Integrated Process Modelling of a Thermophilic Biogas Plant

Anisiji O. E., Chukwuneke J. L., Achebe C. H., Okolie P. C.

Abstract: this work developed a mathematical model of a biogas plant from a mechanistic point of view, for urban area clean energy requirement. It aimed at integrating thermodynamics; which deals with the direction in which a process occurs and Biochemical kinetics; which gives the understanding of the rates of biochemical reaction. The mathematical formulation of the proposed gas plant follows the fundamental principles of thermodynamics, and further analysis were accomplished to develop an algorithm for evaluating the plant performance preferably in terms of daily production capacity. In addition, the capacity of the plant is equally estimated for a given cycle of operation and presented in time histories. A nominal 1500m³ biogas plant was studied characteristically and its performance efficiency evaluated. It was observed that the rate of biogas production is essentially a function of enthalpy ratio, the reactor temperature, pH, substrate concentration, rate of degradation of the biomass, and the accumulation of matter in the system due to bacteria growth. The results of this study conform to a very large extent with reported empirical data of some existing plant and further model validations were conducted in line with classical records found in literature.

Index Terms: Anaerobic Digestion, Biogas Plant, Biogas Production, Bio-reactor, Energy, Fermentation, Rate of Production, Temperature, Thermophilic.

1 INTRODUCTION

Energy is one of the most important factors to global prosperity. The dependence on fossil fuels as primary energy source has led to global climate change, environmental degradation, and human health problems. In the year 2040, the world as predicted by the United Nations will have 9–10 billion people and they must be provided with energy and materials [1]. Moreover, the recent rise in oil and natural gas prices may drive the current economy toward alternative energy sources such as biogas. Anaerobic digestion is the most widely used method of organic waste disposal due to its high performance in volume reduction and stabilization and the production of biogas that makes the process profitable. However, biological hydrolysis, which is the rate-limiting step for the anaerobic degradation [2] has to be improved to enhance the overall process performance and to reduce the associated cost. Several mechanical, thermal, chemical, or biological pre-treatment methods have been considered to improve hydrolysis and anaerobic digestion performance. These pre-treatments result in the ‘lysis’ or disintegration of cells [3, 4] and release of intracellular matter that becomes more accessible to anaerobic micro-organisms [5], thus improving anaerobic digestion [2].

Anaerobic digestion and biogas production are promising means of achieving both global and local environmental benefits. Biogas is a renewable energy carrier, and the introduction of anaerobic digestion of farm residues and municipal organic waste may reduce potentially negative environmental impact of current agricultural practices and waste handling procedures [6]. Biogas is a gaseous mixture of methane, carbon dioxide, hydrogen sulphide and several other gases, produced by anaerobic fermentation of organic materials such as plant, animal and human wastes, under specified conditions. It is very important because of the presence of methane which gives it the property of combustion and makes it suitable for cooking, lighting and powering prime movers. Anaerobic digestion has been suggested as an alternative method of removing the high concentration organic waste. Several research groups have developed anaerobic digestion processes using different organic substrates [7, 8]. The advantages of such processes over conventional aerobic processes are a low energy requirement for operation, a low initial investment cost and a low sludge production [9]. In addition, the anaerobic digestion process produces biogas, which can be used as a clean renewable energy source [10, 11]. It is a common ugly sight in our urban cities to see indiscriminate dumping of wastes. The environmental agencies responsible for clearing these wastes usually dump them in landfills and sometimes, incinerate them in an open air. This has its concomitant effects on our environment. Besides, the wastes are not sorted, that even open air incineration leaves non-biodegradable materials the way they were. Sorting of these solid municipal wastes will provide a better means of putting waste into good economic use. Biodegradable waste sorted can now be used for biogas generation while non-biodegradable ones may be recycled as such the case may be. However, the design of a standard biogas plant capable of utilizing large quantity of refuse emanating from areas of high population densities invariably require detailed analysis and characterisation of the plant based on a well formulated mathematical model in line with the objective of this paper. This paper aims at achieving the following; To Size a biogas plant, To develop a mathematical model for a commercial biogas plant, To determine the rate of biogas production of the commercial biogas plant.

