

# Modeling and Analysis of Queuing Systems in Banks: (A case study of Ghana Commercial Bank Ltd. Kumasi Main Branch)

Wallace Agyei, Christian Asare-Darko, Frank Odilon

**Abstract:** Queues are common sight of many banks in Ghana. The obvious implication of customers waiting in long and winding queues could result to prolonged discomfort and economic cost to them; however increasing the service rate will require additional number of tellers which implies extra cost to management. This study therefore attempts to find the trade-off between minimizing the total economic cost (waiting cost and service cost) and the provision of a satisfactory and reasonably shortest possible time of service to customers, in order to assist management of the bank in deciding the optimal number of tellers needed. Data for this study was collected at the Ghana Commercial Bank Ltd, Kumasi Main Branch for one month through observations, interviews and by administering of questionnaire and was formulated as multi-server single line queuing model. The data was analyzed using TORA optimization Software as well as using descriptive method of analysis. The performance measures of different queuing systems were evaluated and analyzed. The results of the analysis showed using a five teller system was better than a four or a six-teller system in terms of average waiting time and the total economic cost, hence the study recommends that, the management should adopt a five teller model to reduce total economic costs and increase customer satisfaction.

**Keywords:** Queuing theory, Waiting time, Economic cost, Arrival rate, Service rate, Multi-Channel Model, Tellers

## 1. INTRODUCTION

Queues (or waiting lines) are general phenomenon in everyday life. Queues are usually seen at post offices, bus stops, hospitals, bank counters, petrol filling stations etc. Queues are formed when customers (human or object) demanding service have to wait because their number exceeds the number of servers available; or the facility does not work efficiently or takes more than the time prescribed to service a customer [1]. Some customers wait when the total number of customers requiring service exceeds the number of service facilities, some service facilities stand idle when the total number of service facilities exceeds the number of customers requiring service. [2] defines queue as simply a waiting line, while [3] put it in similar way as a waiting line by two important elements: the population source of customer from which they can draw and the service system. The population of customer could be finite or infinite. The banking sector which is the largest and most competitive unit of Ghana's financial sector, acting as a financial intermediary between the surplus and deficit agents of the economy has always been the center of attraction to many customers that want to carry out one transaction or the other through the services provided by these banks. However, the major problem is the inability of the banks to match their service facilities to the needs of customers without much delay. The common experience in Ghana is that most banks do not have the facilities and capacities to service the number of customers without much delay on the part of the customers. The problem in this regard had been that though bank customers for instance, have always been desirous of spending the least possible time in banking transactions, this age-long desire is yet to be met by the banks. Banks on the other hand, want to attract, retain customers and at the same time optimize profit. Profit making in banks is a function of management ability to provide efficient services to customers at little or no time wastage [4]. One major recurring problem in Ghanaian Banks is congestion at the banking halls; this has led to the movement of customers from one bank to the other, where they can obtain banking services without much delay. Despite technological advancement in the banking sector, such as Internet Banking, Mobile or Telephone Banking,

Automated Teller Machine (ATM) etc. by banks in attempt to minimize waiting line problem at over-the counter services have not yielded the much desired result due to frequent breakdown of such computerization and fraudulent activities. Hence, long queue persisted in almost all Ghanaian banks. The danger of keeping customers waiting can cause prolonged discomfort and economic cost to them [5]. The time wasted on the queue would have been judiciously utilized elsewhere (opportunity cost of time spent in queuing). In a queuing system, managers must decide what level of service to offer. A low level of service may be inexpensive, at least in the short run, but may incur high costs of customer dissatisfaction, such as loss of future business. A high level of service will cost more to provide and will result in lower dissatisfaction costs. When considering improvements in services, the management of the bank weighs the cost of providing a given level of service against the potential costs from having customers waiting. The goal of queuing analysis is therefore to strike a balance between minimize the economic cost to the system and provision of satisfactory and reasonable quick service.

## 2. MATERIALS AND METHOD

Data for this study was collected from the banking hall of Ghana Commercial Bank (GCB) Ltd. Kumasi Main Branch. The methods used during the data collection were direct observation and personal interview and questionnaire administering. The data gathered were the daily record of queuing system over a month. The variables measured included arrival and service rate per hour. They were analyzed for simultaneous efficiency in customer satisfaction and economic cost minimization through the use of a multi-channel single-line queuing model, which were compared for a number of queue performances such as; the average number of customers in the queue and in the system, average time each customer spends in the queue and in the system and the probability of the system being idle. The following assumptions were made for modeling the queuing system at Ghana Commercial Bank (GCB) Ltd. Kumasi Main Branch.

