Tuning Of A PD-PI Controller Used With A Highly Oscillating Second–Order Process

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Abstract: High oscillation in industrial processes is something undesired and controller tuning has to solve this problem. PD-PI is a controller type of the PID-family which is suggested to overcome this problem with improved performance regarding the spike characteristics associated with certain types of controllers. This research work has proven that using the PD-PI is capable of solving the dynamic problems of highly oscillating processes. A second order process of 85.45 % maximum overshoot and 8 seconds settling time is controlled using an PD-PI controller (through simulation). The controller is tuned by minimizing the sum of square of error (ISE) of the control system using MATLAB. The MATLAB optimization toolbox is used assuming that the tuning problem is an unconstrained one. The result was cancelling completely the 85.45 % overshoot and producing a step-wise time response without any undershoot. The performance of the control system using an PD-PI controller using the present tuning technique is compared with that using the ITAE standard forms tuning technique.

Index Terms: Controller tuning ; highly oscillating second-order process ; improving control system performance ; PD-PI controller .

1. INTRODUCTION
Highly oscillating response is present in a number of industrial processes incorporating low damping levels. Conventionally, the PID controller is used and tuned for better performance of the control system. The PD-PI controller is one of the next generation of PID controllers where research and application is required to investigate its effectiveness compared with PID controllers. The PD-PI controller is a well known controller industry. Siemens (1999) produced a universal PD-PI controller as a stand alone controller to control various industrial processes [1]. Kaya (2003) extended a work on a modified PI-PI Smith predictor leading to improvements in the control of processes with large time constants or an integrator or unstable plant [2]. Veeraiah, Majhi and Mahanta (2004) proposed a fuzzy PI-PD controller tuning using genetic algorithms. They applied both linear and nonlinear test signals to investigate the validity of the proposed controller [3]. Rodriguez and Coelho (2005) applied the IMC tuning method to the PI-PI controller. Their methodology is assessed by a first-order plus dead-time, a second-order plus dead-time and an integral first-order plus dead-time processes [4]. Siddique and Tokhi (2006) developed a PD-PI-type fuzzy controller using a neural network to tune the scaling factors of the membership functions [5]. Jain and Nigam (2008) explored the idea of model generation and optimization for PD-PI controller. They used the inverted pendulum system as a test system for their approach based on using swarm intelligence [6]. Tan (2009) presented a graphical method for the computation of all stabilizing PI-PD controllers by plotting the stability boundary locus in the parameter plane [7]. Mohan (2010) tried to clarify the misunderstanding and confusion regarding the mathematical modeling of 2-term PI-PD controllers by discussing all relevant aspects with proper information [8].

Magaji, Mustafa and Muda (2011) proposed a fuzzy logic PD-PI to improve the damping inter-area modes of oscillations. They used genetic algorithms in tuning the controller [9]. Palmeira, Magalhaes, Conteate and Ferreira (2012) demonstrated the potential of a fuzzy PI + PD control system compared to classical PID applied to a mobile robot [10]. Hassaan (2014) used a PD-PI controller to control first-order delayed processes resulting in a control system with better performance through tuning the PD-PI controller using an ISE error criterion. He compared his results with those using classical PID controller tuned using two different techniques [11].

1. ANALYSIS
The process is a second order process having the parameters:

\[
\begin{align*}
\text{Natural frequency:} & \quad \omega_n = 10 \text{ rad/s} \\
\text{Damping ratio:} & \quad \zeta = 0.05
\end{align*}
\]

The process has the transfer function:

\[
M_p(s) = \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}
\]  \hspace{1cm} (1)

The time response of this process to a unit step input is shown in Fig.1 as generated by MATLAB:

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![Fig.1 Step response of the uncontrolled process.](image-url)
The severity of the process oscillations is measured by its maximum percentage overshoot. It has a maximum overshoot of 85.4% and an 6 seconds settling time. The controller used in this study is a proportional-derivative (PD) - proportional + integral (PI) controller. In this controller, The PD and PI parts of the controller are connected in series. The input to the PD part is the system error, while the input of the PI part is the output of the PD part [6]. There is a different structure of the PD-PI controller studied by Veerajah and others where the the PI part acts on the system error while the PD part acts on the input [3]. The block diagram of the closed-loop control system incorporating the PD-PI controller is shown in Fig.2 [6].