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2 Method and Material

2.1 Sizing and Design of the Bio-Digester

The design and sizing of the biogas plant was developed using experimental results. The basic materials utilized for the experiment are human waste sample representing a total mass for Awka, a 20-litre mini-bio-digester, water, and Bacillus steatorrhophilus. Basically, 5kg of human waste sample was loaded into the mini-digester mixed in a proportion of 1:1 with water and inoculated with the thermophilic bacteria at various temperatures of 50, 60, and 70°C. The digester was then stirred and left for days as biogas was being produced. Since Glucose makes up the greater part of the human waste, it is assumed in this work that human waste is glucose.

Therefore as the production of biogas from human waste will involve its degradation then, it will be right to say that the decay of this waste is a fermentation process. Traditionally, fermentation was defined as the process for the production of alcohol or lactic acid from glucose (C\textsubscript{6}H\textsubscript{12}O\textsubscript{6}), but this paper will adopt a broader definition of fermentation which according to Webster’s New Collegiate Dictionary, “Fermentation is an enzymatically controlled transformation of an organic compound”. With this, the design proceeds thus: From the 2006 population census, the estimated population of people inhabiting the metropolitan city of Awka by the year 2011 is 97550 persons [National population’s commission]. The average waste per day per person = 0.67kg of feces/day, Total human discharge for Awka = 0.67 × 97550 = 65358.5kg/day, The total fresh discharge = 65358.5kg/day, T.S (Total solid) of fresh discharge = 65358.5 × 0.16 = 10457.36kg. In 8% concentration of T.S (to make favorable condition); 8kg solid → 100kg of influent

\[
1kg \text{ solid} \rightarrow \frac{100}{8} \text{ kg of influent}
\]

10457.36kg of solid = \(\frac{100}{8} \times 10457.36 = 130717\)kg of influent,

The total influent required, \(F_{AO} = \frac{130717\text{kg}}{\text{day}}\).

By mass balance relationship;

\[
F_{AO} - F_A = -r_A V
\]
\[
F_{AO} - F_A = F_{AO}X
\]
\[
V = \frac{F_{AO}X}{-r_A}
\]
\[
-r_A = kC_A \quad \text{(Assuming first order reaction)}
\]
\[
-r_A = \frac{dC_A}{dt}
\]

\[
\frac{dC_A}{dt} = kC_A
\]
2.4 The Biogas Plant Layout

A standard biogas plant layout may be derived by defining an array of components that is capable of executing the major processes (the homogenization or churning of the input mass, the heat rejection and the chemical reaction) involved in biogas production through anaerobic digestion of sewage mass. A typical physical model would seek to accomplish these processes in a minimum number of components. Apparently some existing models execute these processes in one or two components especially those used in gas generation for small scale household utilities like cooking, home heating and other domestic purposes. However, production of biogas for industrial application, power generation and other expanded purposes often requires large amount of biodegradable material whose effective digestion must be enhanced through some thermal and catalytic processes. This fact lead to adaptation of appropriate components designed to achieve this objective in a standard biogas production plant.

Fig. 4. The Physical model of the proposed biogas plant

3 THEORETICAL FORMULATION/ANALYSIS

The analysis for a standard biogas plant of the type whose diagram is given in Fig. 4 follows the application of two fundamental laws of thermodynamics; conservation of mass and energy principles which lead to the well-known continuity and steady flow energy equations (SFEE) respectively. These equations are written simultaneously for individual subsystem considered as an open system exchanging energy and matter with the surroundings to obtain the following reduced Equation [12].

\[
\begin{align*}
&m_1 h_{2,1} + m_1 h_{2,1}' \quad 0 \quad 0 \quad 0 \quad \{Q_A - W_c\}^{-1} \\
&0 \quad (m_1 + m_1') \quad h_{3,2} \quad 0 \quad 0 \\
&0 \quad 0 \quad h_4 \quad h_4' \quad m_4 \quad m_4'
\end{align*}
\]

\[
\begin{align*}
&\begin{cases}
1 \\
(m_1 + m_1') [1 + k] - W_c \\
(m_1 + m_1')[1 + k]
\end{cases}
\end{align*}
\]