**Assumptions of the Model are:**

1. The queue discipline is First-Come, First-Served (FCFS) basis.
2. Arrivals follow a Poisson probability distribution.
3. The service times have an exponential distribution.
4. There is an infinite population from which customers come from.
5. Service time varies from one customer to the next and is independent of another, but their average rate is known.
6. Every customer waits to be served regardless of the length of the queue (i.e. there is no reneging or balking).
7. The service providers are working at their full capacity.
8. The average arrival rate is greater than the average service rate.
9. The waiting space available for customers in the queue is infinite.

**Queuing model notation:**

- $\lambda$  : mean arrival rate
- $\mu$  : mean service rate
- $k$  : number of service channels
- $n$  : number of customers
- $L_s$  : average number of customers in the system (waiting or being served)
- $L_q$  : average number of customers waiting in the queue
- $W_s$  : average time customers spend in the system
- $W_q$  : average time customers wait in the queue
- $\rho$  : system utilization
- $P_0$  : The probability that there are zero customers in the system
- $P_w$  : The probability that a customer has to wait
- $P_n$  : The probability that there are  $n$  customers in the system

**Multiple-Channel Queuing Model with Poisson Arrivals and Exponential Service Times (M/M/k)**

The model adopted in this paper is multiple channel queuing system, in which two or more servers or channels are available to handle arriving customers. Let still assume that customers waiting service form one single line and then proceed to the first available server. For this queuing system, it is assumed that the arrivals follow a Poisson probability distribution with rate  $\lambda$ . Each of these channels has an independent and identical exponential time distribution with mean  $1/\mu$ .

**Equations for Multi-channel queuing Model:**

Utilization factor

$$\rho = \frac{\lambda}{k\mu} \quad (1)$$

The probability that there are zero customers in the system:

$$P_0 = \left[ \sum_{n=0}^{k-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^k \frac{k\mu}{k\mu - \lambda} \right]^{-1} \quad (2)$$

The probability that there are  $n$  number of customers in the system:

$$P_n = \begin{cases} \left(\frac{\rho^n}{n!}\right) P_0, & \text{if } n < k \\ \left(\frac{\rho^n}{k! k^{n-k}}\right) P_0, & \text{if } n \geq k \end{cases} \quad (3)$$

Probability that a customer has to wait:

$$P_w = \frac{1}{k!} \left(\frac{\lambda}{\mu}\right)^k \frac{k\mu}{k\mu - \lambda} P_0 \quad (4)$$

The average number of customers in the system:

$$L_s = \frac{\lambda\mu \left(\frac{\lambda}{\mu}\right)^k}{(k-1)! (k\mu - \lambda)^2} P_0 + \frac{\lambda}{\mu} \quad (5)$$

The average number of customers in the queue:

$$L_q = \frac{\lambda\mu \left(\frac{\lambda}{\mu}\right)^k}{(k-1)! (k\mu - \lambda)^2} P_0 \text{ or } L_q = L_s - \rho \quad (6)$$

The average time a customer spends in the system:

$$W_s = \frac{\mu \left(\frac{\lambda}{\mu}\right)^k}{(k-1)! (k\mu - \lambda)^2} P_0 + \frac{1}{\mu} = \frac{L_s}{\lambda} \quad (7)$$

The average time a customer spends in the queue:

$$W_q = \frac{\mu \left(\frac{\lambda}{\mu}\right)^k}{(k-1)! (k\mu - \lambda)^2} P_0 \text{ or } W_q = W_s - \frac{1}{\mu} = \frac{L_q}{\lambda} \quad (8)$$

**Economic Analysis of Queuing Systems**

Queuing models can be used to determine operating performance of a queuing system. In economic analysis of queuing systems, we seek to use the information provided by the queuing model to develop a cost model for the queuing systems under study. The two basic types of costs associated with queuing systems are the costs involved in operating each service facility like the costs for equipment (including maintenance), materials, labor, etc. these cost increases as the number of service facilities put into operation increases and the opportunity costs associated with causing customers to wait in the system. As the number of service facilities in operation increases, the time the customer has to wait in the system, on the average, decreases, hence the cost associated. The two types of cost are in conflict because an increase in one automatically causes a reduction in the other or vice versa. The total of these two basic types of costs goes to a minimum at some specific number of facilities. This then is the optimum number of service facilities which should be operated by the manager- optimum because it minimizes the total cost of both operating the service facilities and waiting time in the system. The total cost model includes the cost of waiting and the cost of service:

$$TC = C_w L_s + C_s k \quad (9)$$

where:

$C_w$  = the waiting cost per time period for each customer

$L_s$  = average number of customers in the system

$C_s$  = the service cost per time period for each channel

$k$  = the number of channels

$TC$  = the total economic cost per time period

**3. RESULTS AND DISCUSSION**

The multiple channel queuing system at Ghana Commercial Bank Ltd. Kumasi Main Branch with mean arrival rate ( $\lambda$ ) = 75 customers/hr., service rate ( $\mu$ ) = 26 customers/hr. and with number of Tellers ( $k$ ) = 3 has been solved using TORA optimization software to evaluate the measures of

performance of the queuing system at the bank results were obtained as shown in Table 1.

