression Ratio = 5:1
Percent of zeros occurred in the compressed image: 96.4548%

V. CONCLUSIONS AND FUTURE WORKS

This paper is aimed to developing computationally efficient algorithm for various image compression using wavelet techniques. The wavelets based on Haar transform discussed previously is the simplest and crudest member of a large class of possibilities, so it can summarized its properties as:

- It is simple, represent as useful tool for image compression, and easily represented in software and even hardware.
- The main benefit of Haar and their Modified types is the sparse representation, fast transformation, low memory space requirements and possibility of implementation of fast algorithms.

For further work, the tradeoff between the value of the threshold and the image quality can be studied and also fixing the correct threshold value can be of great interest. Furthermore, finding out the exact number of transformation levels required in the case of application for a specific image compression can be studied. To get the better result new wavelets can be blended with other schemes. So new parameters can be considered for the evaluation of compression techniques. Optimization of various compression techniques can be done to preserve energy and better results as much as possible.

VI. REFERENCES

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