Eqn. 14 is solved analytically for the column vectors and rewritten in a more compact form for \( m_4 \) as;

\[
m_4 = \eta m [1 + k] - W_c (h_4 - h_4')^{-1}
\]

Where; \( m = \text{ummation of water and sewage masses (i.e } m = m_1 + m_1') \), \( m_4 = \text{mass of biogas(methane) generated, } \eta = \text{enthalpy ratio } (h_3 - h_4)/(h_4 - h_4') \)

The capacity of a biogas plant is better measured in terms of mass of gas generated per day hence we may estimate the rate of gas production of the model described by evaluating the first derivative of Eqn.(15) to obtain;

\[
m_4 = \eta m + \eta k m + \eta m k - W_c (h_4 - h_4')^{-1}
\]

Where; \( m \), rate of production of biogas, \( m \) is the rate of degradation (depletion) of the biomass, \( k \) is the growth rate of bacteria and, \( W_c \) is the energy dissipation rate due to churning. Assuming the bacteria grow exponentially obeying the natural growth law of the form;

\[
k(t) = k_0 e^{\lambda t}
\]

\(\lambda\) Could be referred as a decay constant which is equivalent to the slope of the bacteria growth curve.

\[
\lambda = (k - k_0)/t = \mu k_0
\]

The evaluation of the degradation rate \( m \) however requires the knowledge of the retention time \( t \). Hence \( t \) is derived from the logical combination of eqn. (17) and the definition of specific growth rate of bacteria \( \mu \) as follows.

We may recall that by definition;

\[
\mu = k - k_0/k_0
\]

Combining eqns. (17) and (19) leads to

\[
t = \frac{ln(\mu + 1)}{\lambda}
\]

Rate of energy dissipation due to scavenging work \( \dot{W}_c \) is numerically equal to zero (for stirring work). Hence,

\[
\dot{W}_c = 0
\]

\[
m = m_1 + m_1' = m_1
\]

Where \( m_1' = 0 \) (assuming water is non reactive)

In view of eqns. (17-21), eqn. (16) can be re-expressed as [12];

\[
m_4 = \frac{\eta d m_1}{ln(\mu + 1)} \left[ 1 + k_0 e^{\lambda t} \right] + \eta m k_0 \mu e^{\lambda t}
\]

Eqn.18 will give the mass of biogas produced per day if \( \mu \) is given in per day, \( t \) in days. To obtain the total mass of biogas obtained during the retention period, eqn. (22) is integrated over time as follows.

\[
m_4 = \frac{\eta d m_1}{ln(\mu + 1)} \left[ \frac{1}{\mu} e^{\lambda t} \right]_{t=0}^{t} + \eta m k_0 \mu e^{\lambda t}
\]

\[
\]
Eqn. (23) shows that the mass of biogas generated can be evaluated completely in terms of the parameters \( \eta, \lambda, m_1, m, k_0 \) and \( \mu \). From eqns. (22), (23) and experimental results, the specific growth rate \( \mu \), can be identified as a key parameter for the production of biogas and is a function of temperature, pH and substrate concentration.

### 3.1 Characterisation of Specific Growth Rate of Thermophilic Bacteria

In biosynthesis, the cells (microorganism/bacteria) also referred to as the biomass, consume nutrients to grow and produce more cells and important products [13].