**Table 1. Measures of Performance of Multiple Channels Queuing Model at Ghana Commercial Bank**

Scenario	c	Lambda	Mu	L'da eff	$P_0$	$L_s$	$L_q$	$W_s$	$W_q$
1	3	75.00000	26.00000	75.00000	0.00892	26.08975	23.20514	0.34786	0.30940
2	4	75.00000	26.00000	75.00000	0.04466	4.07957	1.19496	0.05439	0.01593
3	5	75.00000	26.00000	75.00000	0.05301	3.16899	0.28437	0.04225	0.00379
4	6	75.00000	26.00000	75.00000	0.05514	2.96330	0.07868	0.03951	0.00105
5	7	75.00000	26.00000	75.00000	0.05569	2.90651	0.02189	0.03875	0.00029

**Table 2. Summary of Analysis of Multiple Channels Queuing Model at Ghana Commercial Bank**

Performance Measure	3 Tellers	4 Tellers	5 Tellers	6 Tellers	7 Tellers
Arrival rate ( $\lambda$ )	75	75	75	75	75
Service rate ( $\mu$ )	26	26	26	26	26
System Utilization ( $\rho$ )	96.2%	72.1%	57.7%	48.1%	41.2%
$L_s$	26.0898	4.0796	3.1690	2.9633	2.9065
$L_q$	23.2051	1.1950	0.2844	0.0787	0.0219
$W_s$ (hours)	0.3479	0.0544	0.0423	0.0395	0.0386
$W_q$ (hours)	0.3094	0.0159	0.0038	0.0011	0.0003
$P_0$	0.89%	4.47%	5.30%	5.51%	5.57%
$P_w$	92.80%	46.21%	20.85%	8.49%	3.12%
Total Economic Cost of System/hr	GH¢ 1010.90	GH¢ 715.80	GH¢ 631.69	GH¢ 704.63	GH¢ 779.07

From the queue performance measures, increasing the number of teller points to 4 from the table 1 show a decrease in the waiting time in the queue and system would reduce to 0.0159 hours and 0.0544 hours respectively as against the present situation where each customer has to in the queue and system for 0.3094 and 0.3479 hours respectively. As a result each teller will be busy 72.1% that is 27.9% of the time it is idle. Furthermore, the total economic cost will also decrease from GH¢1010.90 with 3 tellers to GH¢715.80 with 4 tellers. Introduction of additional teller point at the Bank to 5 will also decrease the waiting time in the queue and system to 0.0038 and 0.0423 hours respectively which indicates little or no queue would be experienced as a customer would only have wait for 12.96 seconds in a queue and 2.54 minutes in the system with each teller only busy for 57.7%. Likewise the total economic cost incurred by the system decreases to GH¢631.69 which is economically optimal. Moreover, multiple channels with 6 and 7 teller points at the Bank will also reduce the time customers have to wait in the queue and the system significantly, however this will increase the total economic cost to GH¢704.63 and GH¢779.07 respectively with each teller idle for 51.9% and 58.8% meaning the tellers will be under-utilized. From the analysis, it can observe that the number of tellers necessary to serve customers in the case study of Ghana Commercial Bank, Kumasi Main Branch is 5 teller points. This was proved in table 1 and 2 above. This is the appropriate number of tellers that can serve the customers as and at when due without waiting for long before customers are been served at the actual time necessary for the service.

#### 4. CONCLUSION

In this study, we presented queuing system at the banking hall of Ghana Commercial Bank, Kumasi Main Branch as Multiple-Channel queuing model with Poisson arrivals and Exponential services times to reflect the queue characteristics at the bank and the economic analysis in terms of total expected cost of the system was determined with the aim of finding the optimal number of teller points that will help ease congestion at the banking hall. The analysis of the queuing systems shows increasing the numbers of teller points up to 5 will reduce the waiting time by customers in the queue and system by 98.78% and 87.85% respectively. The total economic cost incurred by system would also reduce to GH¢631.69 as against the present GH¢1010.90. This will also increase the efficiency of the establishment and customer satisfaction. Based on the summary of analysis of different queuing systems, we recommend the management should adopt a five teller model to reduce economic cost.

#### REFERENCES

- [1] Sharma, J. K. Operations Research: Theory and Application, 3rd Ed. (Macmillan Ltd., India 2007)
- [2] Taha, A. H., Operations Research: An Introduction, 7th Ed. (Prentice Hall, India, 2003)
- [3] Hiray, J, Waiting Lines and Queuing System, Article of Business Management, 2008
- [4] Agbadudu, A.B. Elementary Operation Research.

Vol. I, 1996 (Benin City, A.B. Mudiaga Limited).

- [5] Elegalam "Customer Retention Versus Cost Reduction technique" A Paper Presented at the Bankers Forum held at Lagos, pg.9-10.1978.