The controller transfer function is, \( G_c(s) \) is:

\[
G_c(s) = \frac{1}{s} (K_{pc}K_ds^2 + (K_{pc} + K_i)K_d)s + K_i
\]

where:

- \( K_{pc} = \) Proportional gain
- \( K_i = \) Integral gain
- \( K_d = \) Derivative gain

\( i.e. \) the controller has 3 parameters to be identified to control the process and produce a satisfactory performance. The controller and process are cascaded in the forward path of the unity feedback control system. Therefore, the closed-loop transfer function of the control system, \( M(s) \) is given by:

\[
M(s) = \frac{b_0s^2 + b_1s + b_2}{a_0s^3 + a_1s^2 + a_2s + a_3}
\]

where:

- \( b_0 = K_{pc}K_d\omega_n^2 \)
- \( b_1 = \omega_n^2(K_{pc} + K_i) \)
- \( b_2 = K_i\omega_n^2 \)
- \( a_0 = 1 \)
- \( a_1 = 2\zeta\omega_n + K_{pc}K_d\omega_n^2 \)
- \( a_2 = \omega_n^2(1 + K_{pc} + K_i) \)
- \( a_3 = \omega_n^2K_i \)

### 2. SYSTEM STEP RESPONSE

A unit step response is generated by MATLAB using the numerator and denominator of Eq. 3 providing the system response \( c(t) \) as function of time [12].

### 3. CONTROLLER TUNING

The sum of square of error (ISE) is used an objective function, \( F \) of the optimization process. Thus:

\[
F = \int [c(t) - c_{ss}]^2 \, dt
\]

where \( c_{ss} = \) steady state response of the system = 1 for a unit step input. The performance of the control system is judged using two time-based specifications:

- (a) Maximum percentage overshoot, \( OS_{max} \)
- (b) Settling time, \( T_s \)

### 4. TUNING RESULTS

The MATLAB command “fminunc” is used to minimize the optimization objective function given by Eq.4 without any parameters of functional constraints [13]. The results are as follows:

Controller parameters:

- \( K_{pc} = 33.2092 \)
- \( K_i = 34.9363 \)
- \( K_d = 43.1119 \)

The time response of the closed-loop control system to a unit step input is shown in Fig.3.

![Fig.3 Step response of the PD-PI controlled second order process.](image)

Characteristics of the control system using the tuned PD-PI controller:

- Maximum percentage overshoot: 0 \%  
- Maximum percentage undershoot: 0 \%  
- Settling time: 0 \, s

### 5. COMPARISON WITH STANDARD FORMS TUNING

The control system in terms of its transfer function is a fourth order one. The optimal characteristic equation of such a system with a second-order numerator is given using an ITAE criterion by [14]:

\[
s^3 + 2.97\omega_n s^2 + 4.94\omega_n^2s + \omega_n^3
\]

Comparing Eq.5 with the corresponding one in Eq.3 we get 3 equations in \( \omega_n \), \( K_{pc} \), \( K_i \) and \( K_d \). i.e. 4 unknowns and 3 equations. To be able to get the controller parameters using this tuning technique, one of the parameters has to be assumed. It was reasonable from the equations to assign \( K_i \) (it was taken as 34.9363 as obtained in the present tuning
technique using the ISE criterion). The tuned controller parameters using the ITAE standard forms are calculated as:

\[
K_{pc} = 1.3168 \\
K_i = 34.9363 \\
K_d = 0.3346
\]

The time response of the control system using this standard forms tuning technique is shown in Fig.4:

![Step response of the PD-PI controlled second order process using the ITAE standard forms.](image)

**Fig.4 Step response of the PD-PI controlled second order process using the ITAE standard forms.**

Characteristics of the control system using the standard forms tuning technique:

- Maximum percentage overshoot: 0.20 %
- Maximum percentage undershoot: 1.65 %
- Settling time: 0.0027 s

6. CONCLUSIONS

- It was possible to suppress completely the higher oscillations in processes through using the PD-PI controller.
- It was possible to overcome the set-point kick problem associated with the standard PID.
- It was possible using the ISE tuning approach presented in the paper to get a step-wise time response which was not possible in the other PID-types.
- Through using the PD-PI controller it was possible reduce the overshoot, undershoot and settling time to zero.
- Tuning the controller using standard forms produced a time response of the closed loop system having more overshoot, undershoot and settling time (0.2 %, 1.65 % and 0.0027 s respectively).

REFERENCES