The behavior of the biogas plant is modeled to study the specific growth rate of cells inoculated to human waste. The specific growth rate will be the key parameter for the description of the cell growth, human waste consumption and biogas production. Biogas production is sensitive to digester temperature, pH of the liquid manure [14]. The specific growth rate of the cells/bacteria depends on the temperature, pH and concentration of the substrate. The multiplication of these influencing factors gives the model equation/expression. So it could be said that:

\[
\mu(t) = \mu(S). \mu(pH). \mu(T)
\]  

(24)

Since the cells growth rate is proportional to the Cell concentration, and for the simplification of the complicated mechanism of cell growth, the Monod equation was adopted which adequately describes fermentation kinetics when the concentrations of those components which inhibit the cell growth are low. The Monod Model which is analogous to Micheal's-Menten equation will be used to model the behavior of the substrate concentrations against specific growth rate and is written thus:

\[
\mu(S) = \frac{\mu_{max} S}{K_s + S} + K_e
\]  

(25)

Where; \( S \) is the substrate concentration in the medium, \( K_s \) is the system coefficient and \( K_e \) is the cell maintenance factor. The relationship between the specific growth rate and the pH will be treated by the parabolic law as shown:

\[
\mu(pH) = apH^2 + bpH + c
\]  

(26)

Where; \( a, b \) and \( c \) are constants. The influence of temperature will be modeled by an Arrhenius type law:

\[
\mu(T) = A_1 e^{-\frac{E_1}{RT}} - A_2 e^{-\frac{E_2}{RT}} - A_3
\]  

(27)

Where \( T \) is the temperature (K), \( E_1, E_2 \) are activation energies (kJ/Kg mol), \( R \) is the gas constant (kJ/mole K) and \( A_1, A_2, A_3 \) are constants. The mathematical model for the behavior of the system becomes, the expansion of the expression below:

\[
\mu(t) = \left( \frac{\mu_{max} S}{K_s + S} + K_e \right)(apH^2 + bpH + c)(A_1 e^{-\frac{E_1}{RT}} - A_2 e^{-\frac{E_2}{RT}} - A_3)
\]  

(28)

### 4 RESULTS AND DISCUSSION

#### 4.1 Results on Design and Sizing of the Bio-Digester

<table>
<thead>
<tr>
<th>T (days)</th>
<th>( C_A ) (kg/m³)</th>
<th>X</th>
<th>( \ln \left( \frac{C_{AO}}{C_A} \right) )</th>
<th>( C_A ) (kg/m³)</th>
<th>X</th>
<th>( \ln \left( \frac{C_{AO}}{C_A} \right) )</th>
<th>( C_A ) (kg/m³)</th>
<th>X</th>
<th>( \ln \left( \frac{C_{AO}}{C_A} \right) )</th>
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<td>0</td>
<td>0</td>
<td>1000</td>
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<td>1000</td>
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<td>0</td>
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<tr>
<td>1</td>
<td>740</td>
<td>0.26</td>
<td>0.26</td>
<td>700</td>
<td>0.30</td>
<td>0.36</td>
<td>606</td>
<td>0.39</td>
<td>0.5</td>
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<tr>
<td>2</td>
<td>548.8</td>
<td>0.59</td>
<td>0.59</td>
<td>449.3</td>
<td>0.55</td>
<td>0.80</td>
<td>367.9</td>
<td>0.63</td>
<td>1.0</td>
</tr>
<tr>
<td>3</td>
<td>406.6</td>
<td>0.92</td>
<td>0.92</td>
<td>301.2</td>
<td>0.70</td>
<td>1.20</td>
<td>220.1</td>
<td>0.78</td>
<td>1.5</td>
</tr>
<tr>
<td>4</td>
<td>301.2</td>
<td>1.19</td>
<td>1.19</td>
<td>201.9</td>
<td>0.79</td>
<td>1.60</td>
<td>135.3</td>
<td>0.86</td>
<td>2.0</td>
</tr>
<tr>
<td>5</td>
<td>240</td>
<td>1.40</td>
<td>1.40</td>
<td>135.3</td>
<td>0.86</td>
<td>2.00</td>
<td>82.1</td>
<td>0.92</td>
<td>2.5</td>
</tr>
<tr>
<td>6</td>
<td>180.3</td>
<td>1.70</td>
<td>1.70</td>
<td>90.7</td>
<td>0.91</td>
<td>2.40</td>
<td>50.0</td>
<td>0.95</td>
<td>3.0</td>
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<tr>
<td>7</td>
<td>122.5</td>
<td>2.10</td>
<td>2.10</td>
<td>60.8</td>
<td>0.93</td>
<td>2.80</td>
<td>30.2</td>
<td>0.97</td>
<td>3.5</td>
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<tr>
<td>8</td>
<td>90.7</td>
<td>2.40</td>
<td>2.40</td>
<td>40.8</td>
<td>0.96</td>
<td>3.20</td>
<td>22.3</td>
<td>0.97</td>
<td>3.8</td>
</tr>
<tr>
<td>9</td>
<td>57.2</td>
<td>2.86</td>
<td>2.86</td>
<td>27.3</td>
<td>0.97</td>
<td>3.60</td>
<td>15</td>
<td>0.98</td>
<td>4.2</td>
</tr>
</tbody>
</table>

TABLE 1

**EXPERIMENTAL DATA**
From the graph of $\ln \left( \frac{C_A}{C_{A0}} \right)$ Vs T at temperatures of 50°C, 60°C and 70°C, it is found that the plot at 60°C gives the highest regression coefficient. There was better yield of biogas when the very bacteria culture was used. At a conversion, $X = 0.8$, and biogas production rate of $0.4 \text{day}^{-1}$ and from Eqn. (13), the volume of the biogas reactor could be obtained thus;

$$V = \frac{V_0 X}{k(1 - X)} = \frac{130.717 \times 0.8}{0.4 \times 0.2} = 1307.17 \text{m}^3$$

Since the volume of the bacteria culture that would be required to inoculate the waste is 10% of the volume above; $V = 10\%$ of $1307.17 + 1307.17 = 1437.887 \text{m}^3$. New volume of the reactor approximates to $V = 1500 \text{m}^3$. The volumetric and geometric quantification of the bio-reactor is evaluated and expressed in Table (2) below;

<table>
<thead>
<tr>
<th>S/N</th>
<th>Quantity</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>D</td>
<td>15</td>
<td>m</td>
</tr>
<tr>
<td>2.</td>
<td>F₁</td>
<td>3</td>
<td>m</td>
</tr>
<tr>
<td>3.</td>
<td>F₂</td>
<td>1.875</td>
<td>m</td>
</tr>
<tr>
<td>4.</td>
<td>H</td>
<td>6</td>
<td>m</td>
</tr>
<tr>
<td>5.</td>
<td>S₁</td>
<td>205</td>
<td>m²</td>
</tr>
<tr>
<td>6.</td>
<td>S₂</td>
<td>188</td>
<td>m²</td>
</tr>
<tr>
<td>7.</td>
<td>V₁</td>
<td>279</td>
<td>m³</td>
</tr>
<tr>
<td>8.</td>
<td>V₂</td>
<td>169</td>
<td>m³</td>
</tr>
<tr>
<td>9.</td>
<td>V₃</td>
<td>1060</td>
<td>m³</td>
</tr>
<tr>
<td>10.</td>
<td>V₄</td>
<td>75</td>
<td>m³</td>
</tr>
<tr>
<td>11.</td>
<td>V₅</td>
<td>915</td>
<td>m³</td>
</tr>
<tr>
<td>12.</td>
<td>V₆</td>
<td>285</td>
<td>m³</td>
</tr>
<tr>
<td>13.</td>
<td>V₇</td>
<td>225</td>
<td>m³</td>
</tr>
</tbody>
</table>

Fig. 5. Graph of $\ln \left( \frac{C_A}{C_{A0}} \right)$ Vs T

From Fig.6, as concentration of the human waste increases, there will also be increase in the rate of reaction, the reaction rate is proportional to the substrate concentration. This is especially true since the biochemical reaction in the bio-reactor has been assumed to be a first order reaction. From Figs.7 and 8, as the reaction progresses through the retention time and biogas is produced, the rate of reaction will continue to fall as more of the human waste is used up or converted into products.

The rate of production of biogas varies with concentration, time and conversion as shown in Figs. (7), (8) and (9) respectively.

**Fig. 6. The relationship between the rate of reaction and Concentration of the human waste**

**Fig. 7. The relationship between the rate of reaction and the retention time of the human waste**
4.2 Results on Characterisation of the Specific Growth Rate

- **Substrate Concentration**

  The regression analysis was used to determine the Monod equation, which best describes the effect of substrate concentration on specific growth rate \( \mu \). The Monod equation for this model from Eqn. (25) is:

  \[
  \mu(S) = \frac{0.45}{173.94 + S} - 0.00019
  \]  
  (29)

  This relationship is shown graphically in fig. 9 below;

- **pH**

  Also the regression analysis was used to determine \( \mu \) and pH relationship from eqn. (26) as:

  \[
  \mu(pH) = 142pH - 10.2pH^2 - 438
  \]  
  (30)

- **Temperature**

  Used also here, was the regression analysis to establish the relationship between the specific growth rate and temperature. The result is from Eqn. (27) as:

  \[
  \mu(T) = 8540.404 - 3787.26\exp\left(\frac{E_1}{RT}\right) - 4952.322\exp\left(-\frac{E_2}{RT}\right)
  \]  
  (31)
The rate of reaction is described by the modified Arrhenius equation. An increase in temperature increases the rate of reaction. However, the temperature is limited to the optimal biological value of 65°C under thermophilic condition. If the temperature continues to rise, denaturing sets in. This process progressively destroys the activity of the bacteria cells. This causes the overall reaction velocity to drop. From the above individual results for Substrate concentration, pH and Temperature with respect to specific growth rate, we can assemble these results into: μ(t) = μ(S), μ(pH), μ(T)

μ(t) = \left( \frac{K_s}{K_s+5} \right) \cdot (apH^2 + bpH + c) \cdot \left( A_1 \exp \frac{E_f}{RT} - A_2 \exp \frac{-E_f}{RT} - A_3 \right)

(32)

Expanding the above expression, yields

μ(t) = αβ(1 + 0.0233pH^2 - 0.324pH)(0.4\exp \frac{E_f}{RT} - 0.58\exp \frac{-E_f}{RT} - 1)

(33)

μ(t) = \left( \frac{\exp \left( \frac{E_f}{RT} - \frac{-E_f}{RT} \right)}{17.39+5} \right)

(34)

This is the equation that characterises the specific growth rate with respect to temperature, pH and substrate concentration.

where; α = 3.741 × 10^6 and β = μ(S)

TABLE 1

<table>
<thead>
<tr>
<th>S/N</th>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A_1</td>
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</tr>
<tr>
<td>2</td>
<td>A_2</td>
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<td>A_3</td>
<td>8540.404</td>
</tr>
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<td>4</td>
<td>A</td>
<td>-10.2</td>
</tr>
<tr>
<td>5</td>
<td>B</td>
<td>142</td>
</tr>
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<tr>
<td>9</td>
<td>A</td>
<td>3.741×10^6</td>
</tr>
</tbody>
</table>

5 CONCLUSION

The study of biogas production via catalyzed and controlled anaerobic digestion enhanced the successful investigation of the untapped energy potential of the biodegradable industrial or domestic refuse. The survey shows that a commercial quantity of methane-rich biogas can be generated from a municipal waste when it is applied as a substrate for catalytic and heat controlled processes in standard biogas plant. Such equipment may serve as a technological means to protect our city environments from unwanted pollution resulting from unutilized refuse dumps and at the same time alleviate the problem of energy requirements for home and industrial uses. The effort of this work to formulate a mathematical model for the biogas plant leads to the following observations:

I. The rate of biogas production from a given reserve is a function of the energy properties of the system inputs, the specific growth rate of bacteria and the rate of degradation of the biomass.

II. The effective retention period of the biomass elapses after ten awesome days.

III. Maximum daily production of the proposed thermophilic gas plant is identified at the reactor temperature of 60°C.

IV. About 3% of the total sewage mass is convertible to biogas daily and over the retention period from the idealized gas plant.

REFERENCES


